College of Engineering &Technology

University of Sargodha

Department of Electrical Engineering Technology

ET-314

Telecommunication Technology

Lecture 12

Digital Modulation (Part 2)

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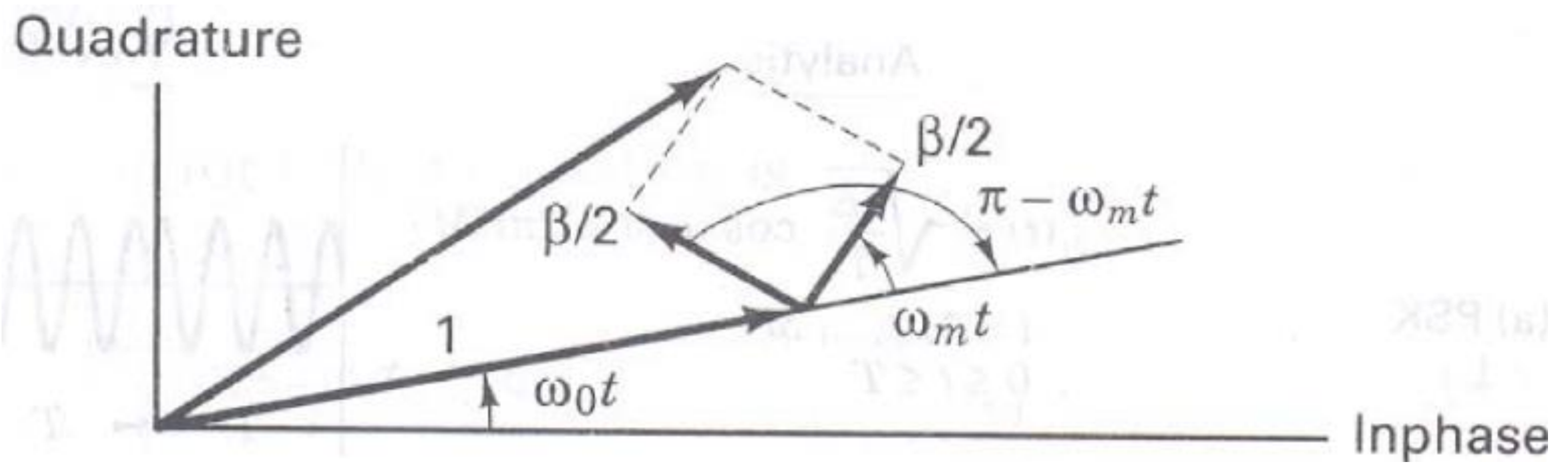
Phasor Representation of a Sinusoid

- $e^{j\omega_0 t}$ is now perturbed by two sideband terms
- $e^{j\omega_m t}/2$ rotating counterclockwise
- $e^{-j\omega_m t}/2$ rotating clockwise
- The sideband phasors are rotating at much slower speed than the carrier-wave phasor.
- The net result of the composite signal is that the rotating carrier-wave phasor now appears to be growing longer and shorter pursuant to the dictates of the sidebands
- Its frequency remains constant

Phasor Representation of a Sinusoid

- Consider the FM in Phasor form

$$s(t) = \mathrm{Re}\left\{ e^{j\omega_0 t}\left(1 - \frac{\beta}{2}e^{-j\omega_m t} + \frac{\beta}{2}e^{j\omega_m t}\right)\right\}$$



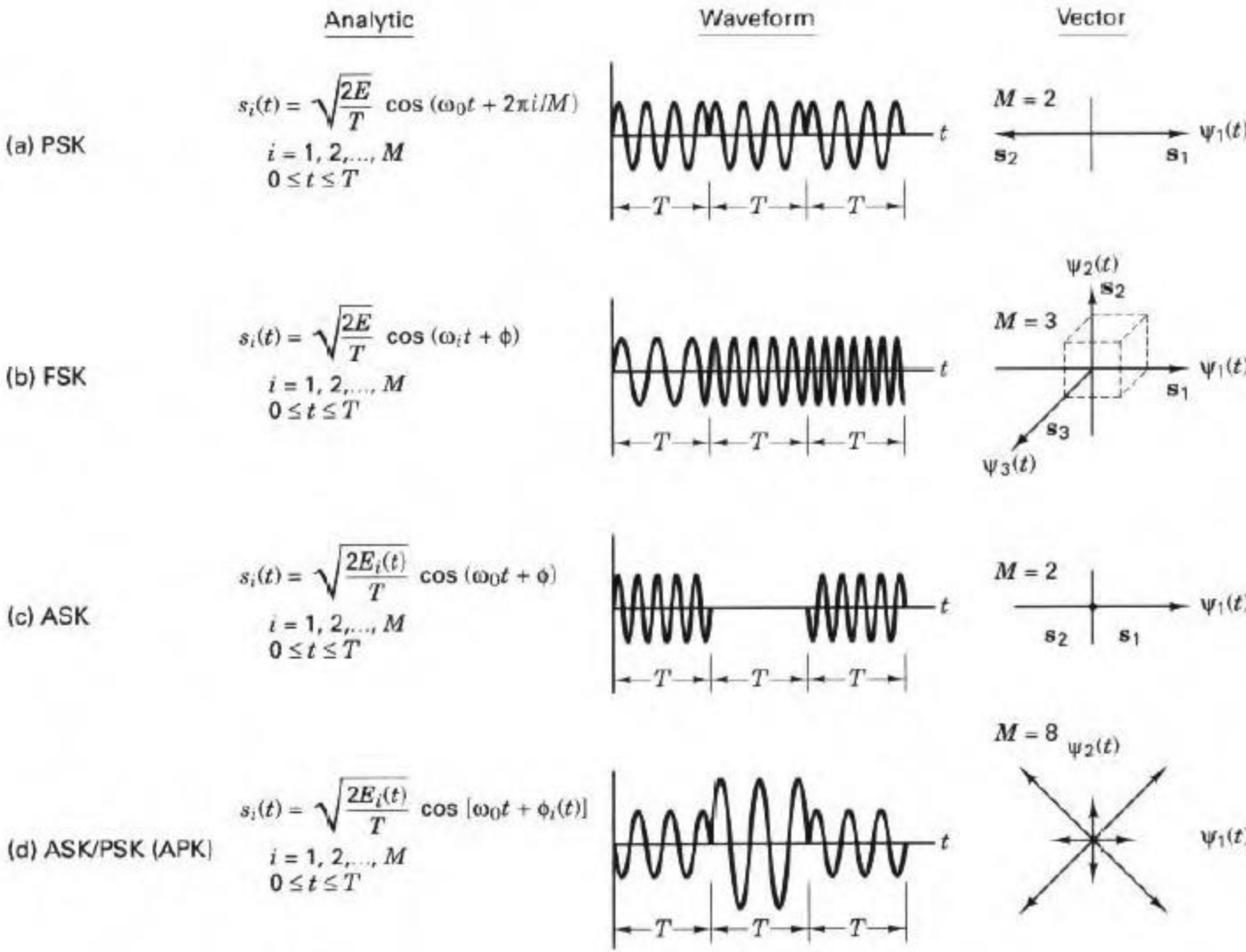


Figure 4.5 Digital modulations. (a) PSK. (b) FSK. (c) ASK. (d) ASK/PSK (APK).

Waveform Amplitude Coefficient

Why $s(t) = \sqrt{\frac{2E}{T}} \cos \omega t$???

Waveform Amplitude Coefficient

- Energy of the received signal is the key parameter in determining the Error Performance of the detection Process

- It facilitates solving directly for the probability of Error $\mathbf{P_E}$ as a function of Signal Energy

Demodulation and Detection

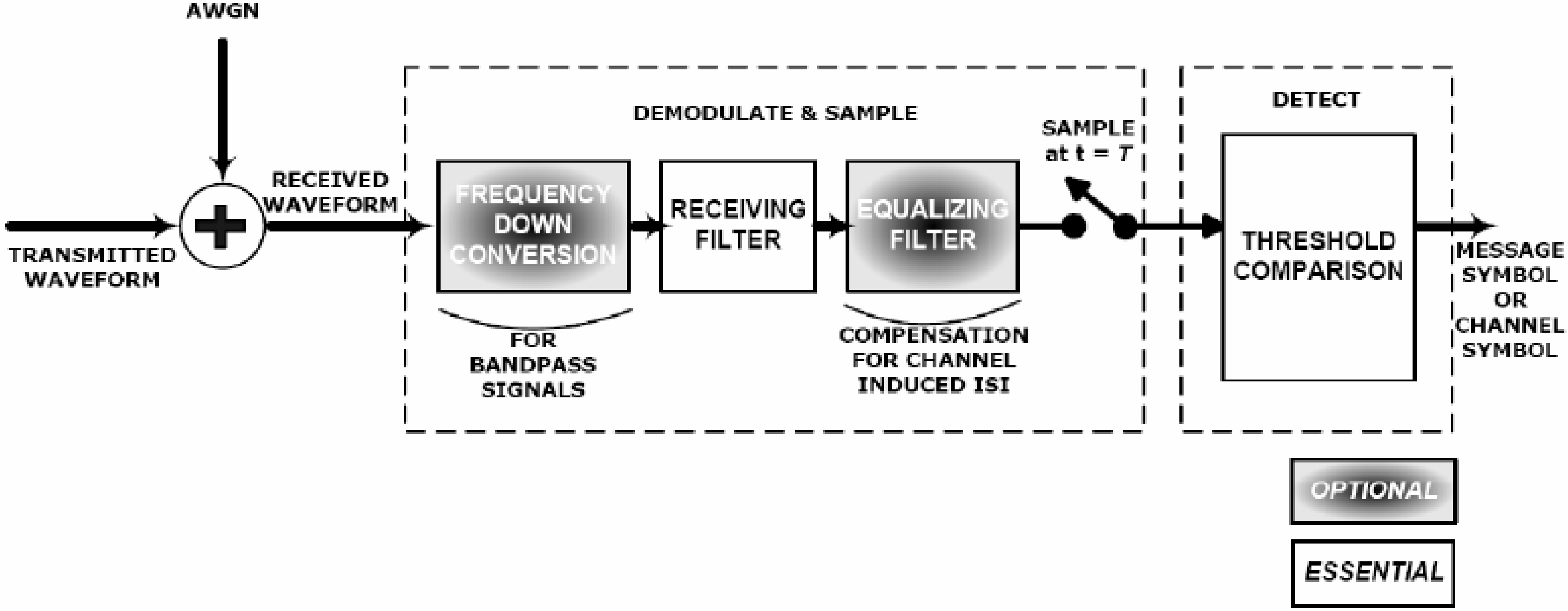
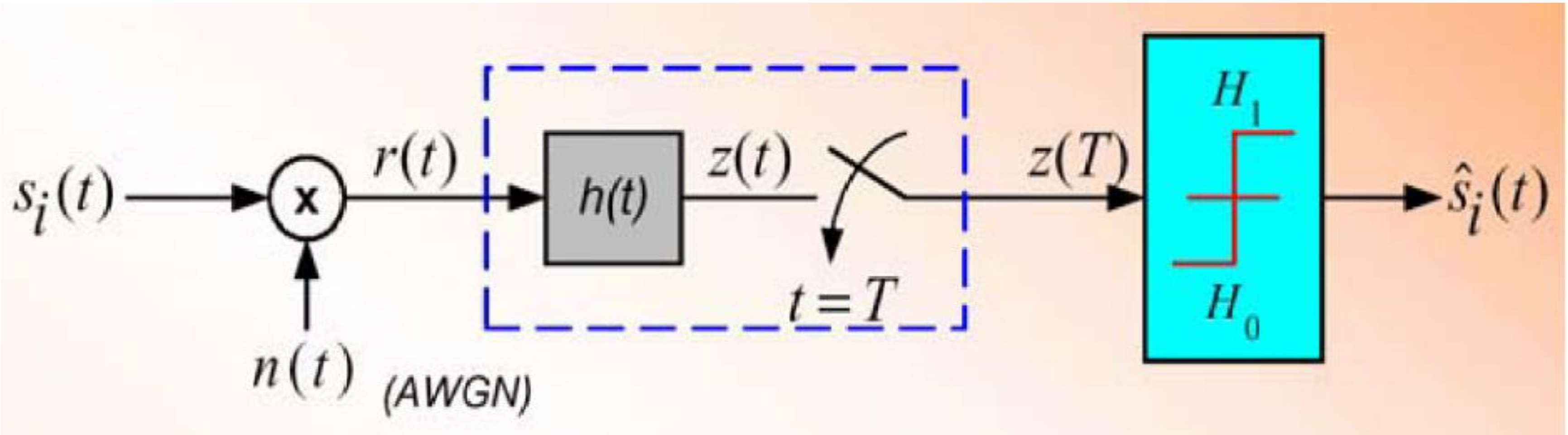


Figure 3.1: Two basic steps in the demodulation/detection of digital signals

The digital receiver performs two basic functions:

- Demodulation, to recover a waveform to be sampled at $t = nT$.
- Detection, decision-making process of selecting possible digital symbol

Detection of Binary Signal in Gaussian Noise



The Matched Filter

- "A linear filter designed to provide the maximum SNR at its output for a given transmitted symbol waveform"

- A filter whose impulse response $h(t)=s(T-t)$, where $s(t)$ is is assumed to be confined to the time interval $0 \leq t \geq T$, is called the Matched Filter to the signal $s(t)$

The Matched Filter

- The impulse response of a filter producing maximum output SNR is the mirror image of message signal *s(t)*, delayed by symbol time duration T

- The filter designed is called as a MATCHED FILTER

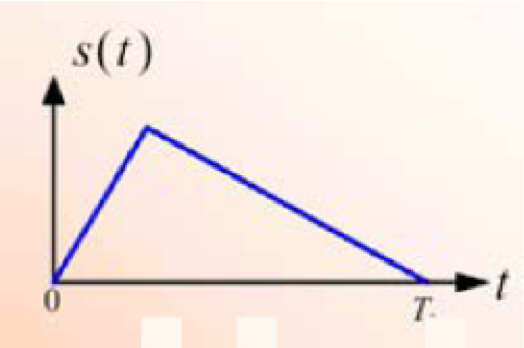
$$h(t) = \begin{cases} kS(T-t) & 0 \le t \le T \\ 0 & else\ where \end{cases}$$

The Matched Filter

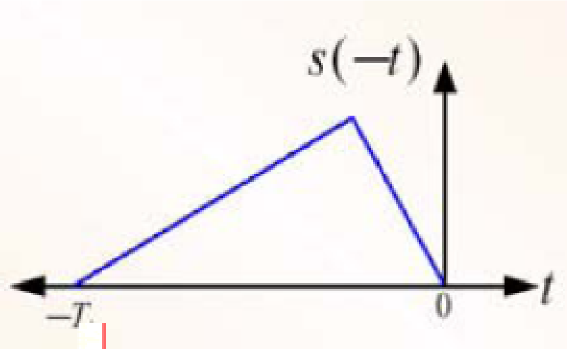
- A filter that is matched to the waveform *s(t)*, has an impulse response

$$h(t) = \begin{cases} kS(T-t) & 0 \le t \le T \\ 0 & else \;\; where \end{cases}$$

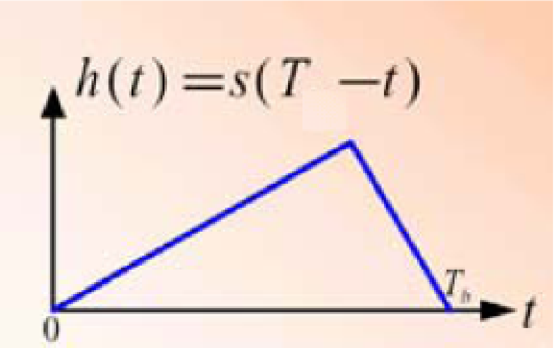
  - *h(t)* is a delayed version of the mirror image (rotated on the *t = 0* axis) of the original signal waveform



Signal Waveform



Mirror image of signal waveform



Impulse response of matched filter

Properties of a Matched Filter

- The performance of all Communication Systems depends upon SNR
- "If a signal *s(t)* is corrupted by AWGN, the filter with an impulse response matched to *s(t)* maximizes the output SNR"
- Matched filter gives the output better than any other filter, as it gives maximum correlation or maximum SNR at the output
- Thus, it is optimal

How Does a Matched Filter Provide Maximum SNR???

Proof

$$\max (S/N)_T = 2Є/N_0$$

The Matched Filter

- Result
  - The maximum output SNR depends on the input signal energy and the power spectral density of the noise
  - And not on the particular shape or detailed characteristics of the waveform used
  - Another important property of a matched filter

Correlation Realization of a Matched Filter

- The impulse response of the filter is the delayed version of the mirror image of the signal waveform

- If the signal waveform is *s(t)*, its mirror image is *s(-t)*, and the mirror image delayed by T seconds is *s(T-t)*

- The output *z(t)* of a causal filter can be described in the time domain as the convolution of the received input waveform *r(t)* with the impulse response of the filter

$$z(t) = r(t) * h(t) = \int_0^t r(\tau)h(t-\tau)\,d\tau$$

Correlation Realization of a Matched Filter

- We know that $$h(t) = \begin{cases} ks(T-t) & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

- By substituting h(t) in h(t-τ) and setting k equal to unity

$$z(t) = \int_0^t r(\tau)s[T-(t-\tau)]\,d\tau$$

$$= \int_0^t r(\tau)s(T-t+\tau)\,d\tau$$

- When t = T

$$z(T) = \int_0^T r(\tau)s(\tau)\,d\tau$$

Correlation Realization of a Matched Filter

$$z(T) = \int_{0}^{T} r(\tau)s(\tau)\, d\tau$$

The product integration of the received signal *r(t)* with a replica of the transmitted waveform *s(t)* over one symbol interval is known as the ***correlation*** of *r(t)* with *s(t)*

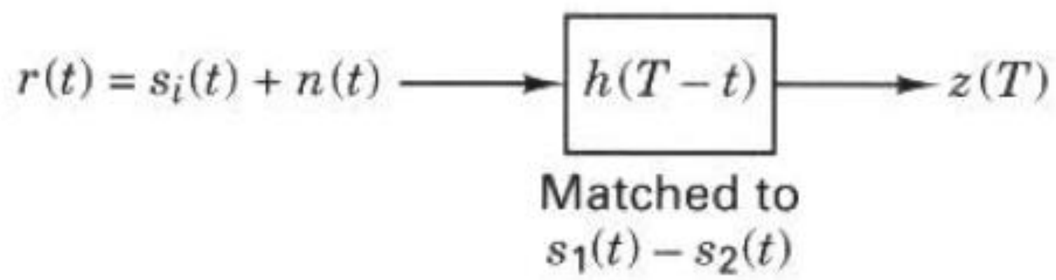
Correlation Realization of a Matched Filter

- Consider that a received signal $r(t)$ is correlated with each prototype signal $s_i(t)$ ($i = 1,\ldots\ldots,M$), using a bank of M correlators

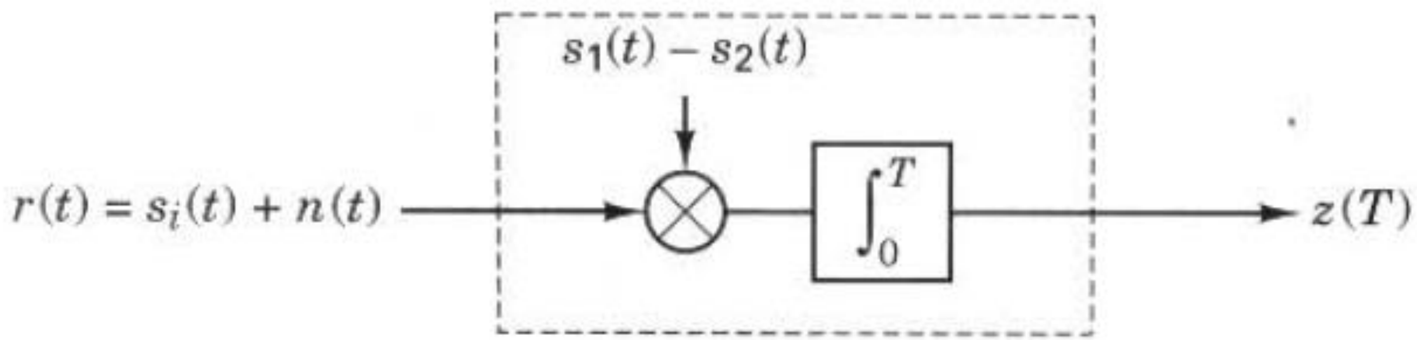
- The signal $s_i(t)$ whose product integration or correlation with $r(t)$ yields the maximum output $z_i(T)$ is the signal that matches $r(t)$ better than all the other $s_j(t)$, $j \neq i$

- We will use this correlation characteristic for the optimum detection of signals

- The function of the Correlator and a Matched Filter is the same
- The equivalence of matched filter and Correlator is shown as under



(a)



(b)

- Thus, the Best Possible Demodulator is

  - A Correlator where we correlate a signal with its own signal shape

    OR

  - A Matched Filter whose impulse response is a flipped and shifted form of the original signal

The Basic SNR Parameter for Digital Communication Systems

Read Page# 117 – 119

(Home Assignment)

Demodulation and Detection

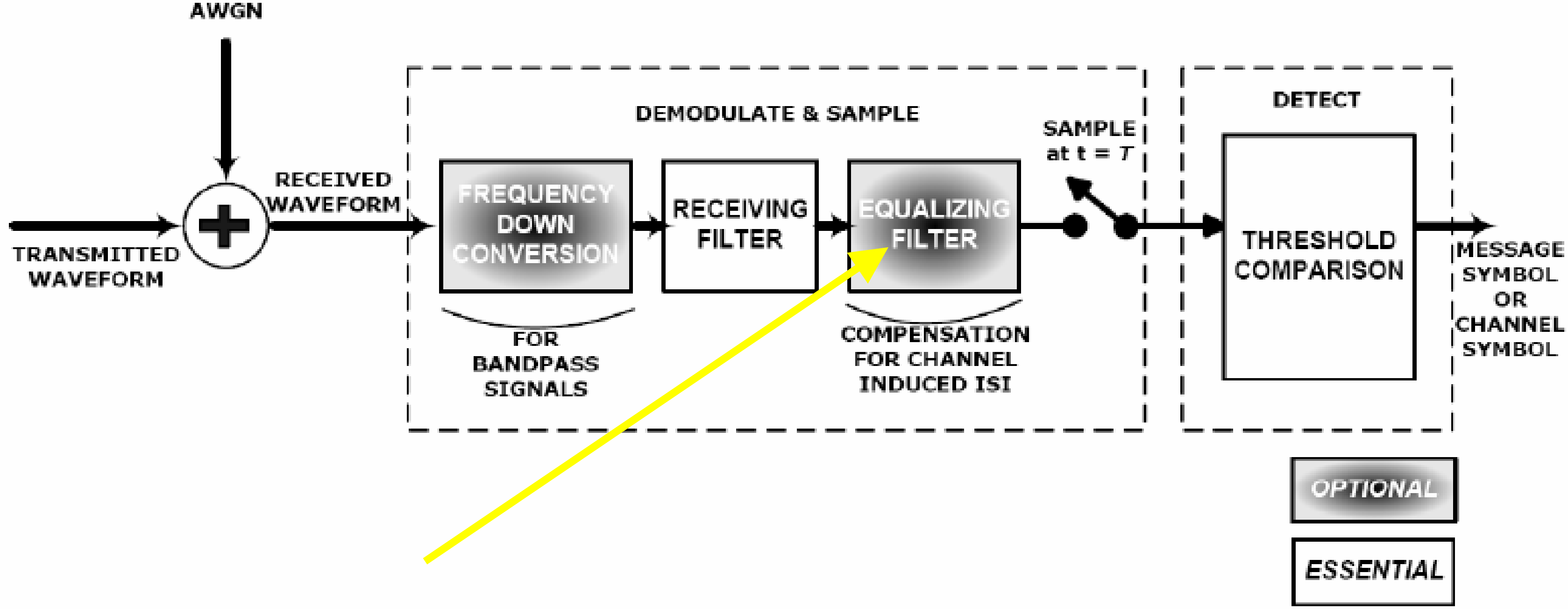


Figure 3.1: Two basic steps in the demodulation/detection of digital signals

Equalization

Channel Characterization

- Many communication channels (e.g. telephone, wireless) can be characterized as band-limited linear filters with an impulse response $h_c(t)$ and a frequency response

$$H_c(f) = |H_c(f)| e^{j\theta_c(f)}$$

Where,

- $|H_c(f)|$ is the channel's amplitude response
- $\theta_c(f)$ is the channel's phase response

Channel Characterization

- In order to have ideal (distortionless) transmission characteristics over a channel, within a signal bandwidth W:
  - $|H_c(f)|$ must be constant
  - $\theta_c(f)$ must be a linear function of frequency

or

  - Delay must be constant for all spectral components of the signal

Channel Characterization

- If $|H_c(f)|$ is not constant within W
  - **Amplitude Distortion**

- If $\theta_c(f)$ is not a linear function of frequency within W
  - **Phase distortion**

- For many channels that exhibit distortion of this type, such as fading channels, amplitude and phase distortion occur together

- For a transmitted sequence of pulses, such distortion causes **ISI**
  - Any one pulse in the received demodulated sequence is not well defined

Equalization

- ISI is one of the major obstacles to reliable high speed data transmission over bandlimited channels

- **Equalization** refers to any signal processing or filtering technique that is designed to eliminate or reduce ISI

Equalization

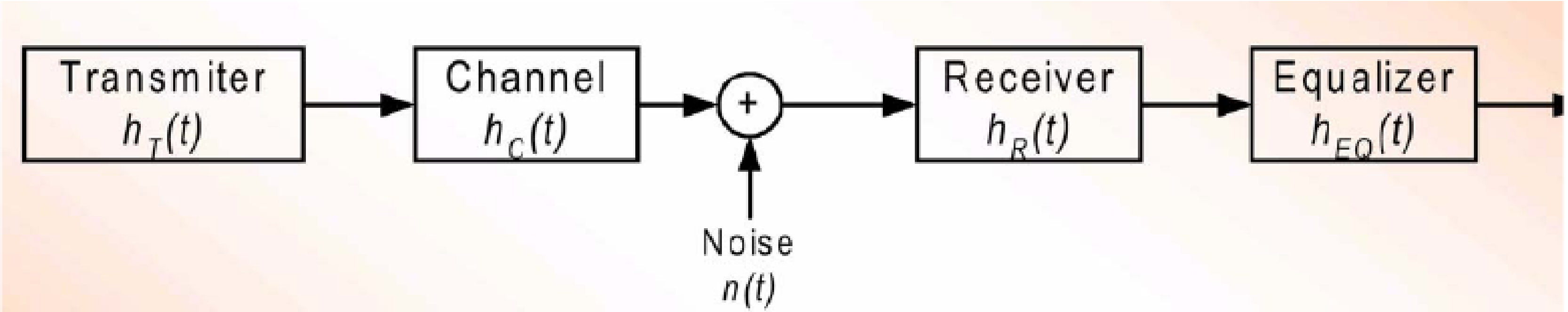
- Nyquist filtering and pulse shaping schemes assumes that the channel is precisely known and its characteristics do not change with time

- However, in practice we encounter channels whose frequency response are either unknown or change with time
  - For example, each time we dial a telephone number, the communication channel will be different because the communication route will be different
  - However, when we make a connection, the channel becomes time-invariant
  - The characteristics of such channels are not known a priori

- Examples of time-varying channels are radio channels
  - These channels are characterized by time-varying frequency response characteristics

Equalization

- To compensate for channel induced ISI we use a process known as ***Equalization:*** a technique of correcting the frequency response of the channel
- The filter used to perform such a process is called an ***equalizer***



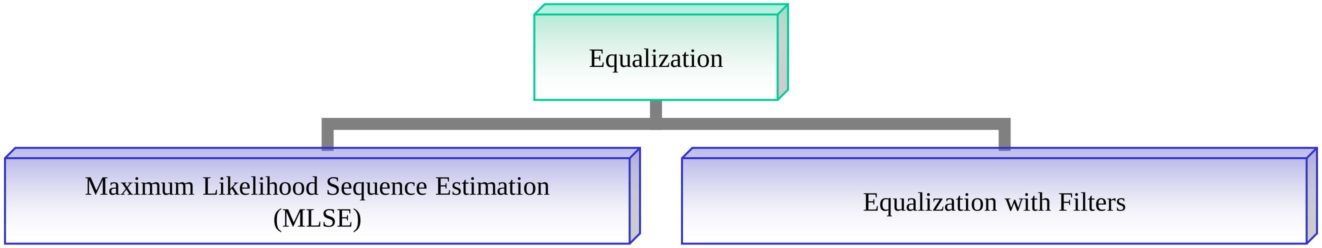
- Since $H_R(f)$ is matched to $H_T(f)$, we usually worry about $H_C(f)$
- The goal is to pick the frequency response $H_{EQ}(f)$ of the ***equalizer*** such that

$$H_c(f)H_{EQ}(f) = 1 \quad \Rightarrow \quad H_{EQ}(f) = \frac{1}{H_C(f)} e^{-j\theta_c(f)}$$

Equalization

- Maximum-likelihood sequence estimation (MLSE)
- Equalization with filters
  - Transversal or decision feedback
  - Preset or Adaptive
  - Symbol spaced or fractionally spaced

Equalization



Maximum Likelihood Sequence Estimation (MLSE)

- **Goal**
  - *To enable the detector to make good estimates from the demodulated distorted pulse sequence*

- Involves making measurements of $h_c(t)$ and then providing a means for adjusting the receiver to the transmission environment

- The distorted samples are not reshaped or directly compensated in any way

- Instead, the mitigating technique for MLSE is to adjust itself in such a way that it can better deal with distorted samples
  - Viterbi Equalization (Section 15.7.1)

Equalization With Filtering

- It uses filters to compensate the distorted pulses

- It can be partitioned into two categories based on:
  - Whether they are linear devices that contain only feedforward elements
    - » **Transversal Equalizers (Preset)**
  - Whether they are nonlinear devices that contain both feedforward and feedback elements
    - » **Decision Feedback Equalizers (Adaptive)**

Transverse Equalizers
or
Preset Equalizer

Linear Transversal Equalizer

- This is simply a linear filter with adjustable parameters
- The parameters are adjusted on the basis of the measurement of the channel characteristics
- A common choice for implementation is the ***transversal filter*** (Tap Delay Line) or the FIR filter with adjustable tap coefficient

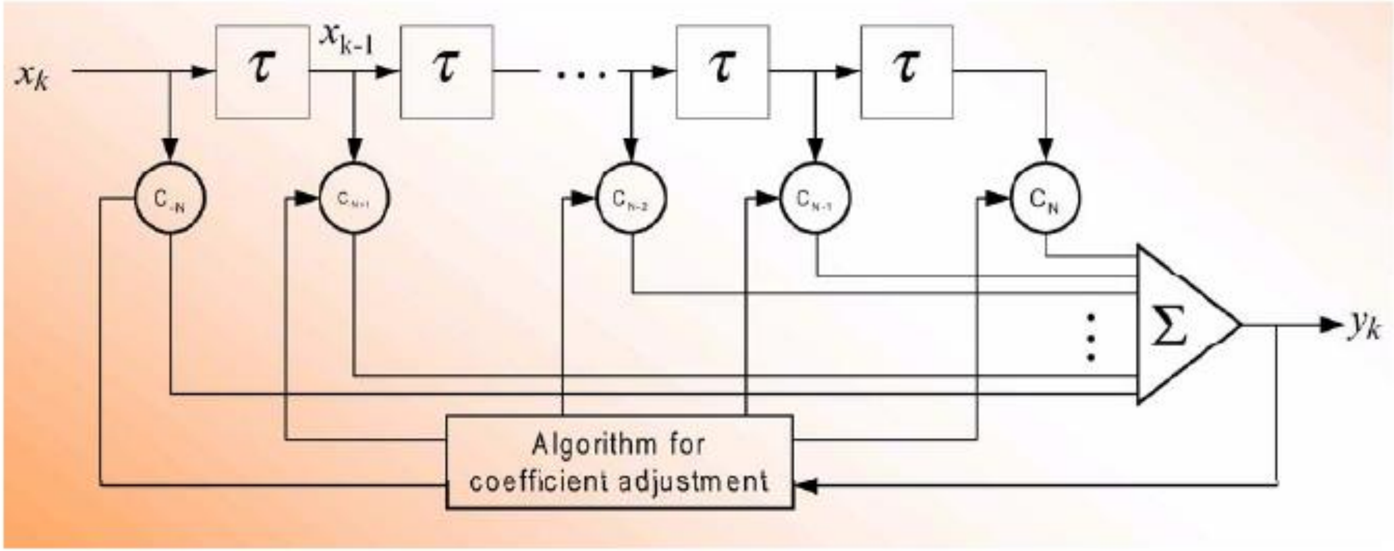


Fig. 3.26

- **Total number of taps = 2N+1**

In Figure 3.26, τ is chosen as high as T

Linear Transversal Equalizer

- It is really not necessary to eliminate or minimize ISI with neighboring pulses for all t

- All that is needed is to eliminate or minimize interference with neighboring pulses at their respective sampling instants only
    - Because the decision is based only on sampled values

Linear Transversal Equalizer

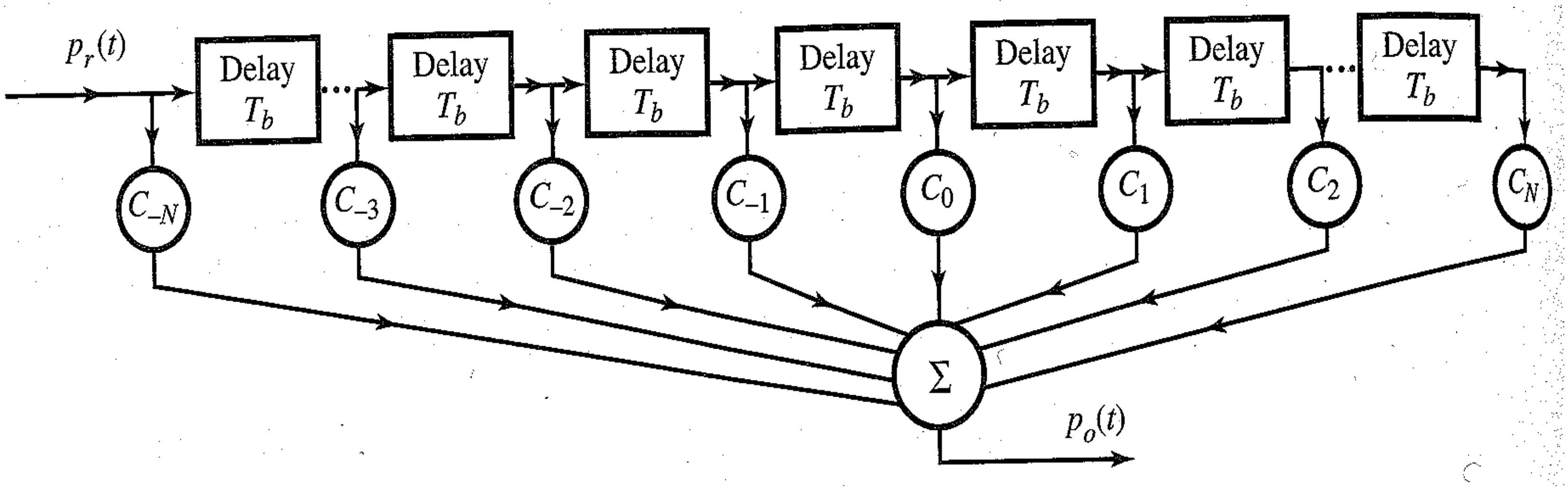
- A transversal-filter equalizer forces the equalizer output to have zero values at the sampling (decision-making) instants

- In other words, the equalizer output pulses should satisfy the Nyquist criterion or the controlled ISI criterion

- The time delay T between successive taps is chosen to be Tb (the interval between pulses)

Linear Transversal Equalizer

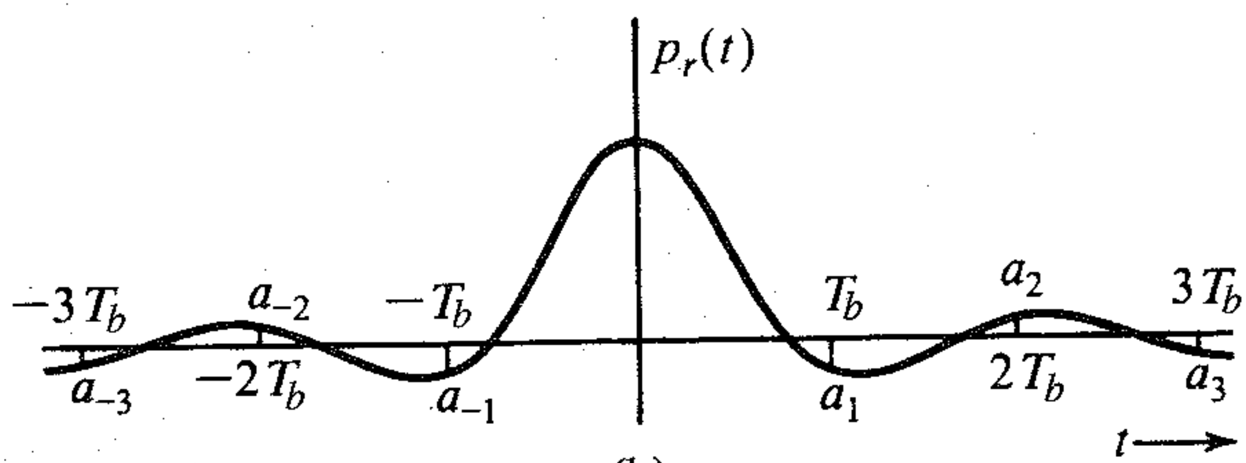
- To begin with, set the tap gains $c_o=1$ and $c_k=0$ for all other values of k in the transversal filter
- Thus the output of the filter will be the same as the input delayed by NT_b



Linear Transversal Equalizer

- For a single pulse $p_r(t)$ at the input of the transversal filter with the previously mentioned tap setting, the filter output will be exactly $p_r(t - NT_b)$ i.e.
    - $P_r(t)$ delayed by NT_b

- But this delay is not relevant to our discussion

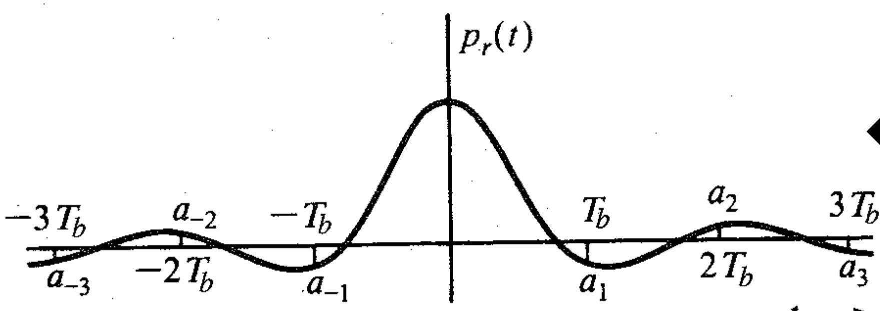


(b)

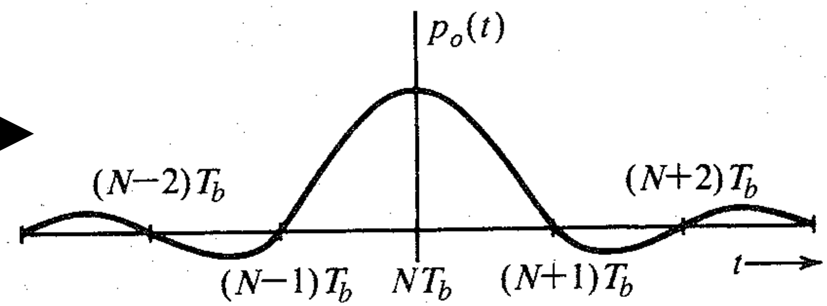
Linear Transversal Equalizer

- We require the output pulse $p_0(t)$ satisfy the Nyquist criterion or the controlled ISI criterion
- For the Nyquist criterion, the output pulse $p_0(t)$ must have zero values at all the multiples of T_b

Linear Transversal Equalizer



(b)



(c)

Linear Transversal Equalizer

- By adjusting the gains (c_k's), we generate additional shifted pulses of proper amplitudes that will force the resulting output pulse to have desired values at $t = 0$, Tb, $2Tb$,…..

- The output $p_0(t)$ is the sum of pulses of the form $c_k p_r(t - kT_b)$, ignoring the delay of NT_b

- Thus,

$$p_o(t) = \sum_{n=-N}^{N} c_n p_r(t - nT_b)$$

The samples of $p_o(t)$ at $t = kT_b$ are

$$p_o(kT_b) = \sum_{n=-N}^{N} c_n p_r[(k - n)T_b] \qquad k = 0, \pm 1, \pm 2, \pm 3, \ldots$$

Using a more convenient notation $p_r[k]$ to denote $p_r(kT_b)$ and $p_o[k]$ to denote $p_o(kT_b)$,

$$p_o[k] = \sum_{n=-N}^{N} c_n p_r[k - n] \qquad k = 0, \pm 1, \pm 2, \pm 3, \ldots$$

The Nyquist criterion requires the samples $p_o[k] = 0$ for $k \neq 0$, and $p_o[k] = 1$ for $k = 0$. Substituting these values into Eq. (7.43b), we obtain a set of infinite simultaneous equations in terms of $2N + 1$ variables. Clearly, it is not possible to solve this set of equations. However, if we specify the values of $p_o[k]$ only at $2N + 1$ points as

$$p_o[k] = \begin{cases} 1 & k = 0 \\ 0 & k = \pm 1, \pm 2, \ldots, \pm N \end{cases} \tag{7.44}$$

then a unique solution exists. This assures that a pulse will have zero interference at the sampling instants of N preceding and N succeeding pulses. Because the pulse amplitude decays rapidly, interference beyond the Nth pulse is not significant for $N > 2$, in general. Substitution of the condition (7.44) into Eq. (7.43b) yields a set of $2N + 1$ simultaneous equations in $2N + 1$ variables:

$$\begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 0 \\ 1 \\ 0 \\ \cdots \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} p_r[0] & p_r[-1] & \cdots & p_r[-2N] \\ p_r[1] & p_r[0] & \cdots & p_r[-2N+1] \\ \cdots & \cdots & \cdots & \cdots \\ & & & \\ p_r[N-1] & p_r[N-2] & \cdots & p_r[-N-1] \\ p_r[N] & p_r[N-1] & \cdots & p_r[-N] \\ p_r[N+1] & p_r[N] & \cdots & p_r[-N+1] \\ \cdots & \cdots & \cdots & \cdots \\ p_r[2N-1] & p_r[2N-2] & \cdots & p_r[1] \\ p_r[2N] & p_r[2N-1] & \cdots & p_r[0] \end{bmatrix} \begin{bmatrix} c_{-N} \\ c_{-N+1} \\ \cdots \\ \\ c_{-1} \\ c_0 \\ c_1 \\ \cdots \\ c_{N-1} \\ c_N \end{bmatrix}$$

The tap-gain c_k's can be obtained by solving this set of equations.

Decision Feedback Equalizers
or
(Adaptive Equalizer)

Automatic And Adaptive Equalization

- The setting of the tap gains of an equalizer can be done automatically by
  - Using an iterative technique to obtain optimum tap gains

- The tap gain are adjusted continuously during transmission