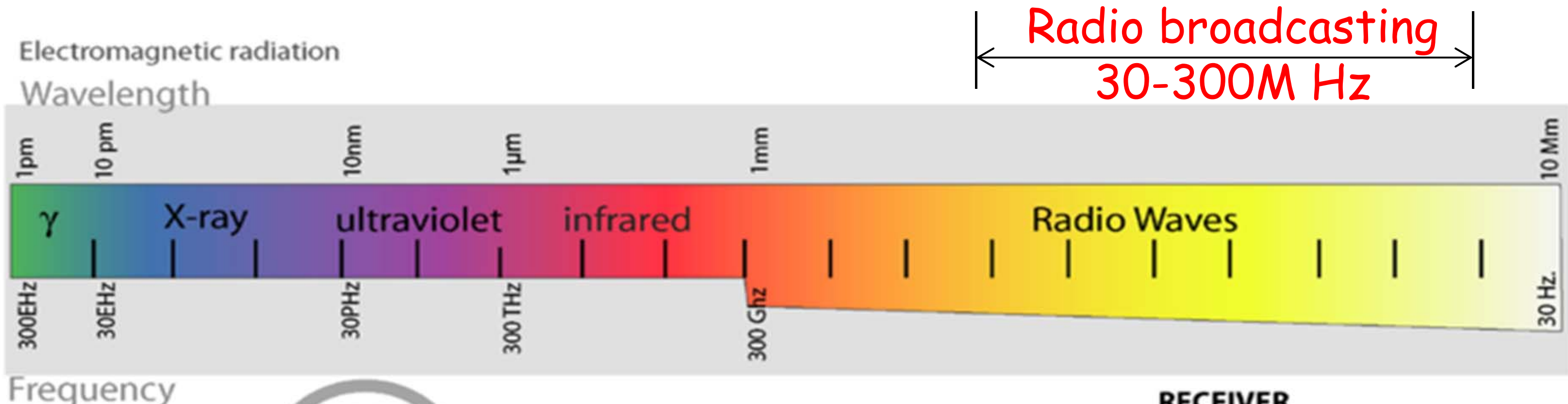
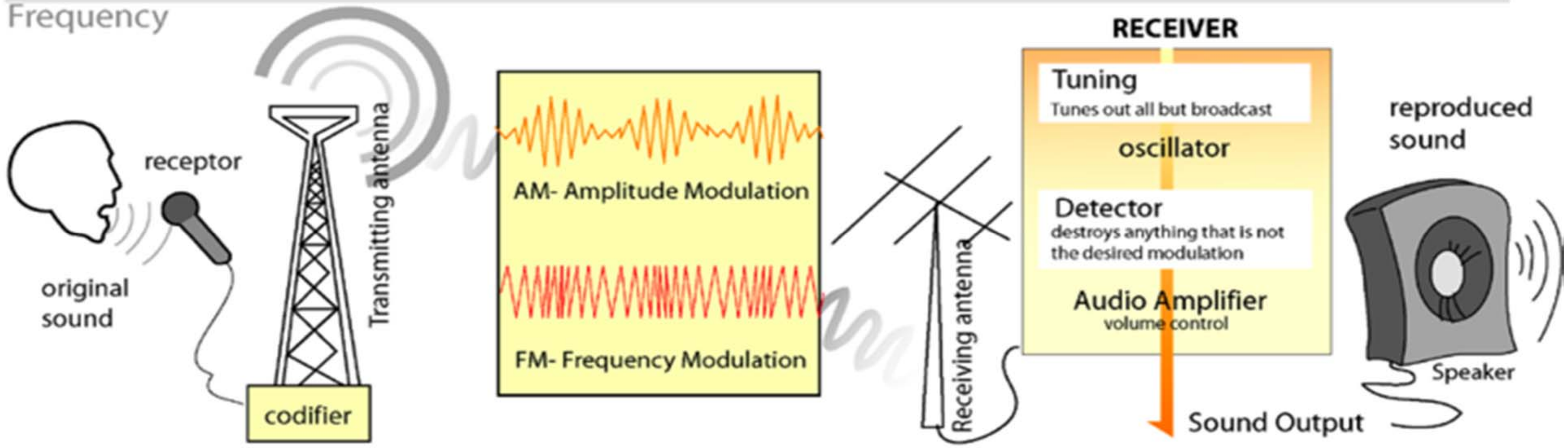
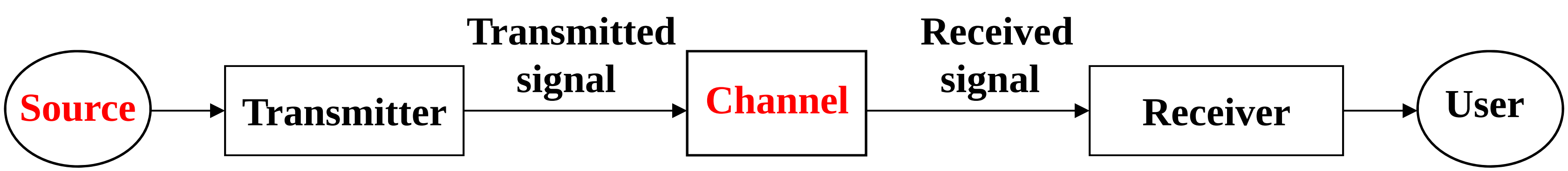
# Analog Communications

- Amplitude Modulation (AM)
- Frequency Modulation (FM)

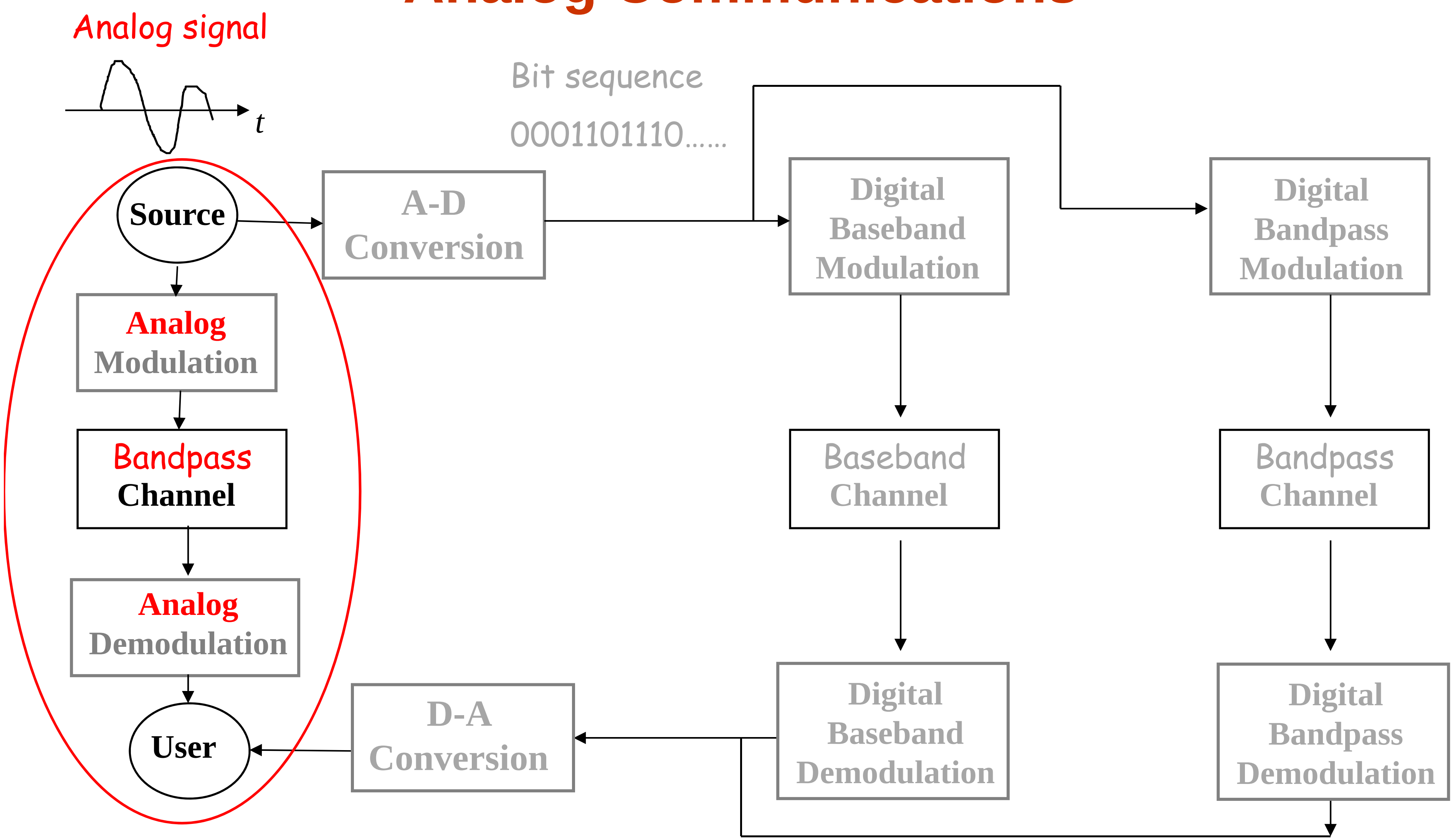


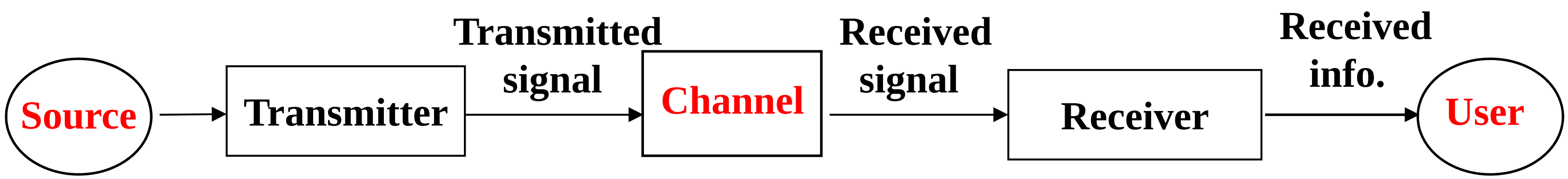




- Analog baseband signal
- Bandpass channel

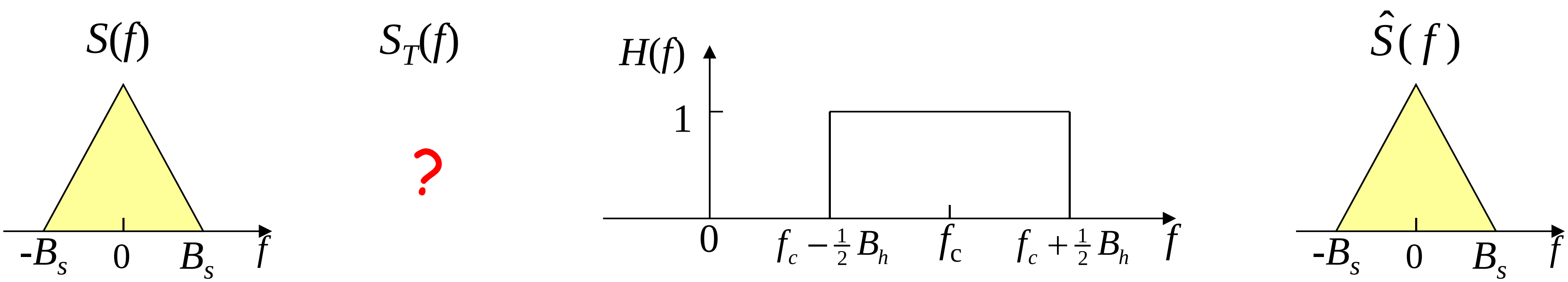
# Analog Communications



# Analog Communications




- Analog baseband signal
- Bandpass channel
- Ignore noise



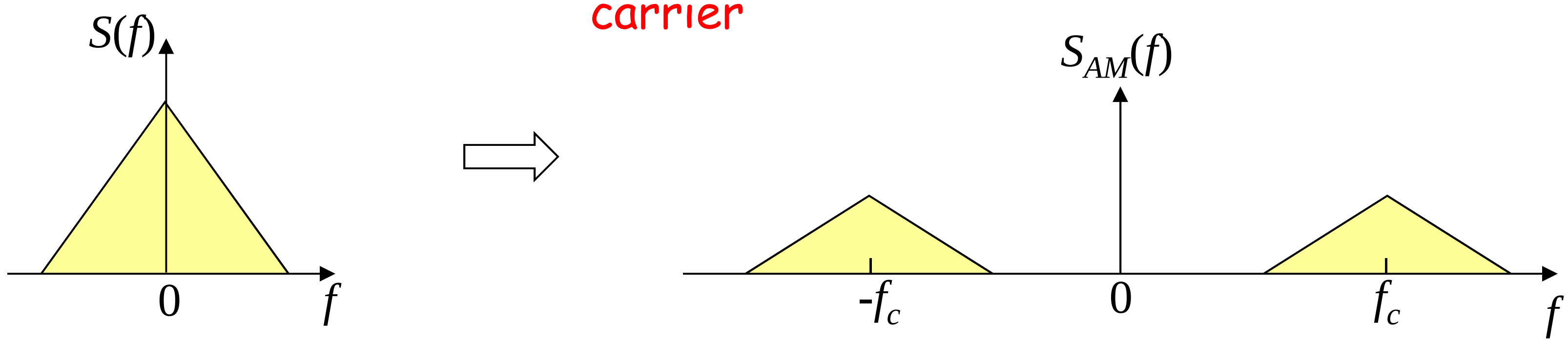

- For reliable communications, i.e., $\hat{S}(f) = S(f)$, all frequency components of the transmitted signal should pass through the channel, which requires:
  - Frequency components of transmitted signal should be centered at $f_c$;
  - The channel bandwidth $B_h$ should be no smaller than the bandwidth of transmitted signal $B_m$.

# Analog Modulation

Modulation property of Fourier Transform:

$$s(t)\cos(2\pi f_c t) \Leftrightarrow \tfrac{1}{2}[S(f-f_c)+S(f+f_c)]$$

carrier




- Amplitude Modulation (AM) $s_{AM}(t) = As(t)\cos(2\pi f_c t)$

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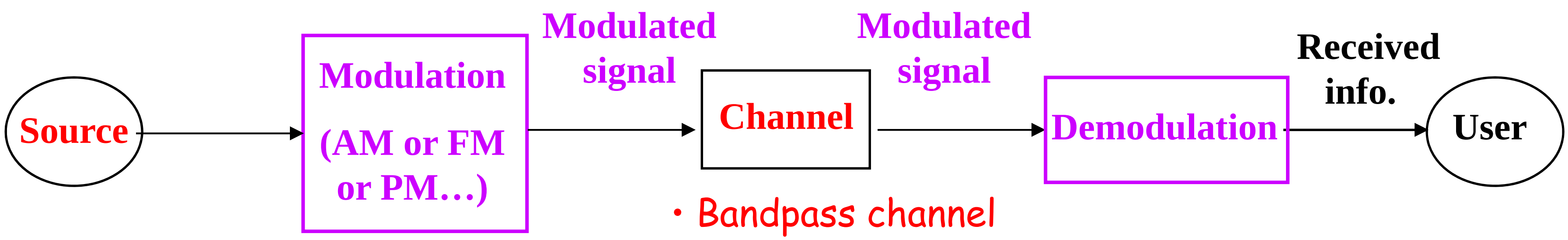
- Phase Modulation (PM) $s_{PM}(t) = A\cos(2\pi f_c t + \alpha s(t))$
- Frequency Modulation (FM) $s_{FM}(t) = A\cos(2\pi(f_c t + k\int_{-\infty}^{t} s(\tau)d\tau))$

# Analog Modulation




- Analog baseband signal
- Bandpass channel
- Ignore noise

- Bandwidth efficiency is an important performance metric, which is defined as:

$$\gamma \triangleq \frac{\text{Information Signal Bandwidth } B_s}{\text{Required Channel Bandwidth } B_h} = \frac{\text{Information Signal Bandwidth } B_s}{\text{Modulated Signal Bandwidth } B_m}$$

  - Required channel bandwidth $B_h$ = Modulated signal bandwidth $B_m$
  - A higher $\gamma$ indicates a better spectral utilization.

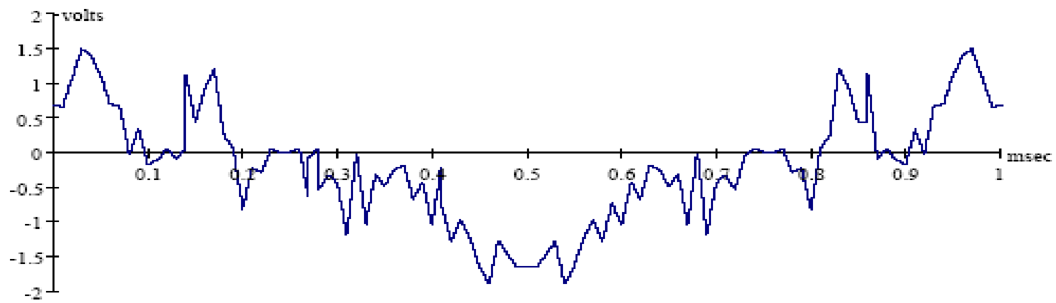
# Lecture 3. Analog Communications
# Part I. Amplitude Modulation (AM)

- AM-DSB-SC
- AM-DSB-C
- AM-SSB
- AM-VSB

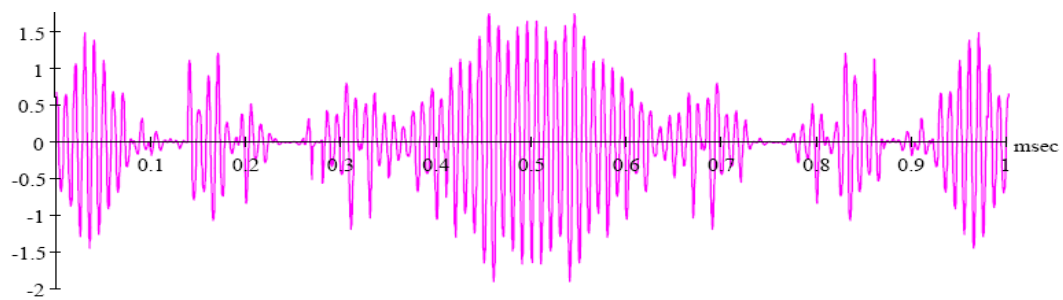
# AM-DSB-SC -- Modulation

- Time Domain: $s_{AM-DSB-SC}(t) = As(t)\cos(2\pi f_c t)$

$s(t)$

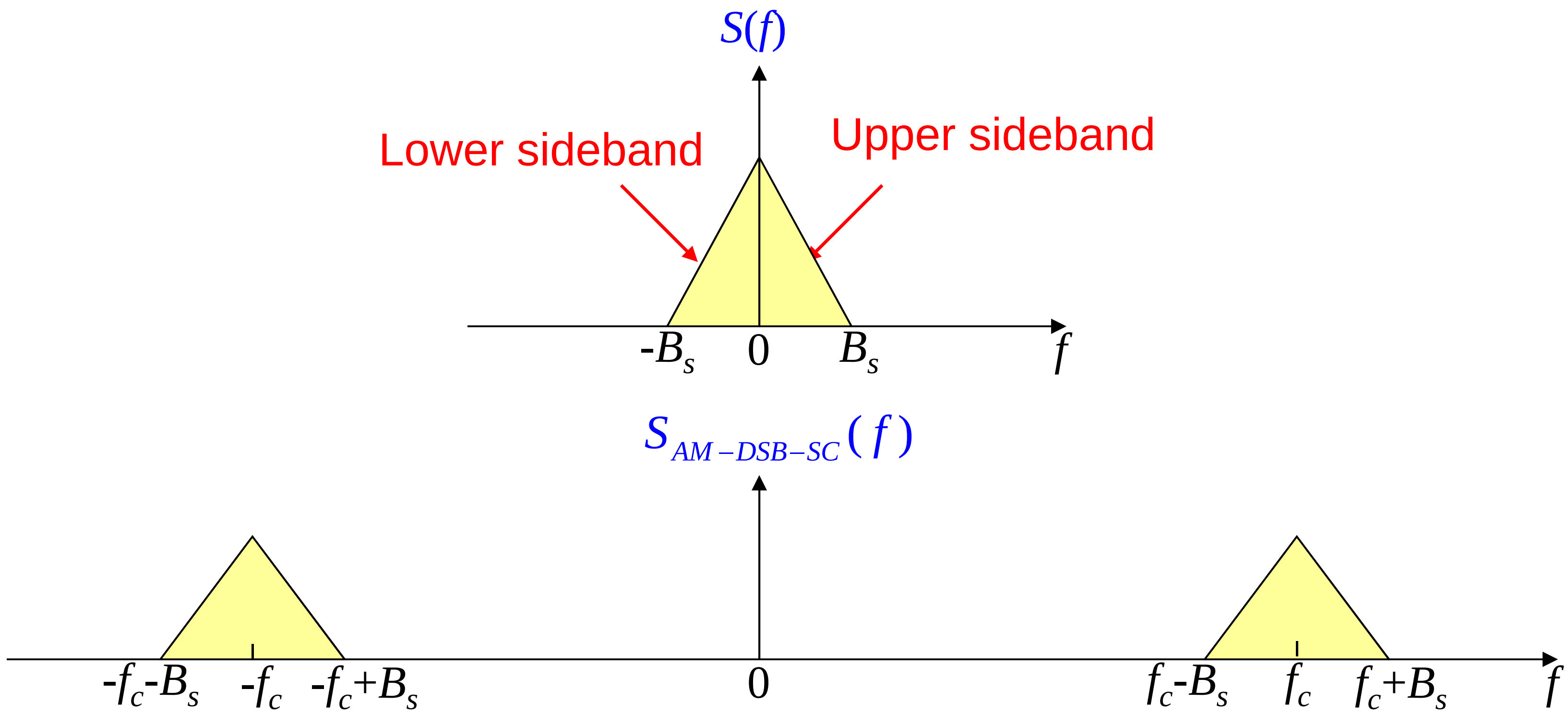



$s_{AM-DSB-SC}(t)$



# AM-DSB-SC -- Modulation

- Frequency Domain: $S_{AM-DSB-SC}(f)=\frac{A}{2}[S(f-f_c)+S(f+f_c)]$



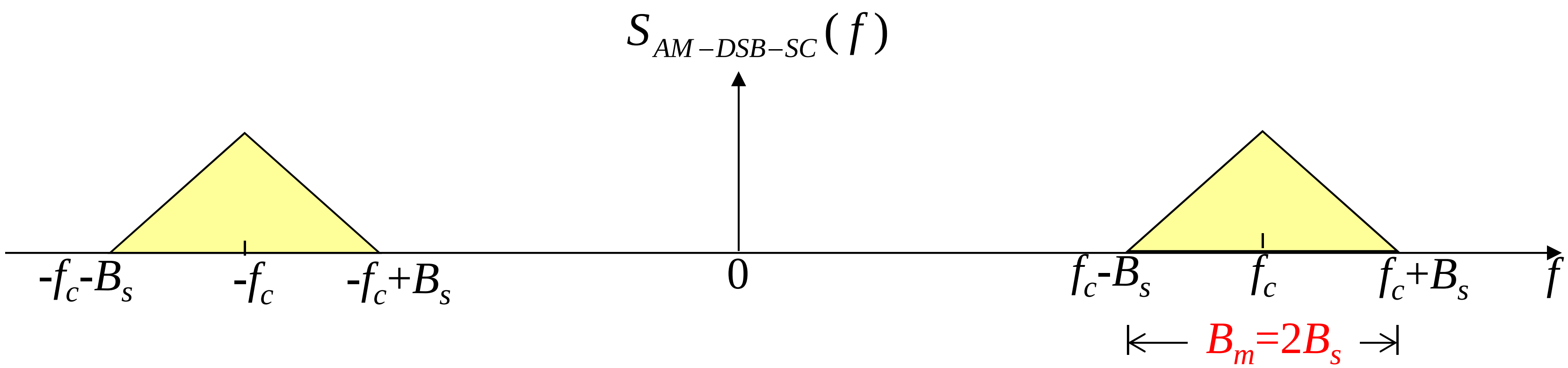

AM-DSB-SC: Amplitude Modulation-Double SideBand-Suppressed Carrier

# Bandwidth Efficiency of AM-DSB-SC

- Bandwidth Efficiency :

$$\gamma = \frac{\text{Information Signal Bandwidth } B_s}{\text{Required Channel Bandwidth } B_h} = \frac{\text{Information Signal Bandwidth } B_s}{\text{Modulated Signal Bandwidth } B_m}$$

With AM-DSB-SC :




$$\gamma_{AM-DSB-SC} = \frac{B_s}{2B_s} = 50\%$$

# AM-DSB-SC -- Demodulation

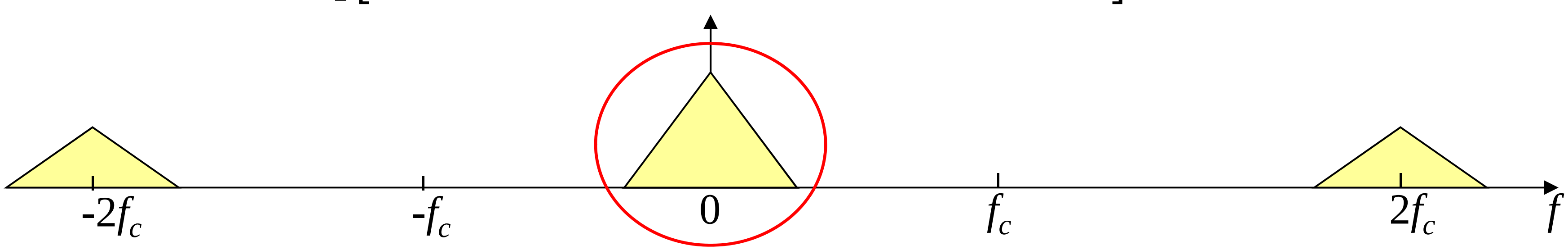
- Time Domain: $s_{AM-DSB-SC}(t) \overset{?}{\Rightarrow} s(t)$

$$s_{AM-DSB-SC}(t)\cos(2\pi f_c t) = As(t)\cos(2\pi f_c t)\cos(2\pi f_c t) = 0.5As(t) + 0.5As(t)\cos(2\pi 2 f_c t)$$

- Frequency Domain: $S_{AM-DSB-SC}(f) \overset{?}{\Rightarrow} S(f)$

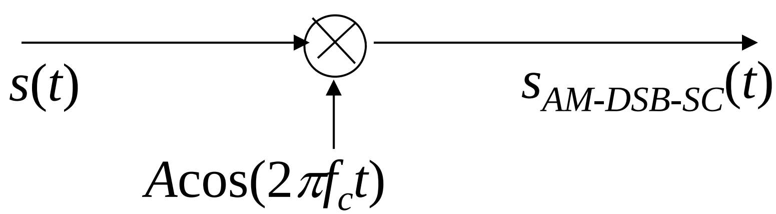
$$\tfrac{1}{2}[S_{AM-DSB-SC}(f-f_c)+S_{AM-DSB-SC}(f+f_c)] = \tfrac{A}{4}[S(f-2f_c)+S(f)]+\tfrac{A}{4}[S(f)+S(f+2f_c)]$$
$$= \tfrac{A}{4}[S(f-2f_c)+S(f+2f_c)]+\tfrac{A}{2}S(f)$$

$$\tfrac{1}{2}[S_{AM-DSB-SC}(f-f_c)+S_{AM-DSB-SC}(f+f_c)]$$



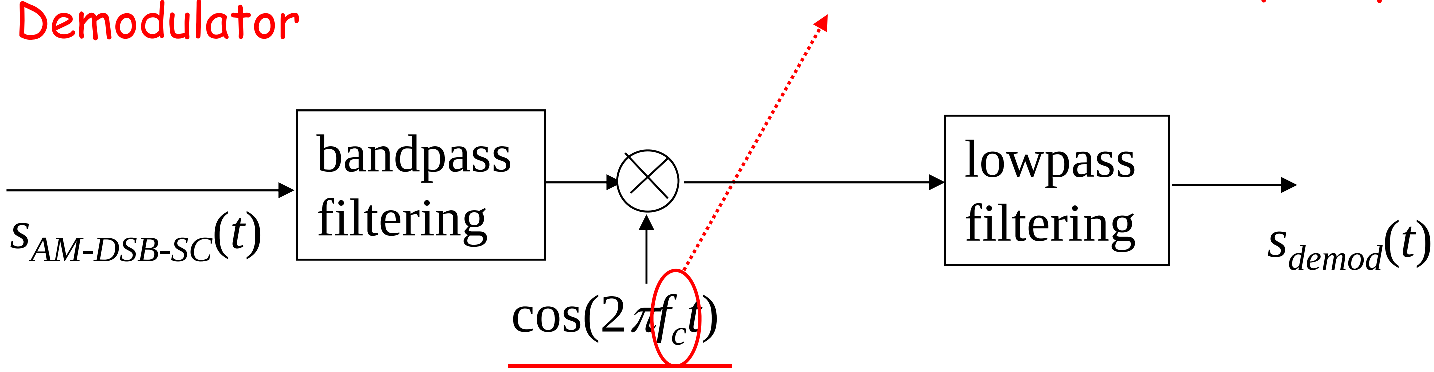
# Modulator and Demodulator of AM-DSB-SC

Modulator




Demodulator

What if there is a frequency error?




Coherent Demodulation: the demodulator requires a reference signal which has exactly the same frequency and phase as the carrier signal.

# Frequency Error of Coherent Demodulator

Consider that the reference signal has a small frequency error, $\Delta f$.

$$w(t) = As(t)\cos(2\pi f_c t)\cos(2\pi(f_c + \Delta f)t)$$
$$= 0.5As(t)\left(\cos(2\pi\Delta f t) + \cos(2\pi(2f_c + \Delta f)t)\right)$$

After lowpass filtering, we have

$$0.5As(t)\cos(2\pi\Delta f t)$$

$$\cos(2\pi\Delta f t) = 1 \quad \text{when } \Delta f = 0$$

$$\cos(2\pi\Delta f t) \text{ changes with } t \text{ when } \Delta f \neq 0$$

The performance of AM-DSB-SC is sensitive to the frequency error of the reference signal.

# Pros and Cons of AM-DSB-SC

- Straightforward

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- Sensitive to frequency and phase error of the reference signal (coherent demodulation)
- Bandwidth inefficient ($\gamma_{AM\text{-}DSB\text{-}SC}$=50%)

# AM-DSB-C

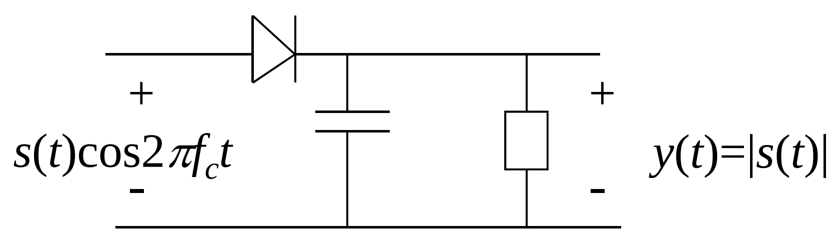
# Envelope and Envelope Detector

- Envelope

Consider a signal $s(t)\cos 2\pi f_c t$. If $s(t)$ varies slowly in comparison with the carrier $\cos 2\pi f_c t$, the envelope of $s(t)\cos 2\pi f_c t$ is $|s(t)|$.

- The envelope $|s(t)| = s(t)$ if $s(t) \geq 0$.

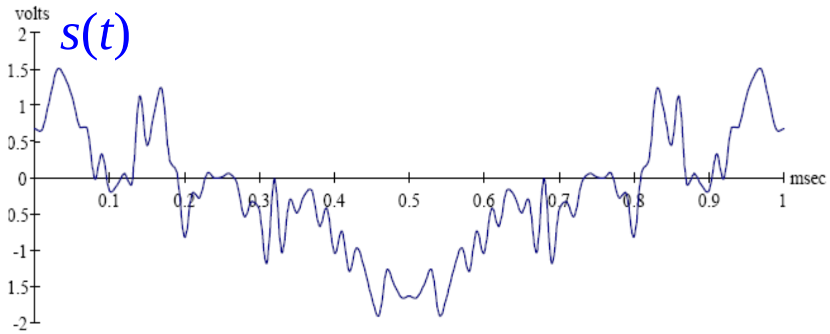
- Envelope Detector:

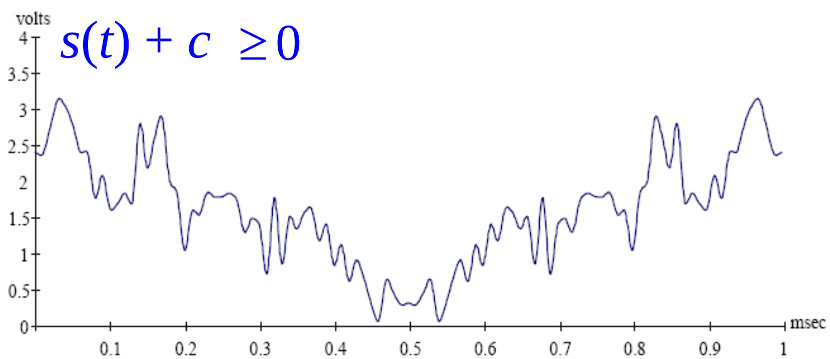


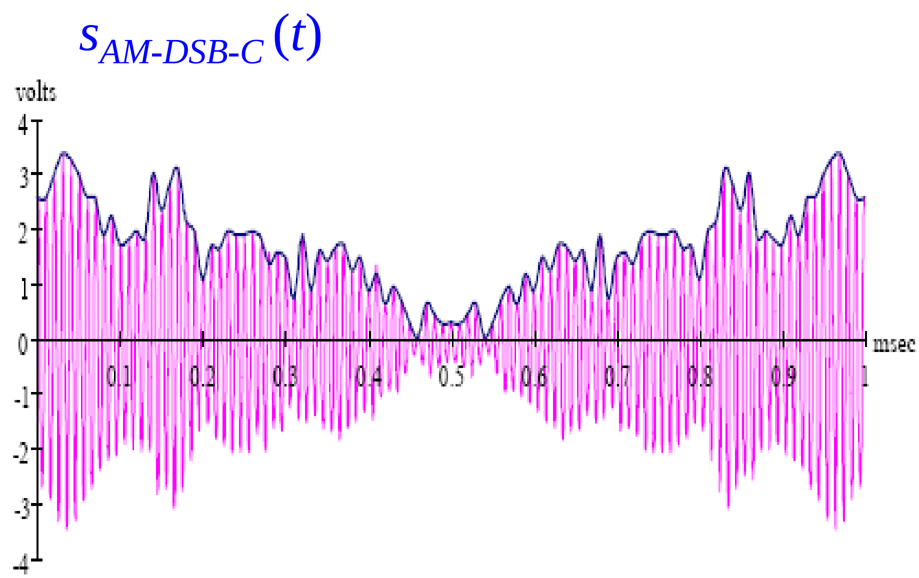

How to apply the Envelope Detector to AM systems?

# AM-DSB-C -- Modulation

- Time Domain: $s_{AM-DSB-C}(t) = A\left(s(t)+c\right)\cos(2\pi f_c t)$




$c$ is a dc offset to ensure $s(t)+c \geq 0$ for any time $t$.

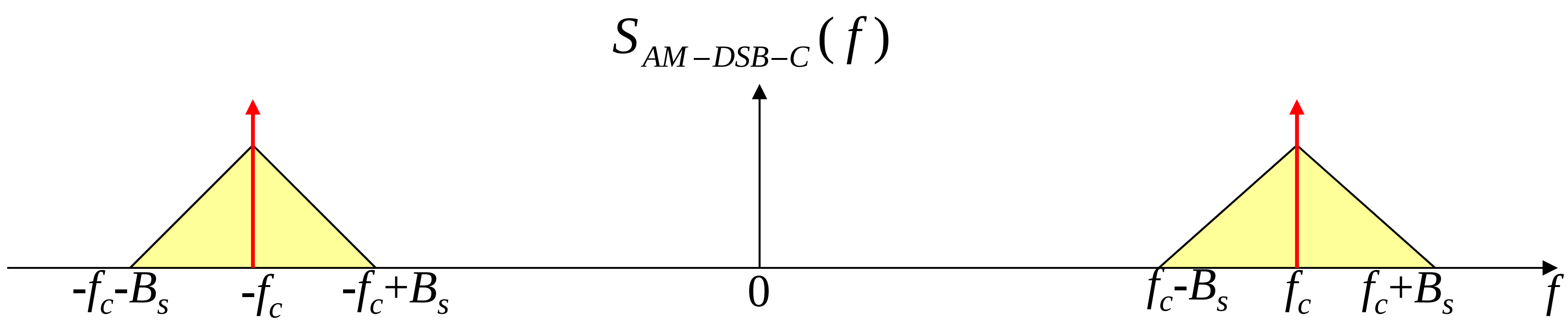
# AM-DSB-C -- Modulation

- Frequency Domain:

$$s_{AM-DSB-C}(t) = A\left(s(t)+c\right)\cos(2\pi f_c t) = As(t)\cos(2\pi f_c t) + Ac\cos(2\pi f_c t)$$

$$\Leftrightarrow$$

$$S_{AM-DSB-C}(f) = \tfrac{A}{2}[S(f-f_c)+S(f+f_c)] + \tfrac{Ac}{2}[\delta(f-f_c)+\delta(f+f_c)]$$



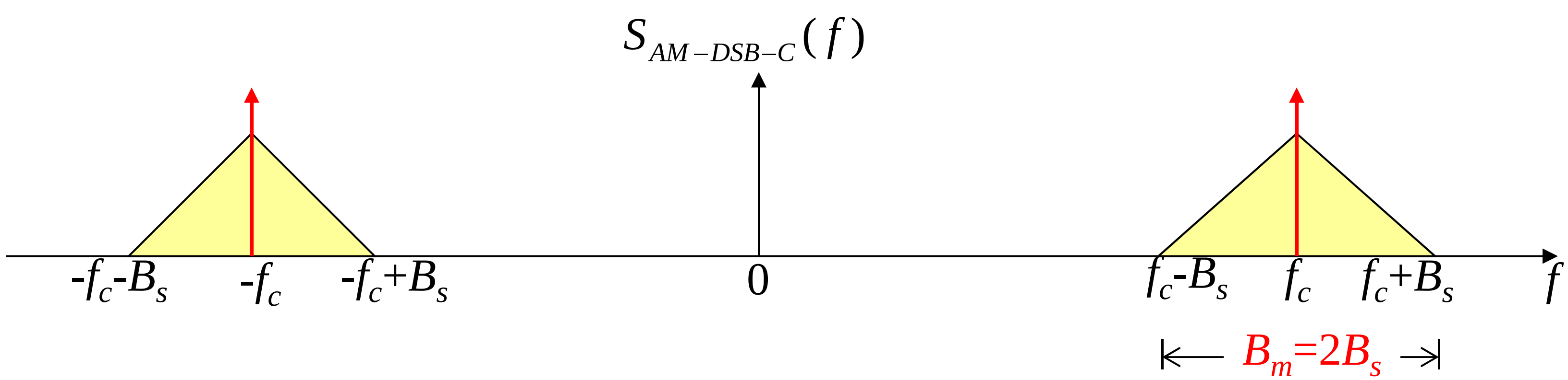

AM-DSB-C: Amplitude Modulation-Double SideBand-Carrier

# Bandwidth Efficiency of AM-DSB-C

- Bandwidth Efficiency :

$$\gamma = \frac{\text{Information Signal Bandwidth } B_s}{\text{Required Channel Bandwidth } B_h} = \frac{\text{Information Signal Bandwidth } B_s}{\text{Modulated Signal Bandwidth } B_m}$$

With AM-DSB-C :



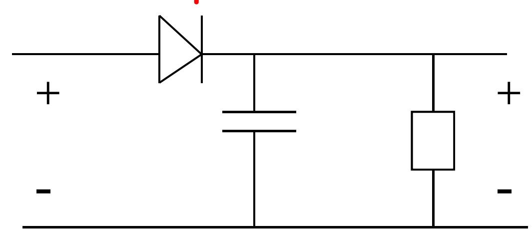

$$\gamma_{AM-DSB-C} = \frac{B_s}{2B_s} = 50\% = \gamma_{AM-DSB-SC}$$

# AM-DSB-C -- Demodulation

- Time Domain: $s_{AM-DSB-C}(t) \overset{?}{\Rightarrow} s(t)$

Envelope Detector:

$s_{AM-DSB-C}(t)$
$= A\big(s(t)+c\big)\cos(2\pi f_c t)$




$y(t) = \left| A\big(s(t)+c\big) \right| = A(s(t)+c)$
$(s(t)+c \geq 0)$

Non-coherent demodulation (no need to generate a reference signal)

- Apply $s_{AM\text{-}DSB\text{-}C}(t)$ to an envelope detector.
- Remove the dc offset c.

- Simple
- Robust
- Any price to pay?

# More about AM-DSB-C

- Define the power efficiency of an AM-DSB-C system as:

$$\eta = \frac{\text{power of information signal } s(t)}{\text{power of modulating signal } s(t)+c} = \frac{P_s}{c^2+P_s} \le 50\%$$

  - $\eta$ increases as the dc offset $c$ decreases.
  - to ensure $s(t)+c \ge 0$, $c^2 \ge P_s$

- Define the modulation index of an AM-DSB-C system as:

$$m = \frac{\max[s(t)+c]-\min[s(t)+c]}{\max[s(t)+c]+\min[s(t)+c]} = \frac{\max s(t)-\min s(t)}{\max s(t)+\min s(t)+2c}$$

  - $m$ increases as the dc offset $c$ decreases.

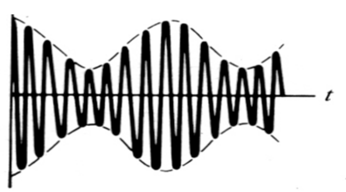
Can $m$ be arbitrarily large?

# Modulation Index $m$ of AM-DSB-C

$$m = \frac{\max[s(t)+c]-\min[s(t)+c]}{\max[s(t)+c]+\min[s(t)+c]} = \frac{\max s(t)-\min s(t)}{\max s(t)+\min s(t)+2c}$$

$m < 1$

when $\min(s(t)+c) > 0$




☺

$m = 1$

when $\min(s(t)+c) = 0$


☺

$m > 1$

when $\min(s(t)+c) < 0$


☹

m should not exceed 1 to avoid over-modulation.

# Pros and Cons of AM-DSB-C

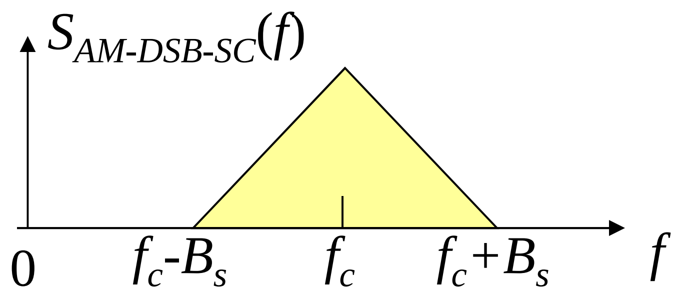
- Simple and robust receiver design (non-coherent demodulation)


- Cost of power efficiency ($\eta$<50%)
- Bandwidth inefficient ($\gamma_{AM\text{-}DSB\text{-}C}=\gamma_{AM\text{-}DSB\text{-}SC}=50\%$)

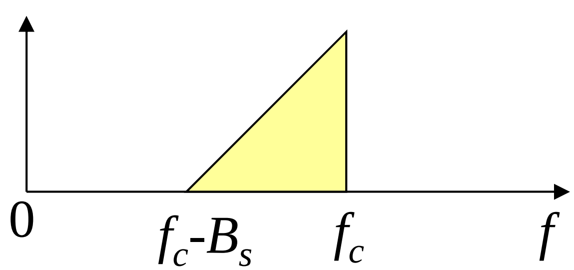
# AM-SSB and AM-VSB

# How to Improve Bandwidth Efficiency?

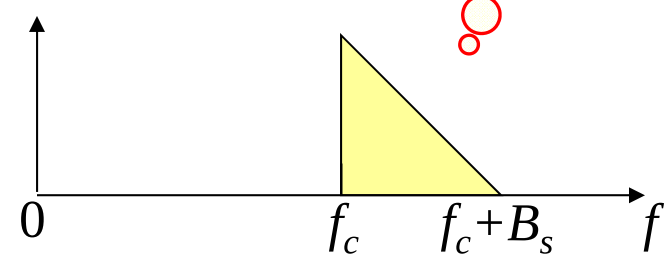
Double sideband (DSB)




Upper sideband carries the same information as the lower sideband!




Lower sideband



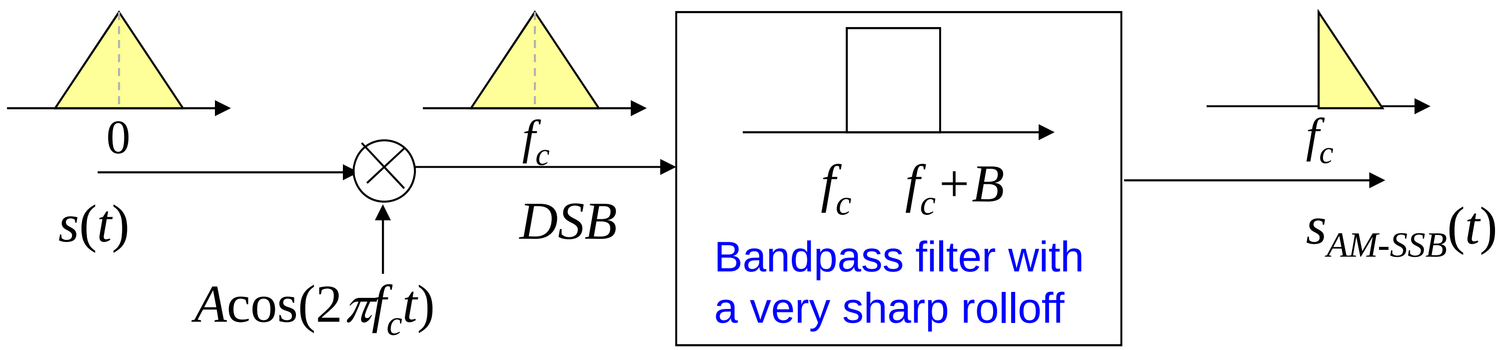

Upper sideband

Use a bandpass filter to select the desired sideband, and only transmit the desired sideband.

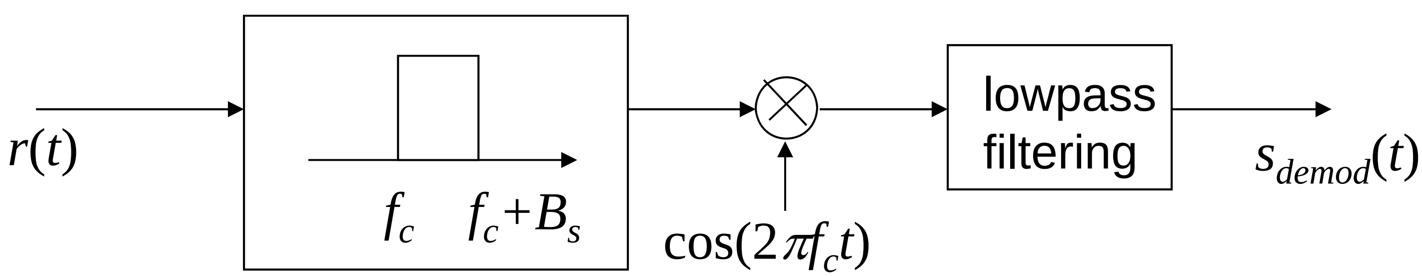
# AM-SSB

## Amplitude Modulation-Single SideBand (AM-SSB)

### Modulation (Frequency Discrimination Method)




### Demodulation (coherent)



# Pros and Cons of AM-SSB

- Bandwidth efficient ($\gamma_{AM\text{-}SSB}$=100%)

Mobile communications,
military communications, …

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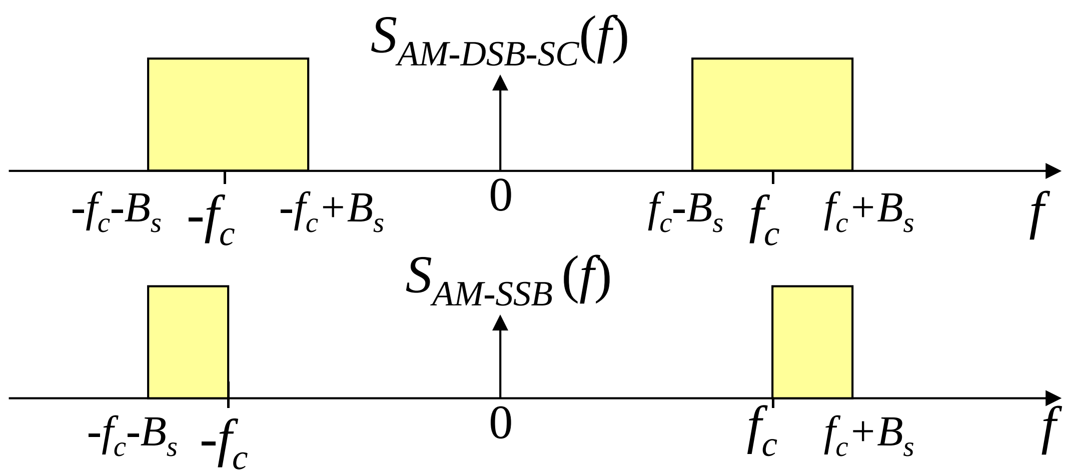
- High requirement on filtering (sharp rolloff)
- Sensitive to frequency and phase error of the reference signal (coherent demodulation)

# AM-VSB

DSB

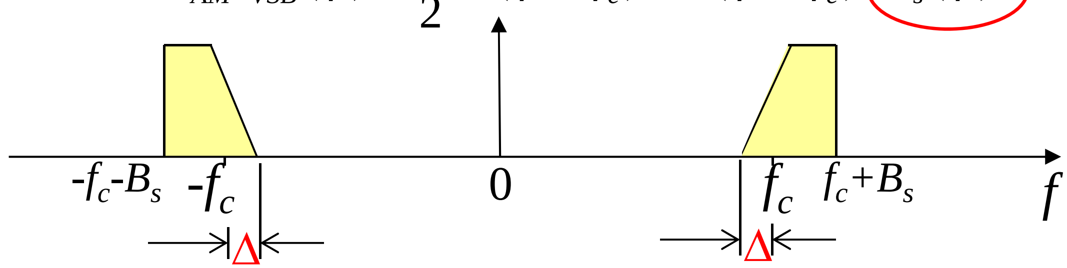
SSB




## Amplitude Modulation-Vestigial SideBand (AM-VSB)

$$S_{AM-VSB}(f) = \frac{1}{2}[S(f-f_c)+S(f+f_c)]H_s(f)$$

VSB




Allow a small portion (or *vestige*) of the lower sideband, $\Delta$, along with the upper sideband.

# Spectrum of AM-VSB Signal

modulation:

$$S_{AM-VSB}(f)=\frac{1}{2}[S(f-f_c)+S(f+f_c)]H_s(f)$$
$$=\frac{1}{2}S(f-f_c)H_s(f)+\frac{1}{2}S(f+f_c)H_s(f)$$

Demodulation:

$$S_{\text{demod}}(f)=\frac{1}{2}[S_{AM-VSB}(f-f_c)+S_{AM-VSB}(f+f_c)]$$
$$=\frac{1}{4}[S(f-2f_c)+S(f)]H_s(f-f_c)+\frac{1}{4}[S(f)+S(f+2f_c)]H_s(f+f_c)$$
$$=\frac{1}{4}S(f)[H_s(f-f_c)+H_s(f+f_c)]+\frac{1}{4}S(f-2f_c)H_s(f-f_c)+\frac{1}{4}S(f+2f_c)H_s(f+f_c)$$

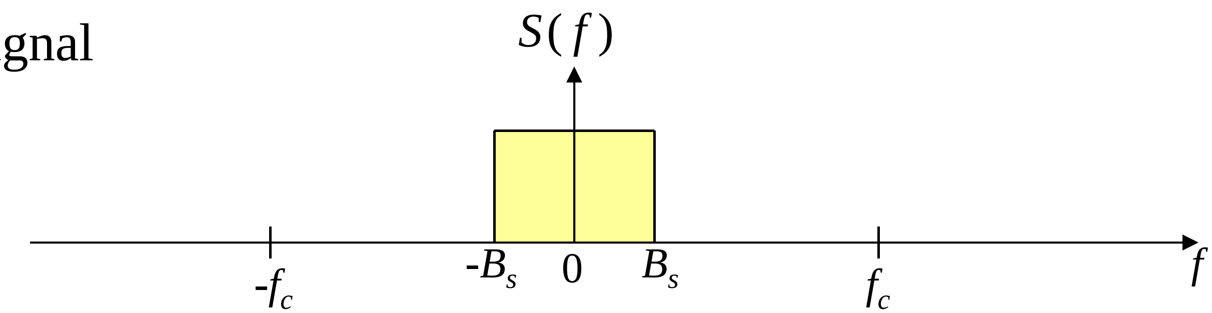
After lowpass filtering:

$$S_{\text{demod}}(f)=S(f)[H_s(f-f_c)+H_s(f+f_c)]$$

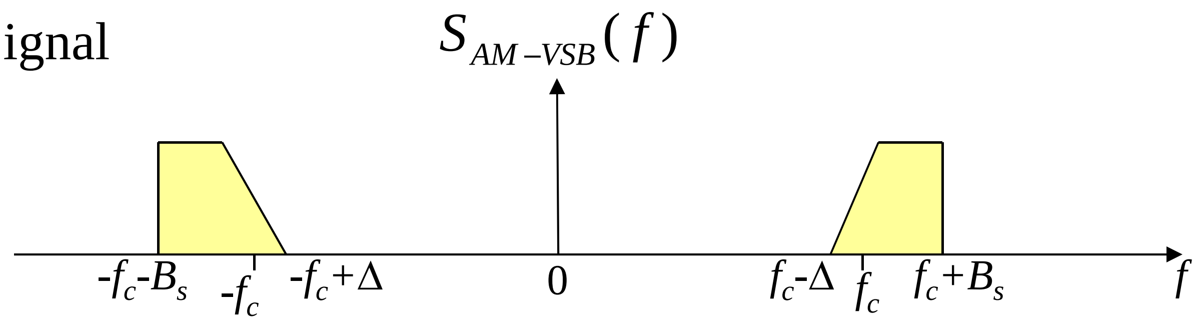
$$H_s(f-f_c)+H_s(f+f_c)=k \qquad 0\le|f|\le B$$

# Spectrum of AM-VSB Signal

Baseband signal

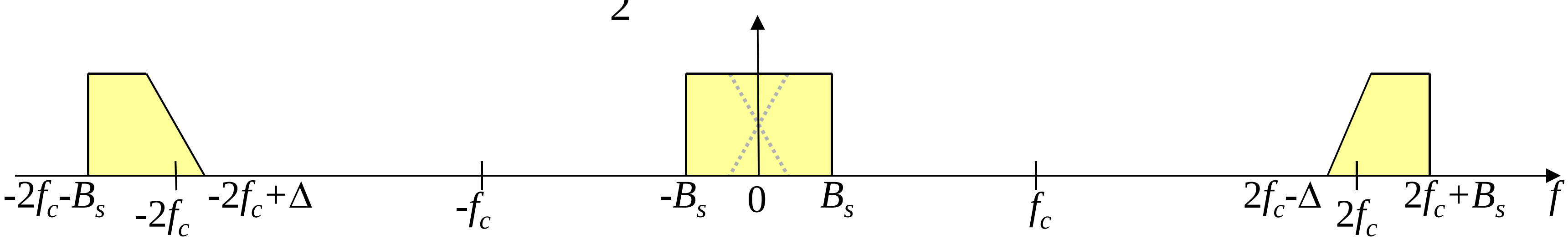



Modulated Signal

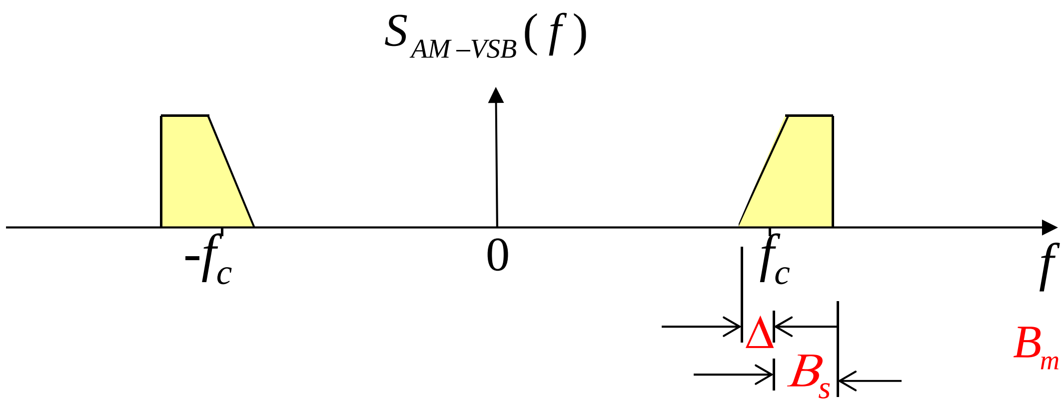



De-modulation

$$S_{\mathrm{demod}}(f)=\frac{1}{2}[S_{AM-VSB}(f-f_c)+S_{AM-VSB}(f+f_c)]$$



# Tradeoff between Complexity and Bandwidth Efficiency




$B_m = B_s + \Delta$

$$50\% < \gamma_{AM-VSB} = \frac{B_s}{B_s + \Delta} < 100\%$$

$0 < \Delta < B_s$

- The bandwidth efficiency $\gamma$ decreases as the vestige $\Delta$ increases.
- The larger vestige $\Delta$, the lower receiver complexity.

# Summary of AM

| | | | |
|---|---|---|---|
| AM-DSB-SC | Power efficient | Coherent demodulation | Bandwidth inefficient $\gamma$=50% |
| AM-DSB-C<br>Commercial radio broadcasting | Power inefficient | Non-coherent demodulation (simple and robust) | Bandwidth inefficient $\gamma$=50% |
| AM-SSB<br>Mobile and military communications | Power efficient | Coherent demodulation | Bandwidth efficient $\gamma$=100% |
| AM-VSB<br>Public television systems | Power efficient | Coherent demodulation | Tradeoff between bandwidth and complexity $50\% < \gamma = \frac{B_s}{B_s + \Delta} < 100\%$ |