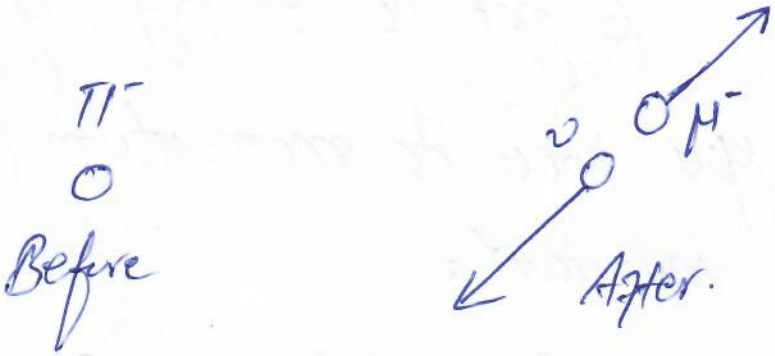
Example:

A pion at rest decays into a muon plus a neutrino; what is the speed of the muon?

Sol:



§ Decay of the charged pion.

Conservation of energy requires

$$E_\pi = E_\mu + E_\nu .$$

Conservation of Momentum Gives

$$P_\pi = P_\mu + P_\nu$$

but $P_\pi = 0$

hence $P_\mu = -P_\nu$

Thus the muon and the neutrino fly off back-to-back, with equal & opposite momenta.

⇒ To proceede, we need a formula relating ie energy of a particle to its momentum.

so

To get ie energy of a particle, when you knoeo its momentum (or vice versa) use ie invariant:

$$\frac{E^2}{c^2} - \mathbf{P}^2 = m^2c^2 \qquad \text{——— ①}$$

or $$E^2 - \mathbf{P}^2c^2 = m^2c^4$$

In ie present case, we have.

$$E_\pi = m_\pi c^2 \qquad \text{—(i)}$$

$$E_\mu = c\sqrt{m_\mu^2 c^2 + \mathbf{P}_\mu^2} \quad \text{(using ① with subscript 'μ')}$$

$$E_\nu = |\mathbf{P}_\nu|c = |\mathbf{P}_\mu|c$$

Using equation for Conservation of Energy we have (i.e.)

$$E_\pi = E_\mu + E_\nu \checkmark$$

(2)

$$m_\pi c^2 = c\sqrt{m_\mu^2 c^2 + P_\mu^2} + |P_\mu| c$$

$$(m_\pi c - |P_\mu|)c = c\sqrt{m_\mu^2 c^2 + P_\mu^2}$$

$$(m_\pi c - |P_\mu|)^2 = m_\mu^2 c^2 + P_\mu^2$$

Solving for $|P_\mu|$

i.e. $|P_\mu| = ?$

$$(m_\pi c)^2 + P_\mu^2 - 2m_\pi |P_\mu| c = m_\mu^2 c^2 + P_\mu^2$$

$$2m_\pi |P_\mu| c = m_\pi^2 c^2 - m_\mu^2 c^2$$

$$|P_\mu| = \frac{(m_\pi^2 - m_\mu^2)\, c^2}{2 m_\pi c}$$

$$|P_\mu| = \frac{m_\pi^2 - m_\mu^2}{2 m_\pi} c \qquad \text{——} (2)$$

Energy of the muon can be calculated using ② in (i)

$$E_\mu = c\sqrt{m_\mu^2 c^2 + \left(\frac{m_\pi^2 - m_\mu^2}{2m_\pi} c\right)^2}$$

which leads to

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} c^2 \qquad \text{③}$$

Now we can find the velocity of the particle, If

$$E = \gamma m c^2, \quad P = \gamma m v$$

Dividing the below

$$P/E = v/c^2$$

and

$$v = PC^2/E \checkmark$$

So the answer to our problem is

$$v_\mu = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2} C \quad \text{---} *$$

Putting the masses, we can get.

$$v_\mu = 0.271C$$