Problem :-

Cosmic ray muons are produced high in the atmosphere (at 8000 m, say) and travel toward the earth at very nearly the speed of light (0.998 c).
1 say

(a) Given the life time of the muon (2.2×10^{-6} sec), how far would it go before disintegrating, according to prerelativistic physics? Would the muons make it to ground level?

(b) Now answer the same question using relativistic physics. (Because of time dilation, the muons last longer, so they travel farther.)

(c) Pions are also produced in the upper

atmosphere. In fact, the sequence is proton (from outer space) hits proton (in atmosphere) $\longrightarrow$ p + p + pions. The pions then decay into muons:

$$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu ;$$

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu .$$

But the lifetime of the pion is much shorter (2.6×10^{-8} s). Assuming the pions have the same speed (0.998 c), will they reach ground level?

Solution:-

(a) Classically, we know that

$$d = vt$$
$$= (0.998 \times 3 \times 10^8 \text{ m/s})(2.2 \times 10^{-6} \text{ s})$$
$$= 659 \text{ m} , \text{ No.}$$

(b) we also know that, Relativistically,

$$d = v(\gamma t) \quad \text{——— (1)}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{0.998c^2}{c^2}}}$$

$$= 15.8.$$

Rearranging & solving eqn (1)

$$d = \gamma(vt)$$

$$= \gamma(659\ m)$$

$$= (15.8)(659) = 10,400\ m, \text{ Yes}$$

(c) they only travel

$$\frac{10,400(2.6 \times 10^{-8})}{(2.2 \times 10^{-6})} = 123\ m, \text{ No!}$$