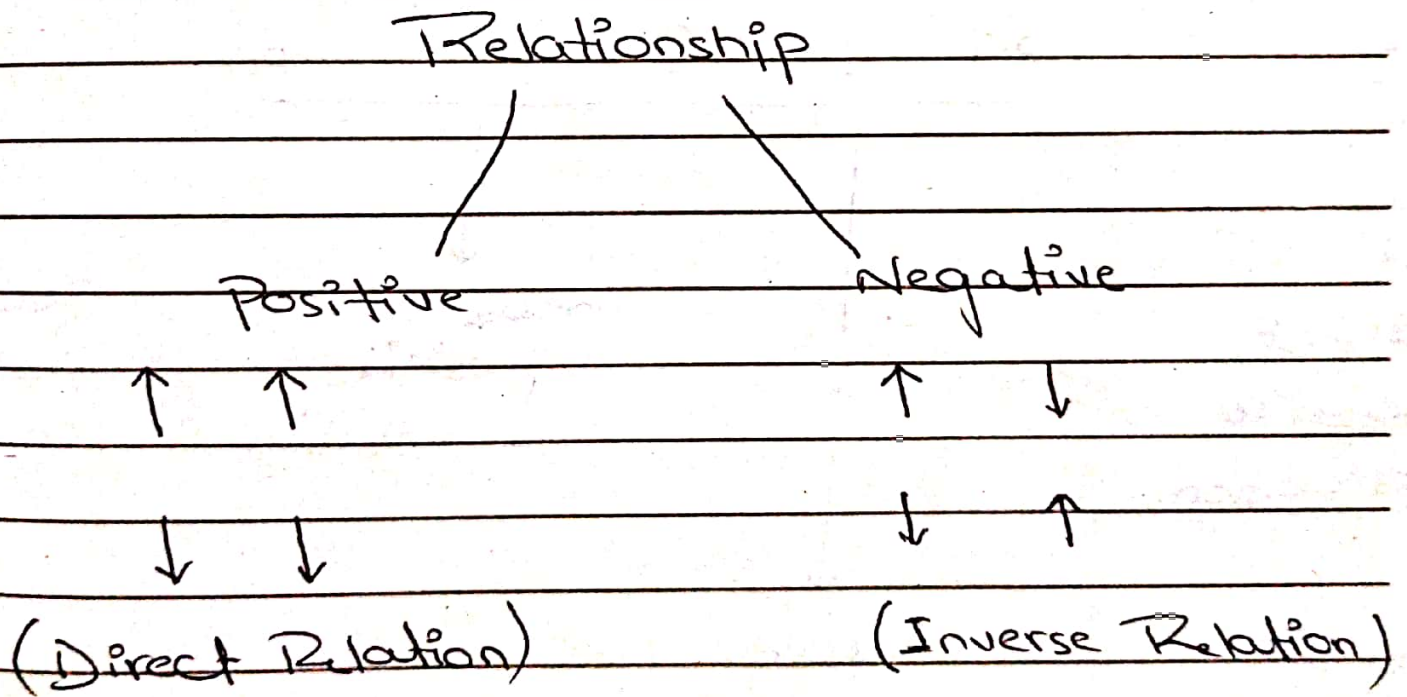


CORRELATION:

⇒ Definition:

Correlation is a technique, which measures the strength of relationship b/w two variables. It is denoted by ' r '.

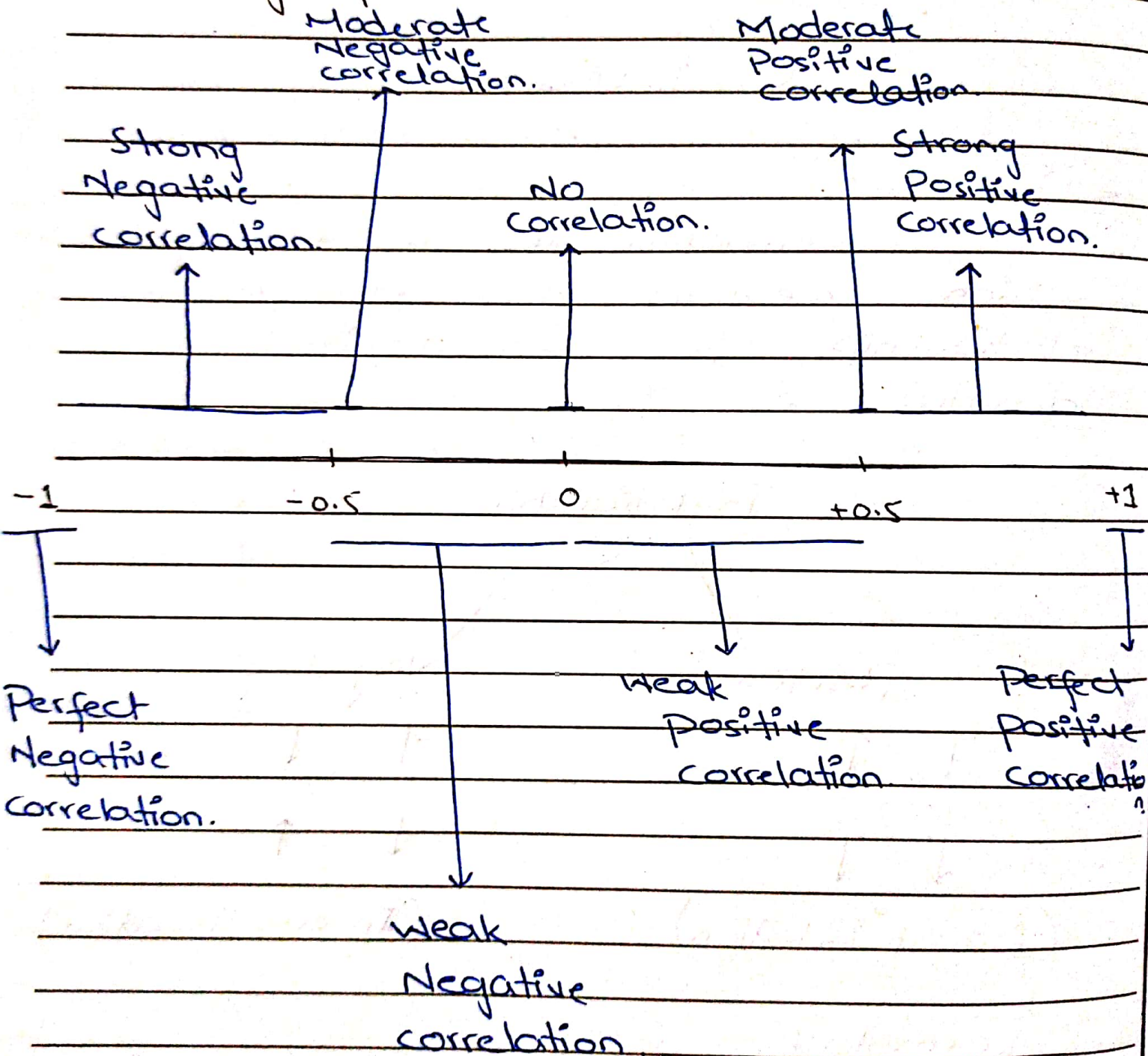


⇒ Formula:

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$$

Correlation coefficient.

⇒ Range of Correlation :



⇒ Positive Correlation:

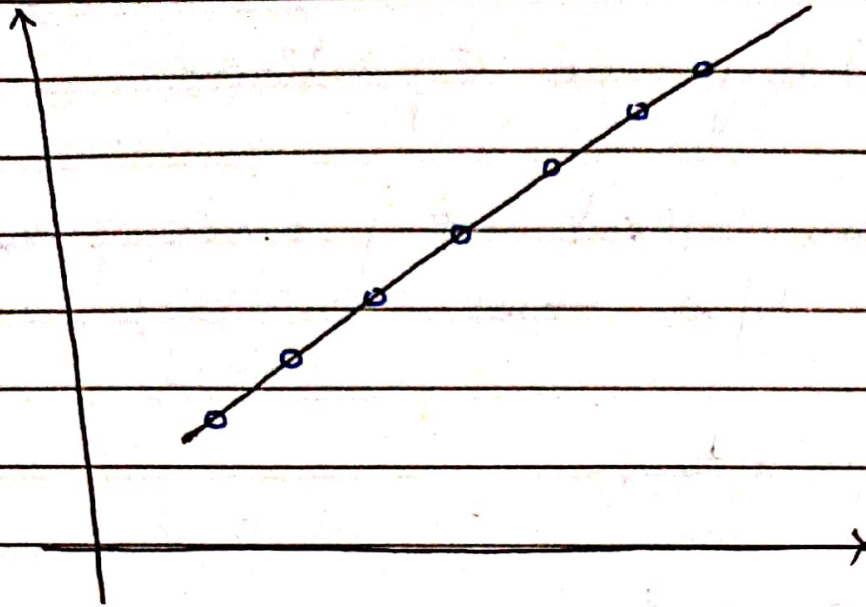
If two variables are related in such a way that they increase or decrease together. It means that their change is in same direction. Then such a correlation is called positive correlation or Direct correlation. e.g.,

Relation b/w age & weights of children

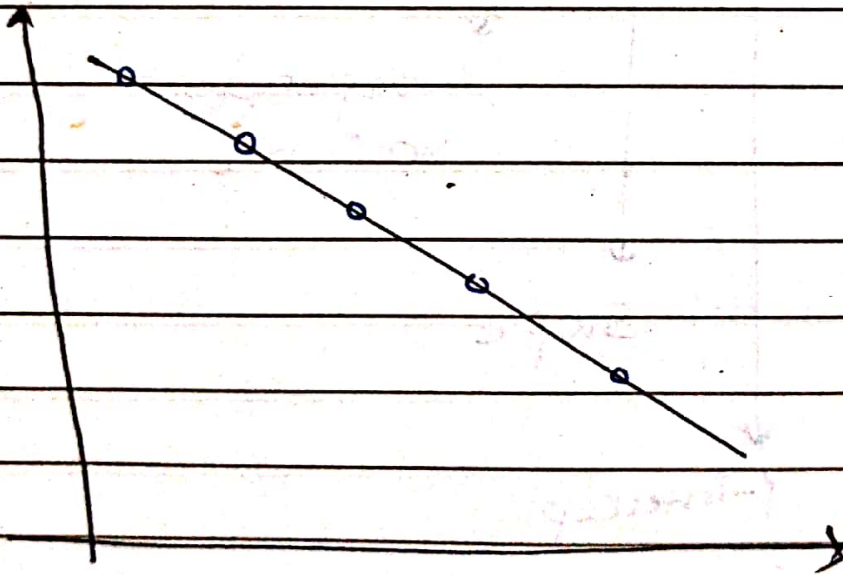
⇒ Negative Correlation:

If two variables are related in such a way that one variable increases & the other decreases. It means that the change of the variables are in opposite direction. Then such a relation b/w variables is called Negative correlation. e.g.,

Relation b/w supply & price will be negative.



Perfect
Positive
Correlation



Perfect
Negative
Correlation

⇒ Example:

Calculate co-efficient of correlation b/w X & Y from the following data:

| | | | | | |
|---|---|---|---|---|---|
| X | 1 | 2 | 3 | 4 | 5 |
| Y | 2 | 5 | 3 | 8 | 7 |

$$\bar{X} = \frac{\sum X}{n} = \frac{15}{5}$$

$$\bar{X} = 3$$

$$\bar{Y} = \frac{\sum Y}{n} = \frac{25}{5}$$

$$\bar{Y} = 5$$

| X | Y | $X - \bar{X}$ | $Y - \bar{Y}$ | $(X - \bar{X})^2$ | $(Y - \bar{Y})^2$ | $(X - \bar{X})(Y - \bar{Y})$ |
|-----------|-----------|---------------|---------------|-------------------|-------------------|------------------------------|
| 1 | 2 | -2 | -3 | 4 | 9 | 6 |
| 2 | 5 | -1 | 0 | 1 | 0 | 0 |
| 3 | 3 | 0 | -2 | 0 | 4 | 0 |
| 4 | 8 | 1 | 3 | 1 | 9 | 3 |
| 5 | 7 | 2 | 2 | 4 | 4 | 4 |
| <u>15</u> | <u>25</u> | <u>0</u> | <u>0</u> | <u>10</u> | <u>26</u> | <u>13</u> |

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$$

$$= \frac{13}{\sqrt{10 \times 26}}$$

$$r = 0.8$$

There is strong positive correlation
b/w x & y .

→ Example:

Find coefficient of correlation b/w the variables X & Y represented in the following table

| | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|----|
| X | 25 | 29 | 30 | 30 | 31 | 32 | 33 | 35 | 37 | 38 |
| Y | 20 | 22 | 24 | 29 | 23 | 31 | 29 | 31 | 30 | 31 |

$$\bar{X} = \frac{\sum X}{n}$$

$$= \frac{320}{10}$$

$$\bar{X} = 32$$

$$\bar{Y} = \frac{\sum Y}{n}$$

$$= \frac{270}{10}$$

$$\bar{Y} = 27$$

| X | Y | $x - \bar{x}$ | $y - \bar{y}$ | $(x - \bar{x})^2$ | $(y - \bar{y})^2$ | $(x - \bar{x})(y - \bar{y})$ |
|----|----|---------------|---------------|-------------------|-------------------|------------------------------|
| 25 | 20 | -7 | -7 | 49 | 49 | 49 |
| 29 | 22 | -3 | -5 | 9 | 25 | 15 |
| 30 | 24 | -2 | -3 | 4 | 9 | 6 |
| 30 | 29 | -2 | 2 | 4 | 4 | -4 |
| 31 | 23 | -1 | -4 | 1 | 16 | 4 |
| 32 | 31 | 0 | 4 | 0 | 16 | 0 |
| 33 | 29 | 1 | 2 | 1 | 4 | 2 |
| 35 | 31 | 3 | 4 | 9 | 16 | 12 |
| 37 | 30 | 5 | 3 | 25 | 9 | 15 |
| 38 | 31 | 6 | 4 | 36 | 16 | 24 |
| | | | | <u>138</u> | <u>164</u> | <u>123</u> |

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$$

$$= \frac{123}{\sqrt{138 \times 164}}$$

$$= \frac{123}{\sqrt{138 \times 164}}$$

$$r = 0.82$$

Strong Positive Correlation.

⇒ Example :

Find coefficient of correlation b/w
 X & Y

| | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|
| X | 400 | 200 | 700 | 100 | 500 | 300 | 600 |
| Y | 50 | 60 | 20 | 70 | 40 | 30 | 10 |

$$\bar{X} = \frac{2800}{7} = \underline{\underline{400}}$$

$$\bar{Y} = \frac{280}{7} = \underline{\underline{40}}$$

| X | Y | $X - \bar{X}$ | $Y - \bar{Y}$ | $(X - \bar{X})^2$ | $(Y - \bar{Y})^2$ | $(X - \bar{X})(Y - \bar{Y})$ |
|-----|-----|---------------|---------------|-------------------|-------------------|------------------------------|
| 400 | 50 | 0 | 10 | 0 | 100 | 0 |
| 200 | 60 | -200 | 20 | 40000 | 400 | -4000 |
| 700 | 20 | 300 | -20 | 90000 | 400 | -6000 |
| 100 | 70 | -300 | 30 | 90000 | 900 | -9000 |
| 500 | 40 | 100 | 0 | 10000 | 0 | 0 |
| 300 | 30 | -100 | -10 | 10000 | 100 | 1000 |
| 600 | 10 | 200 | -30 | 40000 | 900 | -6000 |
| | | | | <u>280000</u> | <u>2800</u> | <u>-24000</u> |

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$$

$$= \frac{-24000}{\sqrt{280000 \times 2800}}$$

$$= -0.857$$

$$= -0.857$$

There is strong negative correlation
b/w X & Y!

→ Example :

| | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|
| X | 30 | 35 | 40 | 45 | 50 | 60 | 70 | 80 | 90 |
| Y | 2 | 4 | 5 | 5 | 8 | 15 | 24 | 30 | 32 |

$$\gamma = ?$$

$$\bar{X} = \frac{500}{9} = \boxed{55.55}$$

$$\bar{Y} = \frac{125}{9} = \boxed{13.89}$$

| x | y | (x - \bar{x}) | (y - \bar{y}) ² | (y - \bar{y}) ² | (x - \bar{x}) ² | (x - \bar{x})(y - \bar{y}) |
|----|----|------------------|-------------------------------|-------------------------------|-------------------------------|----------------------------------|
| 30 | 2 | -25.55 | -11.89 | 141.37 | 652.80 | 303.7895 |
| 35 | 4 | -20.55 | -9.89 | 97.81 | 422.30 | 203.2395 |
| 40 | 5 | -15.55 | -8.89 | 79.03 | 241.80 | 138.24 |
| 45 | 5 | -10.55 | -8.89 | 79.03 | 111.30 | 93.79 |
| 50 | 8 | -5.55 | -5.89 | 34.69 | 30.80 | 32.69 |
| 60 | 15 | 4.45 | 1.11 | 1.23 | 19.80 | 4.94 |
| 70 | 24 | 14.45 | 10.11 | 102.21 | 208.80 | 146.09 |
| 80 | 30 | 24.45 | 16.11 | 259.53 | 597.80 | 393.89 |
| 90 | 32 | 34.45 | 18.11 | 327.97 | 1186.80 | 623.89 |
| | | | | <u>1122.87</u> | <u>3472.2</u> | <u>1940.56</u> |

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$$

$$= \frac{1940.56}{\sqrt{3472.2 \times 1122.87}}$$

$$r = 0.98$$

$$r = 0.98$$

There is strong positive correlation
b/w X & Y.

→ Example:

$$r = ?$$

| | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|
| x | 137 | 209 | 113 | 189 | 178 | 200 | 219 |
| y | 23 | 47 | 22 | 40 | 39 | 51 | 49 |

$$\bar{x} = \frac{1245}{7} = 177.8$$

$$\bar{y} = \frac{271}{7} = 38.7$$

| x | y | $x - \bar{x}$ | $(y - \bar{y})$ | $(x - \bar{x})^2$ | $(y - \bar{y})^2$ | $(x - \bar{x})(y - \bar{y})$ |
|-----|----|---------------|-----------------|-------------------|-------------------|------------------------------|
| 137 | 23 | -40.8 | -15.7 | 1664.64 | 246.49 | 640.56 |
| 209 | 47 | 31.2 | 8.3 | 973.44 | 68.89 | 258.96 |
| 113 | 22 | -64.8 | -16.7 | 4199.04 | 278.89 | 1082.16 |
| 189 | 40 | 11.2 | 1.3 | 125.44 | 1.69 | 14.56 |
| 178 | 39 | 0.2 | 0.3 | 0.04 | 0.09 | 0.06 |
| 200 | 51 | 22.2 | 12.3 | 492.84 | 151.29 | 273.06 |
| 219 | 49 | 41.2 | 10.3 | 1697.44 | 106.09 | 424.36 |
| | | | | <u>9152.88</u> | <u>853.43</u> | <u>2693.7</u> |

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$$

$$= \frac{2693.7}{\sqrt{(9152.88)(853.43)}}$$

$$r = \frac{2693.7}{2794.88}$$

$$r = 0.964$$

Strong positive correlation.