Filters

Major filter categories

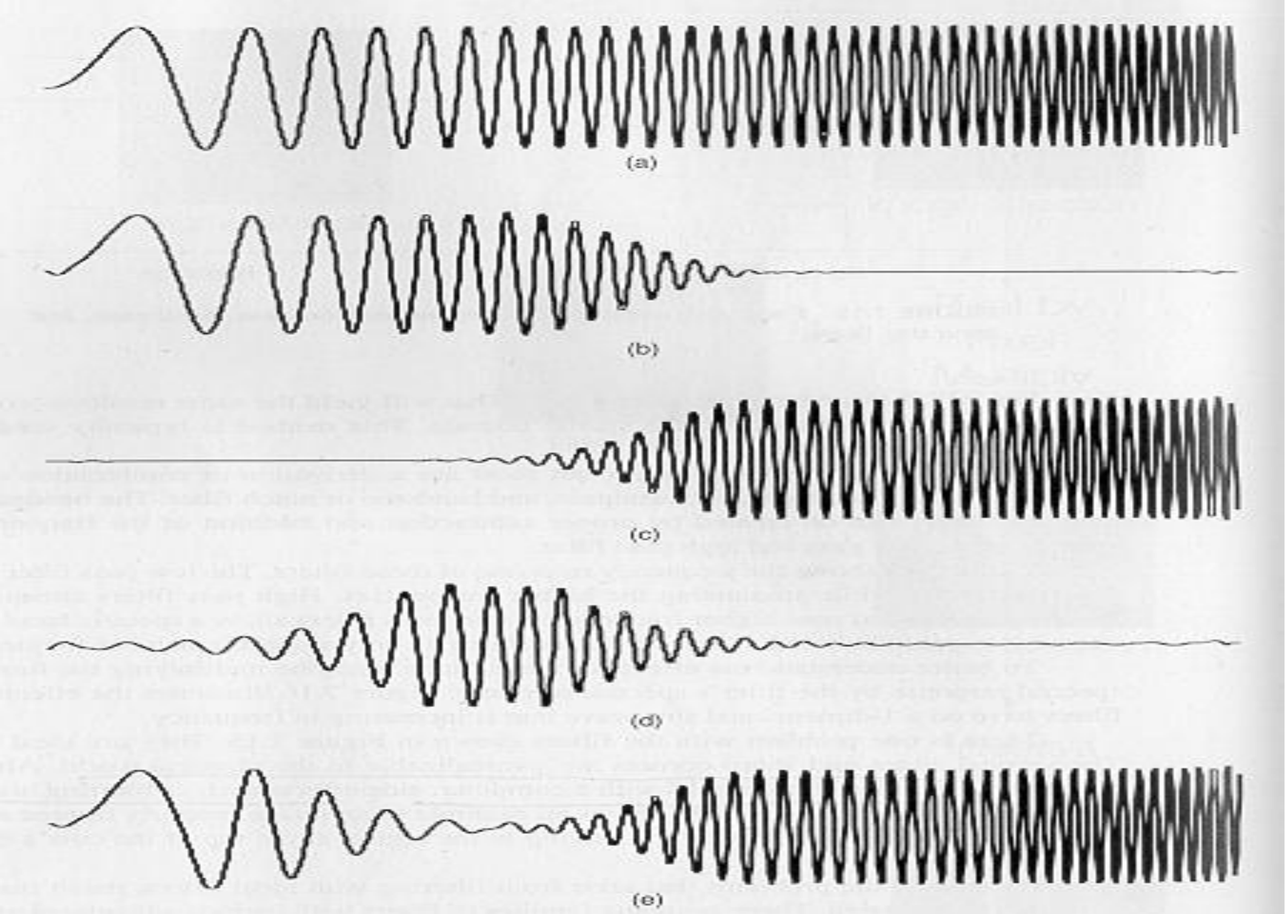
- Typically, filters are classified by examining their properties in the frequency domain:

  (1) Low-pass

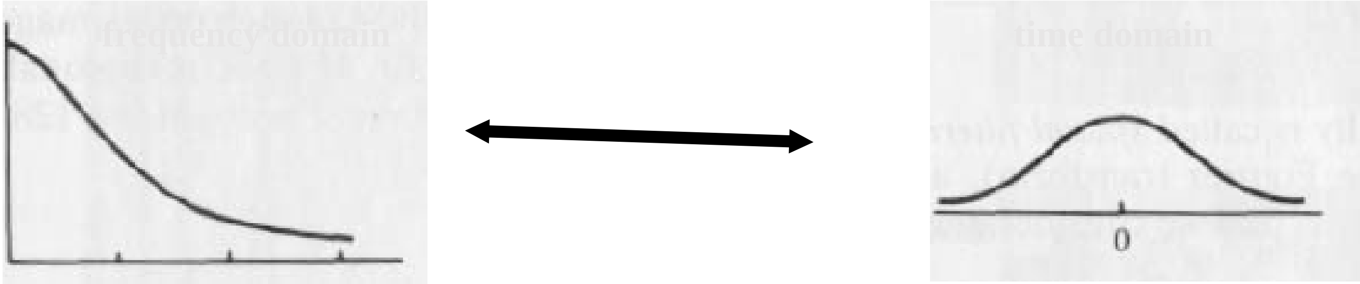
  (2) High-pass

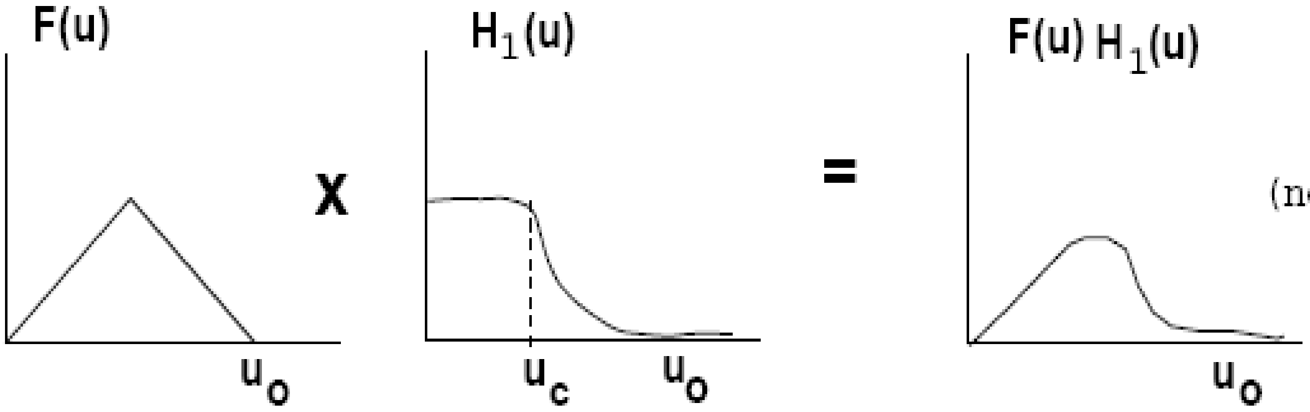
  (3) Band-pass

  (4) Band-stop


(a)
(b)
(c)
(d)
(e)

Low Pass filter

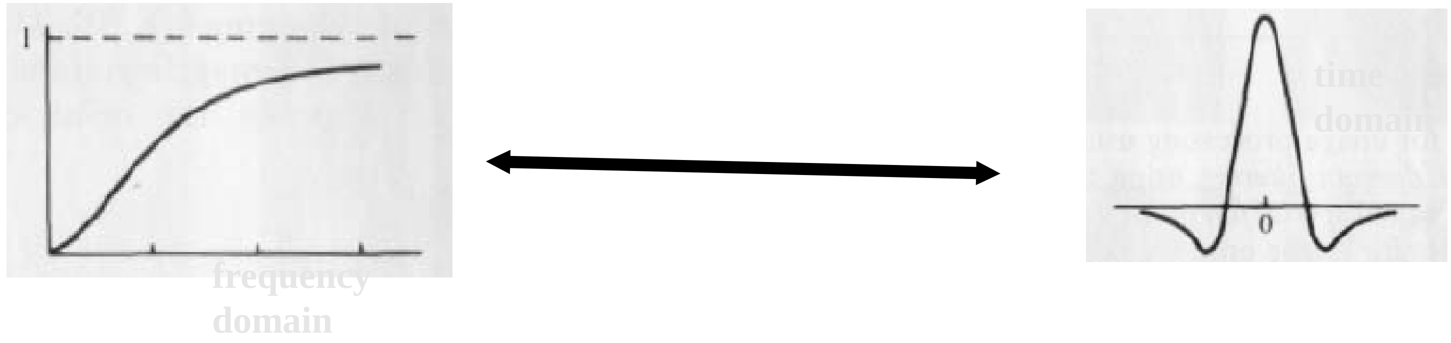


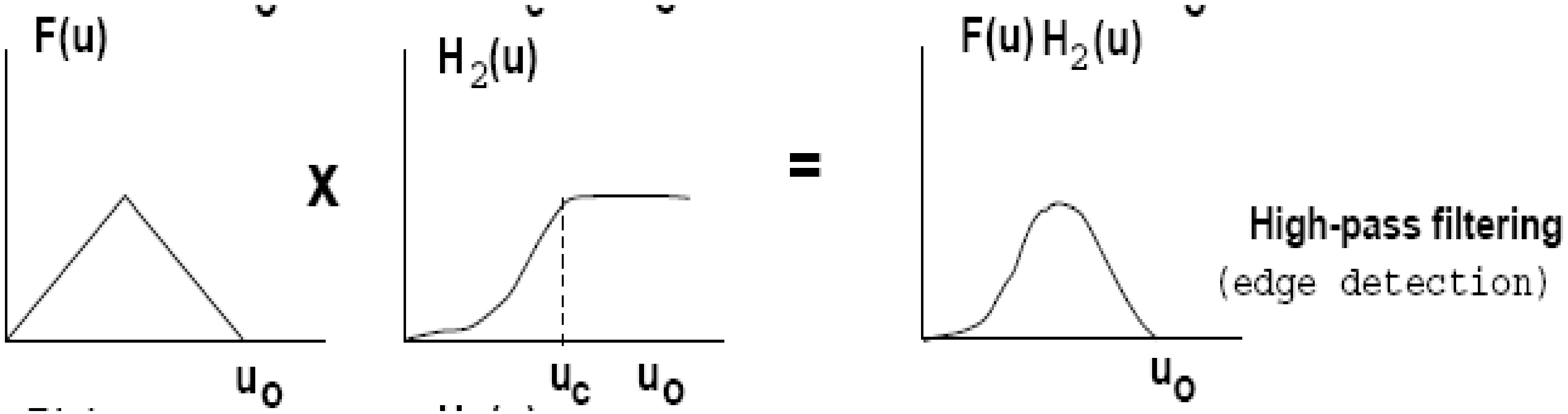


Low-pass filtering

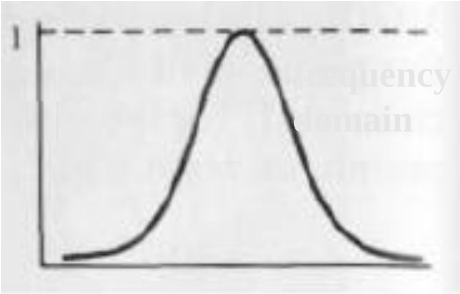
(noise suppression)

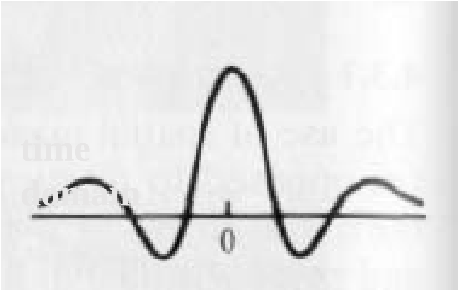
High-pass filters

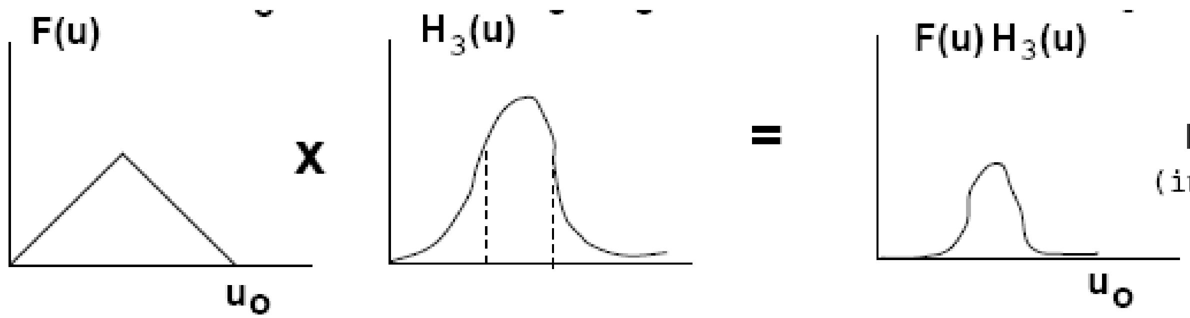




Band-pass filters

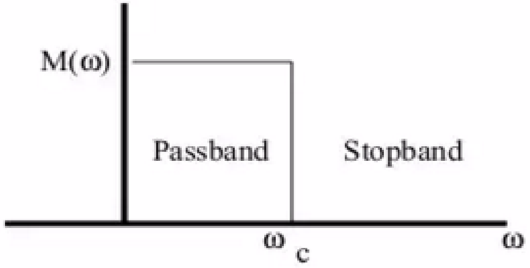




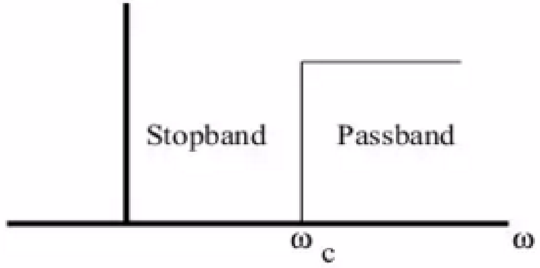


Band-pass filtering
(image restoration)

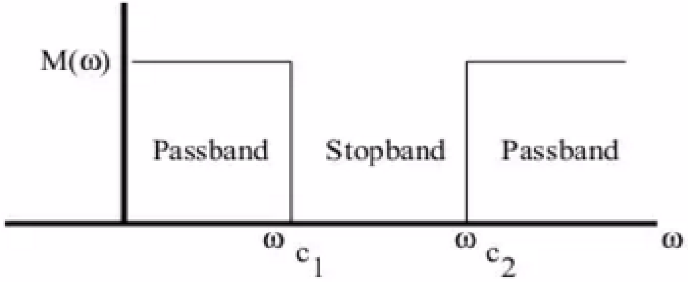
Lowpass Filter



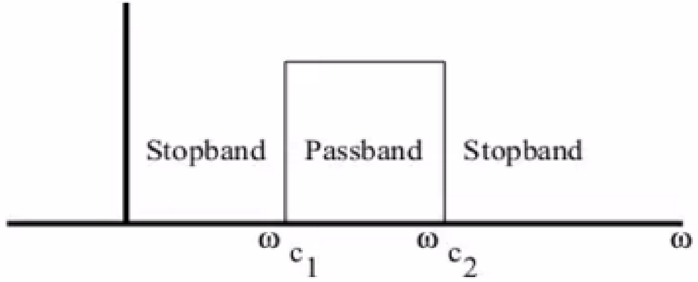
Highpass Filter

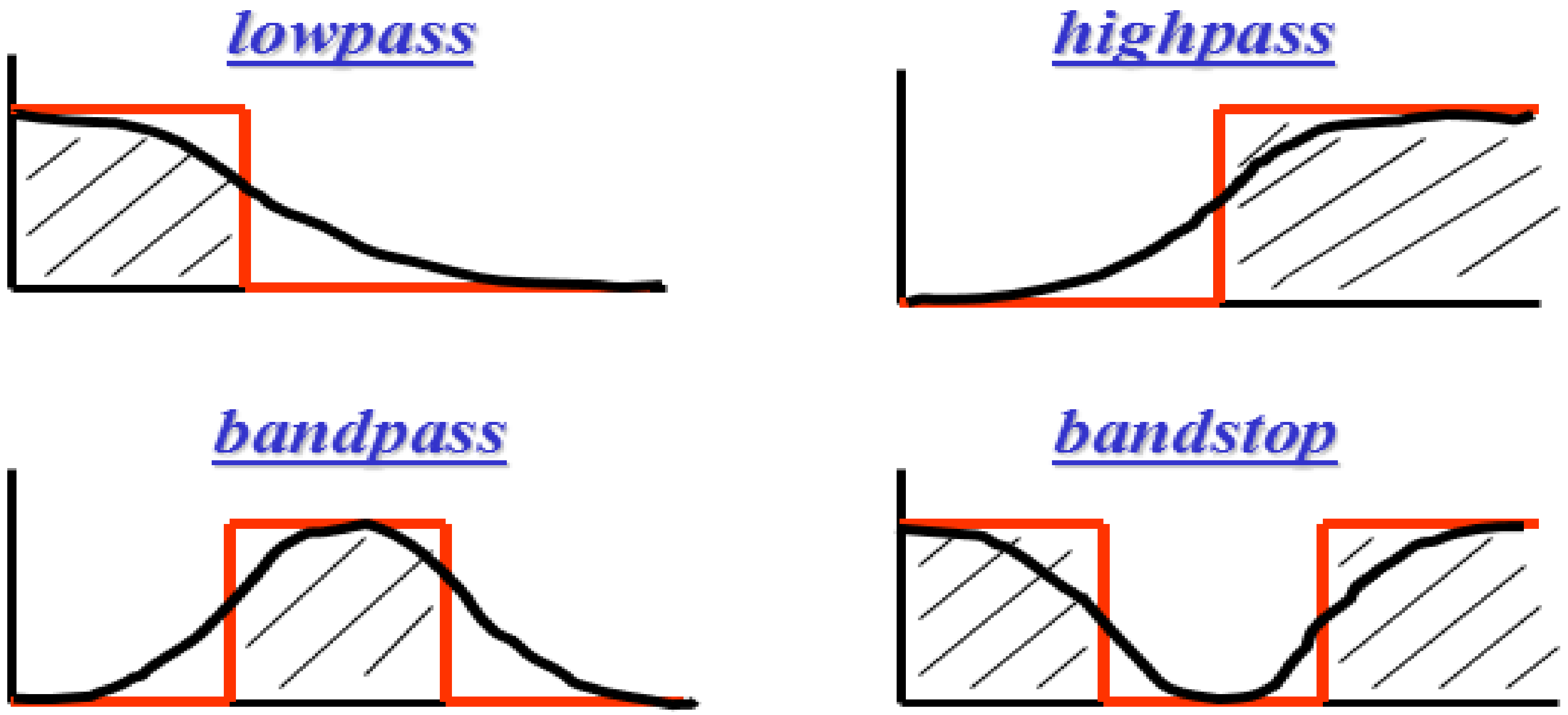


Bandstop Filter



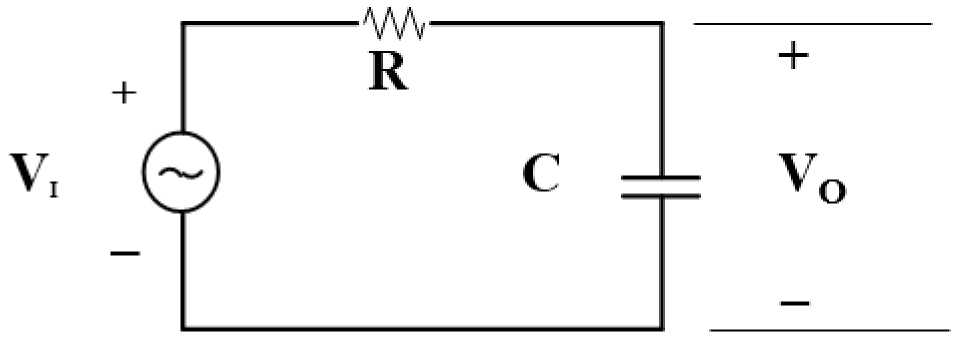
Bandpass Filter




lowpass
highpass
bandpass
bandstop

Low Pass Filter

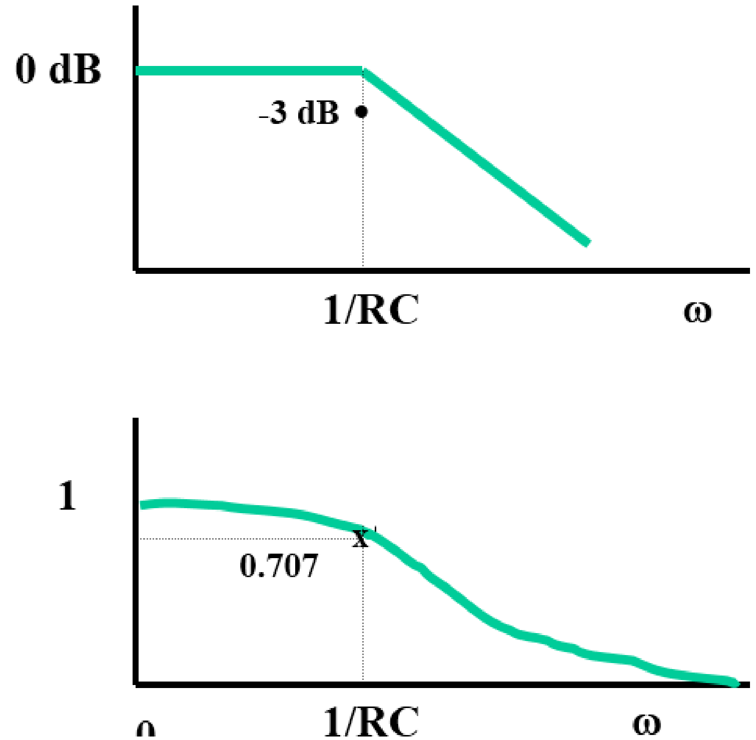
Consider the circuit below.



Low pass filter circuit

$$\frac{V_o(jw)}{V_i(jw)} = \frac{\frac{1}{jwC}}{R+\frac{1}{jwC}} = \frac{1}{1+jwRC}$$

Low Pass Filter



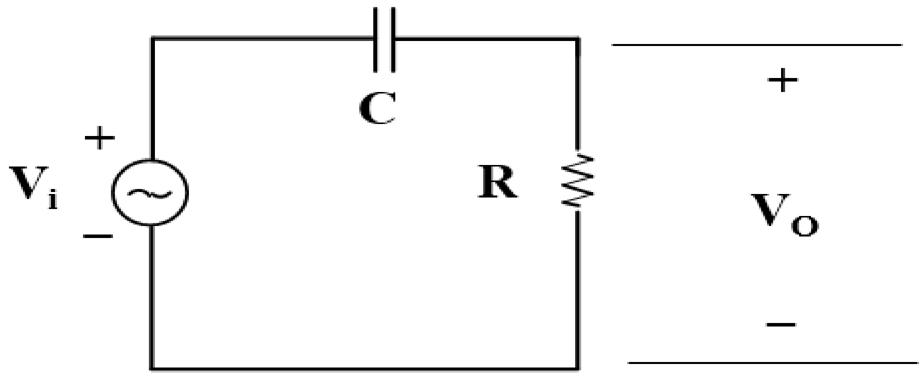
Bode

Linear Plot

Passes low frequencies
Attenuates high frequencies

High Pass Filter

Consider the circuit below.



High Pass Filter

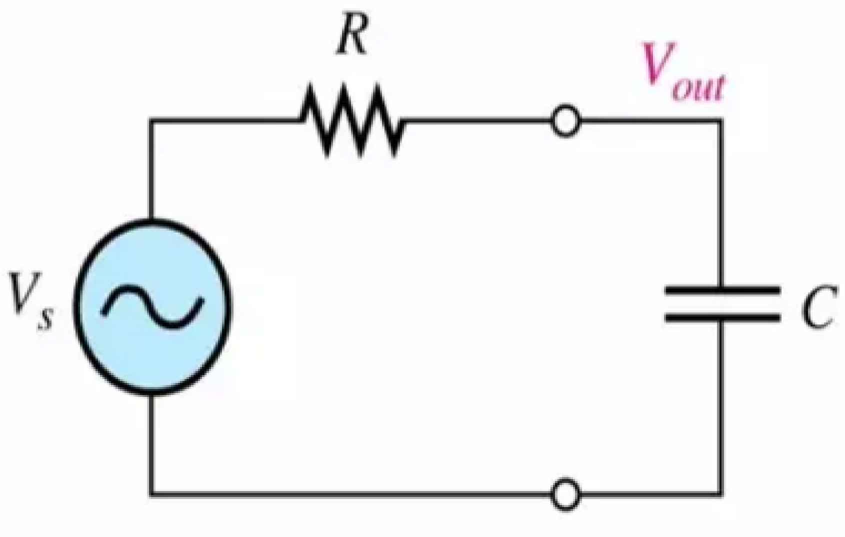
$$\frac{V_o(jw)}{V_i(jw)} = \frac{R}{R+\frac{1}{jwC}} = \frac{jwRC}{1+jwRC}$$

The **bandwidth** of an **ideal** low-pass filter is equal to f_c:

$$BW = f_c$$

The critical frequency of a low-pass RC filter occurs when **X_c = R** and can be calculated using the formula below:

$$f_c = \frac{1}{2\pi RC}$$



(b) Basic low-pass circuit

At critical frequency,

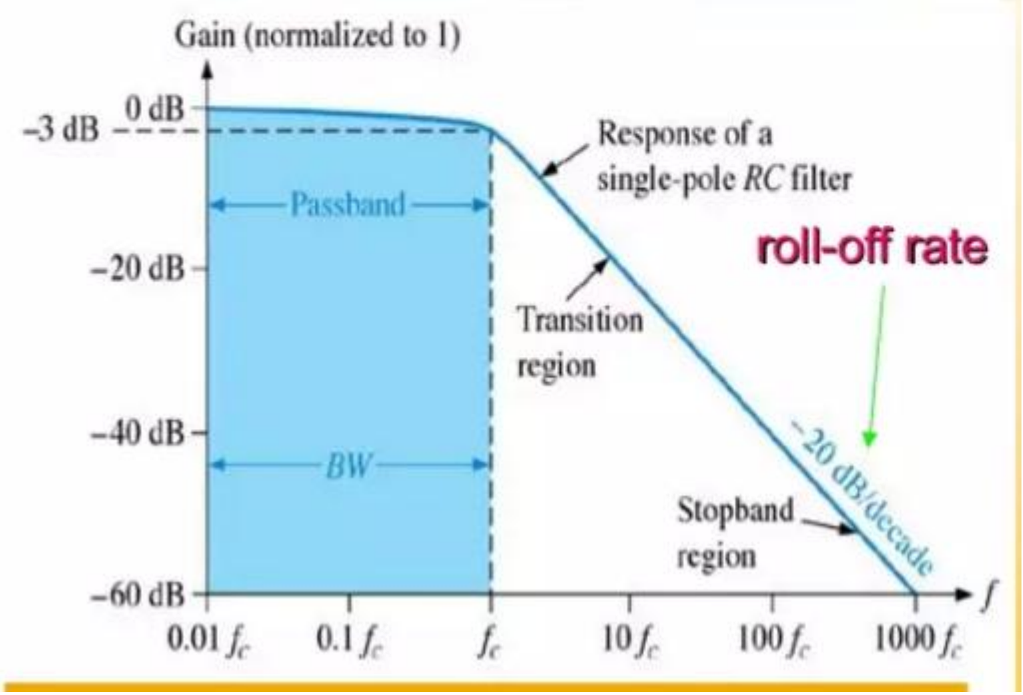
Resistance = Capacitance

$$R = X_c$$

$$R = \frac{1}{\omega_c C}$$

$$R = \frac{1}{2\pi f_c C}$$

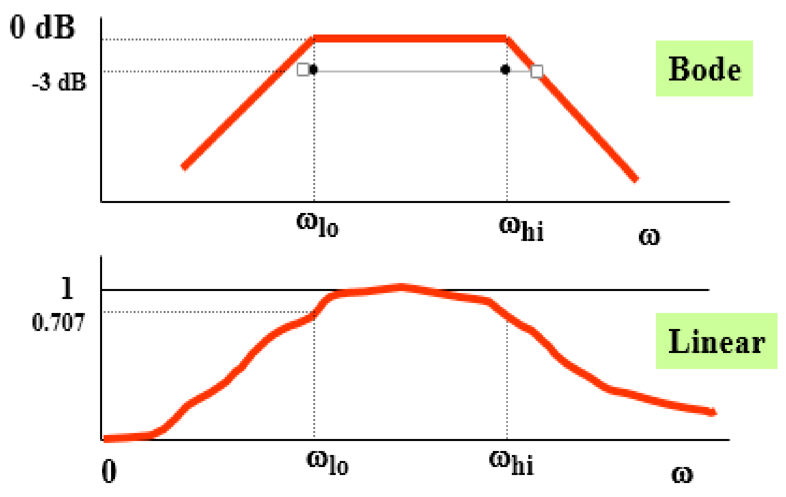
Critical frequency, f_c, (also called the cutoff frequency) defines the end of the passband and normally specified at the point where the response drops – 3 dB (70.7%) from the passband response.

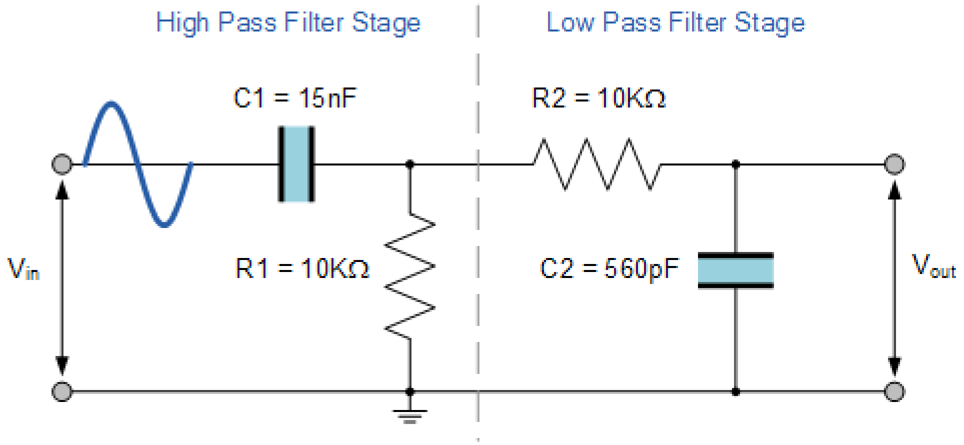


Band Pass Filter

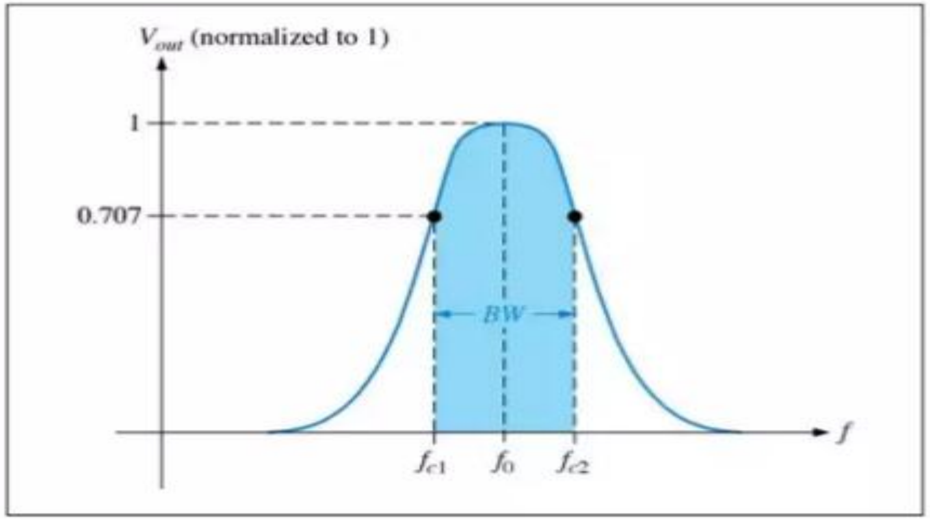
Bandpass Pass Filter

We can make a bandpass from the previous equation and select the poles where we like. In a typical case we have the following shapes.

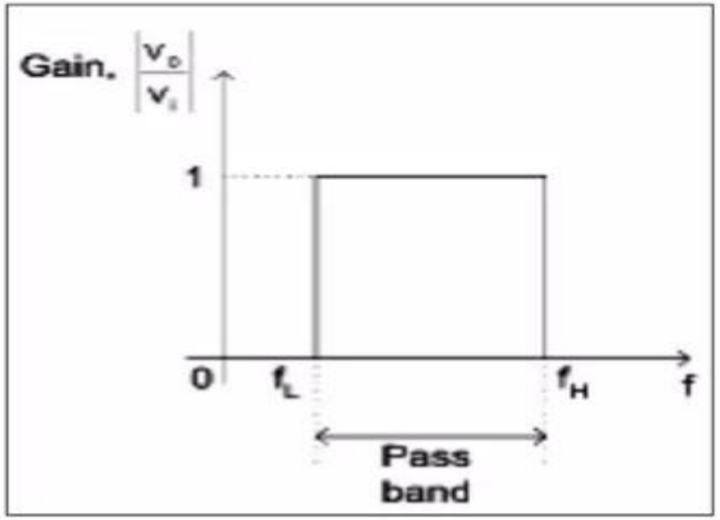




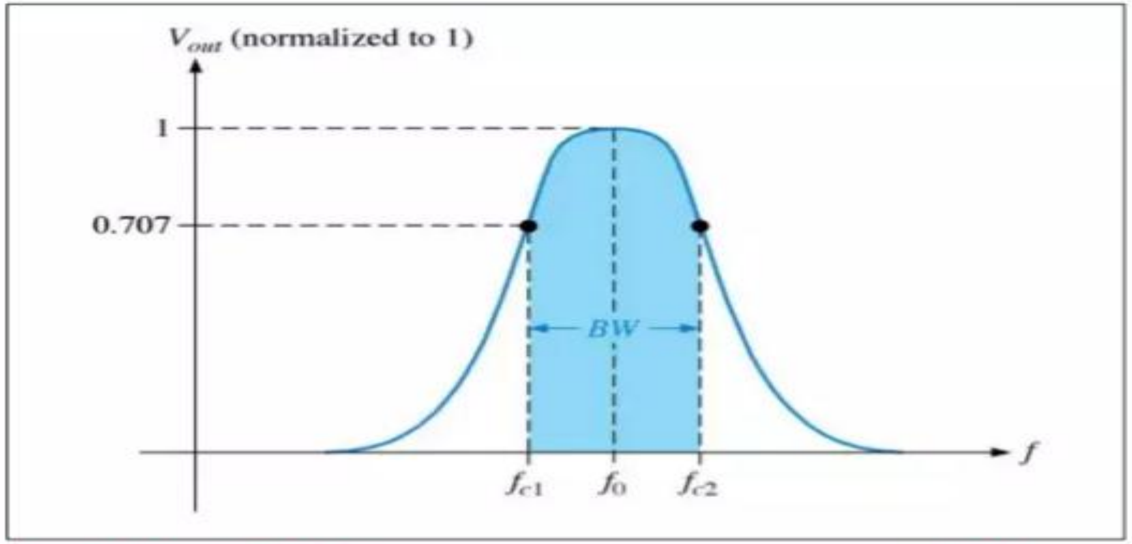
➢ A **band-pass filter** passes all signals lying within a band between a **lower-frequency limit** and **upper-frequency limit** and essentially rejects all other frequencies that are outside this specified band.



Actual response



Ideal response



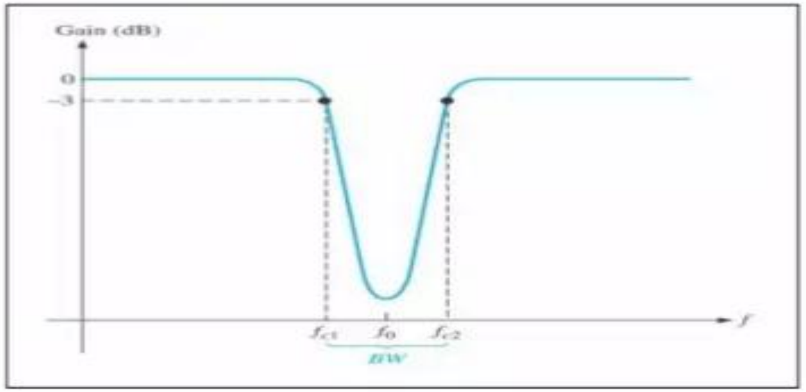
➢ The **bandwidth (BW)** is defined as the **difference** between the **upper critical frequency (f_{c2})** and the **lower critical frequency (f_{c1})**.

$$BW = f_{c2} - f_{c1}$$

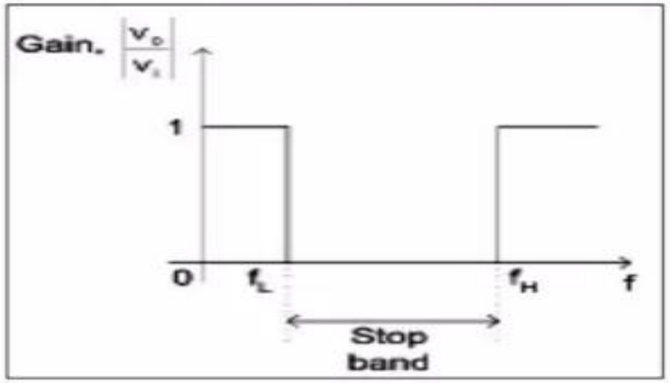
- The frequency about which the pass band is centered is called the ***center frequency***, ***f_o*** and defined as the geometric mean of the critical frequencies.

$$f_o = \sqrt{f_{c1} f_{c2}}$$

Band Stop Filter



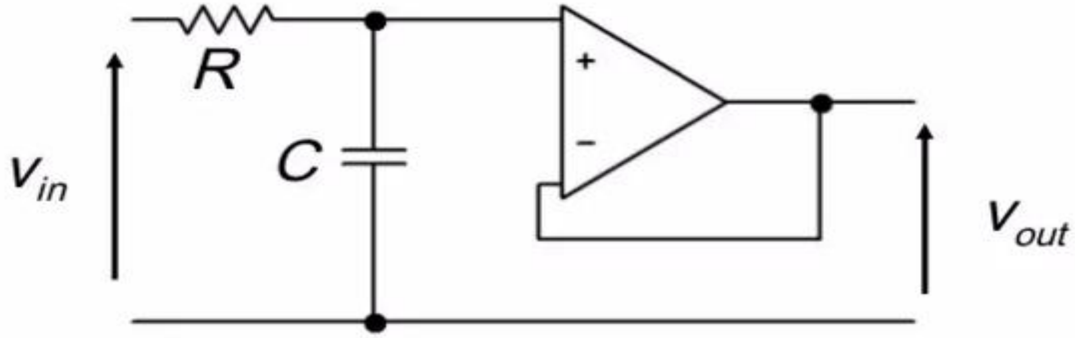
Actual response



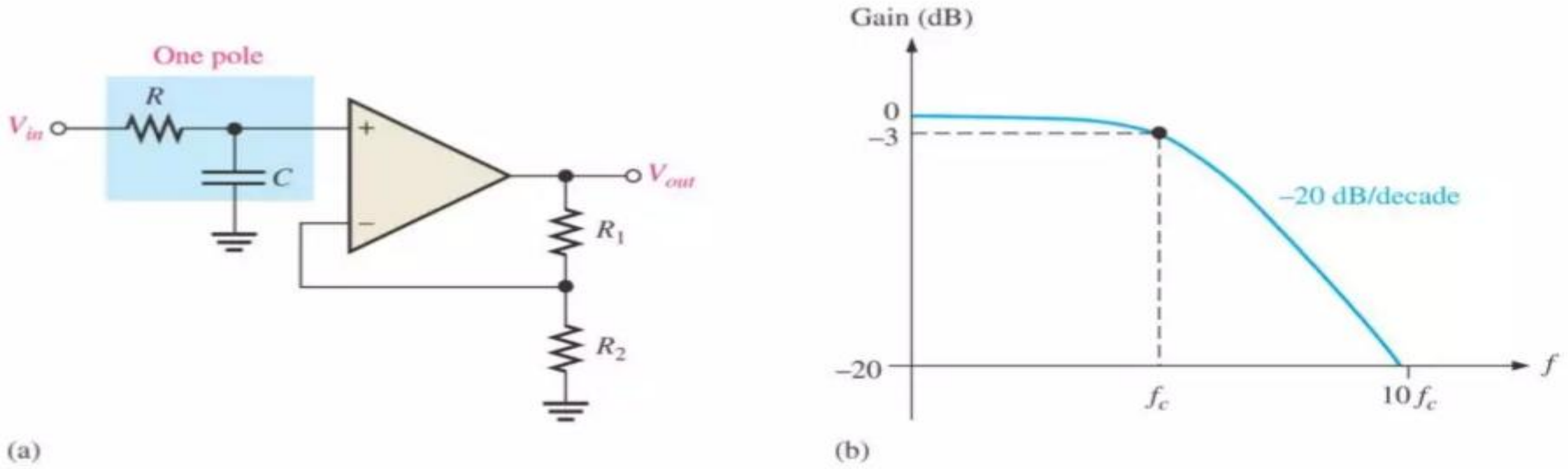
Ideal response

- **Band-stop filter** is a filter which its operation is **opposite** to that of the band-pass filter because the frequencies **within** the bandwidth are **rejected**, and the frequencies above f_{c1} and f_{c2} are **passed**.

- For the band-stop filter, the **bandwidth** is a band of frequencies between the 3 dB points, just as in the case of the band-pass filter response.



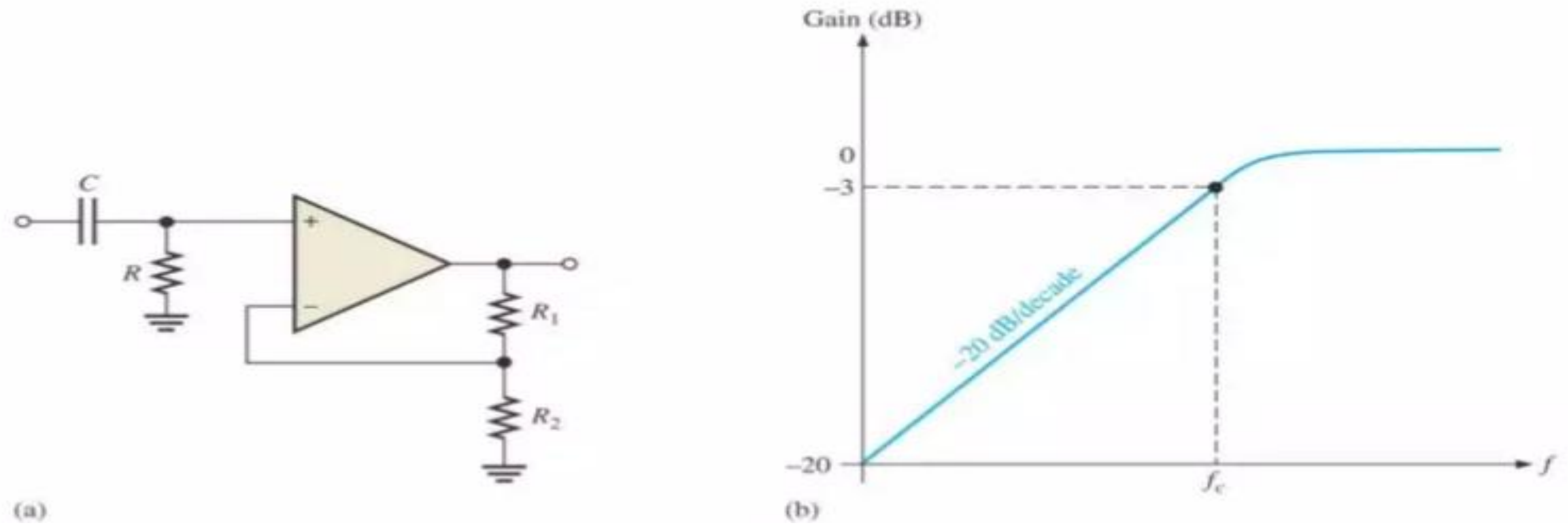
- Same frequency response as passive filter.
- Buffer amplifier does not load RC network.
- Output impedance is now zero.

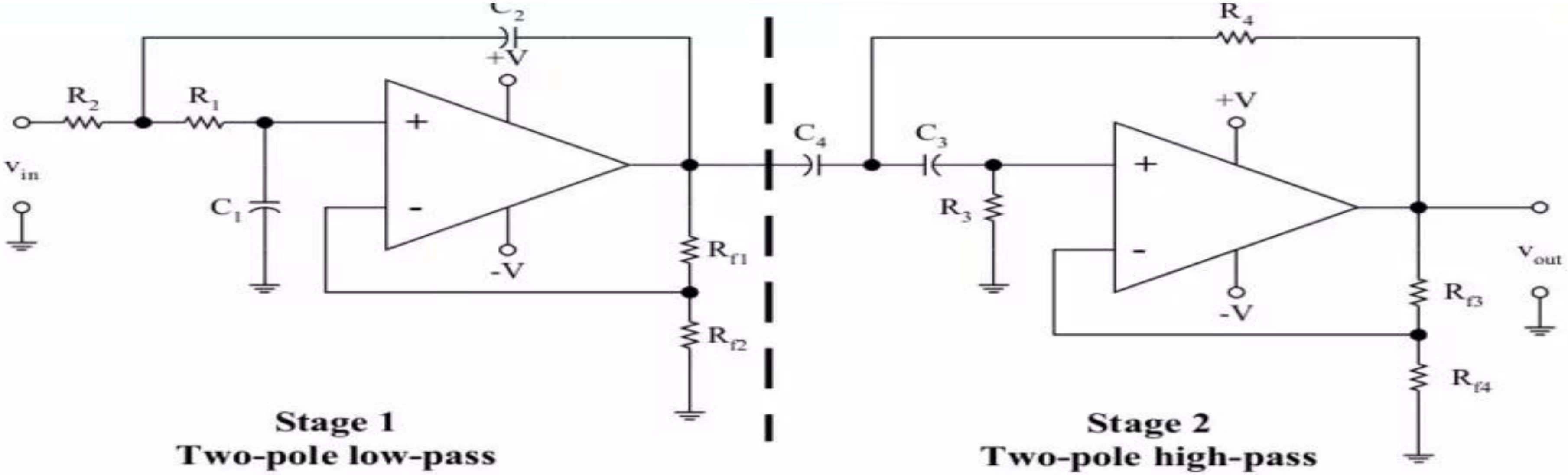


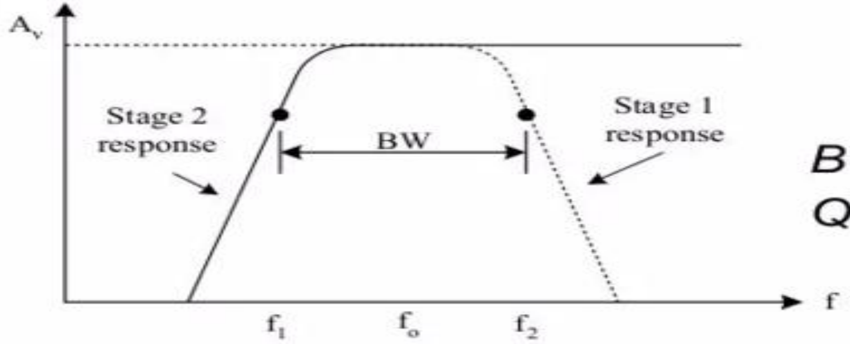
Single-pole active low-pass filter and response curve.

➢ This filter provides a roll-off rate of -20 dB/decade above the critical frequency.

- In high-pass filters, the roles of the **capacitor** and **resistor** are **reversed** in the RC circuits as shown from Figure (a). The negative feedback circuit is the same as for the low-pass filters.
- Figure (b) shows a high-pass active filter with a -20dB/decade roll-off





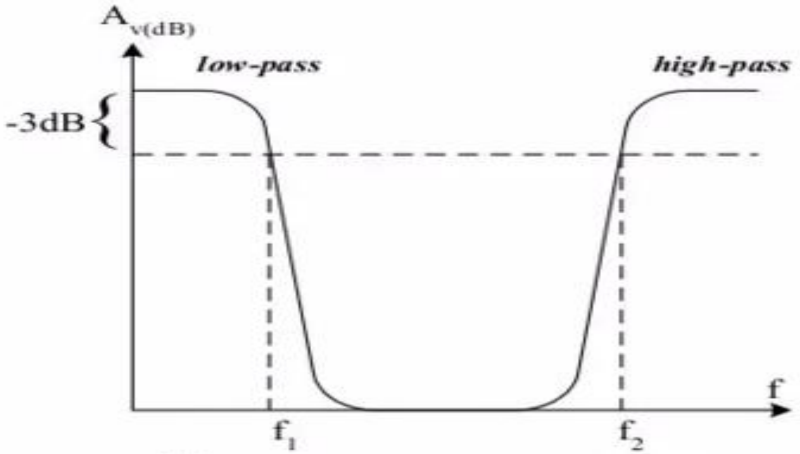


$$BW = f_2 - f_1$$

$$Q = f_0 / BW$$

The notch filter is designed to block all frequencies that fall within its bandwidth. The circuit is made up of a ***high pass filter***, a ***low-pass filter*** and a ***summing amplifier***. The summing amplifier will have an output that is equal to the sum of the filter output voltages.

Block diagram



Frequency response

(I) Low - Pass Filter

$$|H(j\omega)| = 1 \qquad f < f_o$$

$$|H(j\omega)| = 0 \qquad f > f_o$$

(II) High - Pass Filter

$$|H(j\omega)| = 0 \qquad f < f_o$$

$$|H(j\omega)| = 1 \qquad f > f_o$$

(III) Band - Pass Filter

$$|H(j\omega)| = 1 \qquad f_L < f < f_H$$

$$|H(j\omega)| = 0 \qquad f < f_L \text{ and } f > f_H$$

(IV) Band - Stop (Notch) Filter

$$|H(j\omega)| = 0 \qquad f_L < f < f_H$$

$$|H(j\omega)| = 1 \qquad f < f_L \text{ and } f > f_H$$

(V) All - Pass (or phase - shift) Filter

$$|H(j\omega)| = 1 \qquad \text{for all } f$$

has a specific phase response

Advantages of active filters over passive filters (R, L, and C elements only):

1. By containing the op-amp, active filters can be designed to provide required gain, and hence **no signal attenuation** as the signal passes through the filter.
2. **No loading problem**, due to the high input impedance of the op-amp prevents excessive loading of the driving source, and the low output impedance of the op-amp prevents the filter from being affected by the load that it is driving.
3. **Easy to adjust over a wide frequency range** without altering the desired response.