19

BASIC FILTERS

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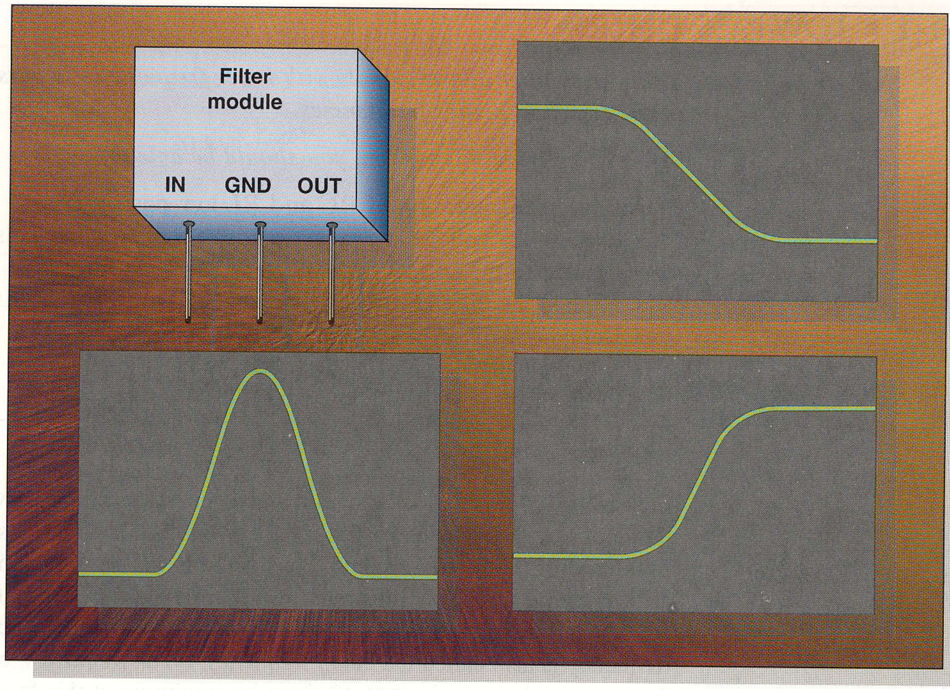
INTRODUCTION

The concept of filters was introduced in Chapters 16, 17, and 18 to illustrate applications of *RC, RL,* and *RLC* circuits. This chapter is essentially an extension of the earlier material and provides additional coverage of the important topic of filters.

Passive filters are discussed in this chapter. Passive filters use various combinations of resistors, capacitors, and inductors. In a later course, you will study active filters which use passive components combined with amplifiers. You have already seen how basic *RC, RL,* and *RLC* circuits can be used as filters. Now, you will learn that passive filters can be placed in four general categories according to their response characteristics: low-pass, high-pass, band-pass, and band-stop. Within each category, there are several common types that will be examined.

In the TECH TIP assignment in Section 19–5, you will plot the frequency responses of filters based on oscilloscope measurements and identify the types of filters.

■ **TECHnology**
Theory
Into
Practice



■ CHAPTER OBJECTIVES

- Analyze the operation of *RC* and *RL* low-pass filters
- Analyze the operation of *RC* and *RL* high-pass filters
- Analyze the operation of band-pass filters
- Analyze the operation of band-stop filters

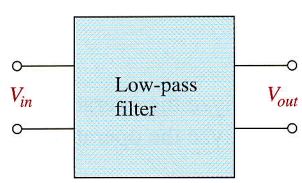
19–1 ■ LOW-PASS FILTERS

A low-pass filter allows signals with lower frequencies to pass from input to output while rejecting higher frequencies.

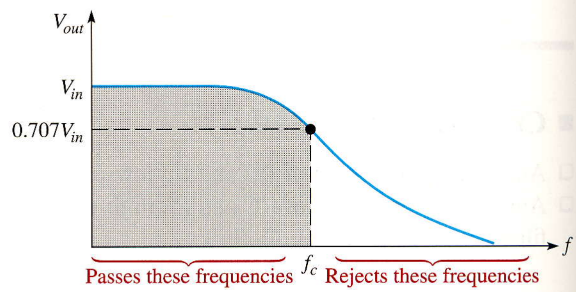
After completing this section, you should be able to

- **Analyze the operation of *RC* and *RL* low-pass filters**
  - Express the voltage and power ratios of a filter in decibels
  - Determine the critical frequency of a low-pass filter
  - Explain the difference between actual and ideal low-pass response curves
  - Define *roll-off*
  - Generate a Bode plot for a low-pass filter
  - Discuss phase shift in a low-pass filter

Figure 19–1 shows a block diagram and a general response curve for a low-pass filter. The range of low frequencies passed by a low-pass filter within a specified limit is called the **passband** of the filter. The point considered to be the upper end of the passband is at the critical frequency, f_c, as illustrated in Figure 19–1(b). The **critical frequency** is the frequency at which the filter's output voltage is 70.7% of the maximum. The filter's critical frequency is also called the *cutoff frequency, break frequency,* or *−3 dB frequency* because the output voltage is down 3 dB from its maximum at this frequency. The term *dB (decibel)* is a commonly used one that you should understand because the decibel unit is used in filter measurements.



(a)



(b)

FIGURE 19–1
Low-pass filter block diagram and general response curve.

Decibels

The basis for the decibel unit stems from the logarithmic response of the human ear to the intensity of sound. The **decibel** is a logarithmic measurement of the ratio of one power to another or one voltage to another, which can be used to express the input-to-output relationship of a filter. The following equation expresses a power ratio in decibels:

$$\text{dB} = 10 \log\left(\frac{P_{out}}{P_{in}}\right) \quad \textbf{(19–1)}$$

From the properties of logarithms, the following decibel formula for a voltage ratio is derived. This formula is correct only when both voltages are measured in the same impedance.

$$\text{dB} = 20 \log\left(\frac{V_{out}}{V_{in}}\right) \quad \textbf{(19–2)}$$

EXAMPLE 19–1 At a certain frequency, the output voltage of a filter is 5 V and the input is 10 V. Express the voltage ratio in decibels.

Solution

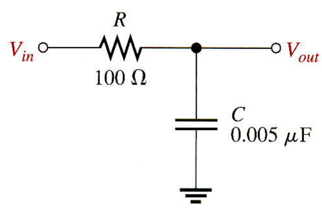
$$20\log\left(\frac{V_{out}}{V_{in}}\right) = 20\log\left(\frac{5\text{ V}}{10\text{ V}}\right) = 20\log(0.5) = \mathbf{-6.02\ dB}$$

Related Problem Express the ratio $V_{out}/V_{in} = 0.85$ in decibels.

RC Low-Pass Filter

A basic *RC* low-pass filter is shown in Figure 19–2. Notice that the output voltage is taken across the capacitor.

FIGURE 19–2



When the input is dc (0 Hz), the output voltage equals the input voltage because X_C is infinitely large. As the input frequency is increased, X_C decreases and, as a result, V_{out} gradually decreases until a frequency is reached where $X_C = R$. This is the critical frequency, f_c, of the filter.

$$X_C = R$$

$$\frac{1}{2\pi f_c C} = R$$

$$f_c = \frac{1}{2\pi RC} \qquad \textbf{(19–3)}$$

At any frequency, the output voltage magnitude is

$$V_{out} = \left(\frac{X_C}{\sqrt{R^2 + X_C^2}}\right)V_{in}$$

by application of the voltage-divider formula. Since $X_C = R$ at f_c, the output voltage at the critical frequency can be expressed as

$$V_{out} = \left(\frac{R}{\sqrt{R^2 + R^2}}\right)V_{in} = \left(\frac{R}{\sqrt{2R^2}}\right)V_{in} = \left(\frac{R}{R\sqrt{2}}\right)V_{in} = \left(\frac{1}{\sqrt{2}}\right)V_{in} = 0.707V_{in}$$

These calculations show that the output is 70.7% of the input when $X_C = R$. The frequency at which this occurs is, by definition, the critical frequency.

The ratio of output voltage to input voltage at the critical frequency can be expressed in decibels as follows:

$$V_{out} = 0.707V_{in}$$

$$\frac{V_{out}}{V_{in}} = 0.707$$

$$20\log\left(\frac{V_{out}}{V_{in}}\right) = 20\log(0.707) = -3\text{ dB}$$

EXAMPLE 19–2 Determine the critical frequency for the low-pass RC filter in Figure 19–2.

Solution

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(100\ \Omega)(0.005\ \mu\text{F})} = \mathbf{318\ kHz}$$

The output voltage is 3 dB below V_{in} at this frequency (V_{out} has a maximum value of V_{in}).

Related Problem A certain low-pass RC filter has $R = 1.0\ \text{k}\Omega$ and $C = 0.022\ \mu\text{F}$. Determine its critical frequency.

"Roll-Off" of the Response Curve

The blue line in Figure 19–3 shows an actual response curve for a low-pass filter. The maximum output is defined to be 0 dB as a reference. Zero decibels corresponds to $V_{out} = V_{in}$, because $20 \log(V_{out}/V_{in}) = 20 \log 1 = 0$ dB. The output drops from 0 dB to −3 dB at the critical frequency and then continues to decrease at a fixed rate. This pattern of decrease is called the **roll-off** of the frequency response. The red line shows an ideal output response that is considered to be "flat" out to f_c. The output then decreases at the fixed rate.

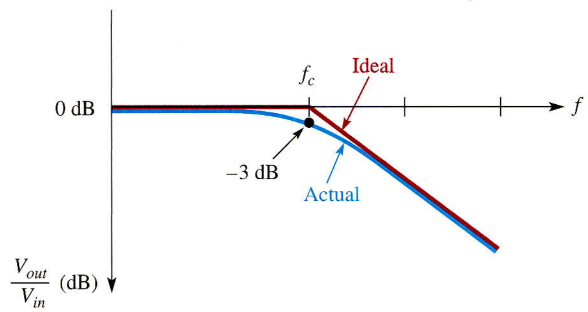


FIGURE 19–3
Actual and ideal response curves for a low-pass filter.

As you have seen, the output voltage of a low-pass filter decreases by 3 dB when the frequency is increased to the critical value f_c. As the frequency continues to increase above f_c, the output voltage continues to decrease. In fact, for each tenfold increase in frequency above f_c, there is a 20 dB reduction in the output, as shown in the following steps.

Let's take a frequency that is ten times the critical frequency ($f = 10f_c$). Since $R = X_C$ at f_c, then $R = 10X_C$ at $10f_c$ because of the inverse relationship of X_C and f.

The **attenuation** of the RC circuit is the ratio V_{out}/V_{in} and is developed as follows:

$$\frac{V_{out}}{V_{in}} = \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{X_C}{\sqrt{(10X_C)^2 + X_C^2}}$$

$$= \frac{X_C}{\sqrt{100X_C^2 + X_C^2}} = \frac{X_C}{\sqrt{X_C^2(100 + 1)}} = \frac{X_C}{\sqrt{X_C}101} = \frac{1}{\sqrt{101}} \cong \frac{1}{10} = 0.1$$

The dB attenuation is

$$20 \log\left(\frac{V_{out}}{V_{in}}\right) = 20 \log(0.1) = -20\ \text{dB}$$

A tenfold change in frequency is called a **decade.** So, for the *RC* circuit, the output voltage is reduced by 20 dB for each decade increase in frequency. A similar result can be derived for the high-pass circuit. The roll-off is a constant –20 dB/decade for a basic *RC* or *RL* filter. Figure 19–4 shows the ideal frequency response plot on a semilog scale, where each interval on the horizontal axis represents a tenfold increase in frequency. This response curve is called a **Bode plot.**

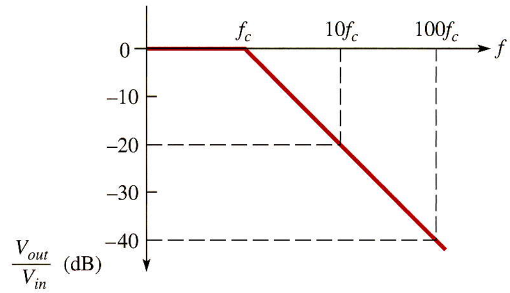
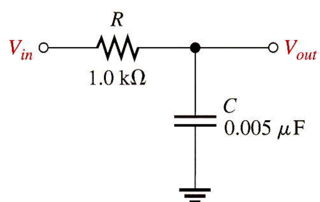


FIGURE 19–4
Frequency roll-off for a low-pass RC filter (Bode plot).

EXAMPLE 19–3 Make a Bode plot for the filter in Figure 19–5 for three decades of frequency. Use semilog graph paper.

FIGURE 19–5



Solution The critical frequency for this low-pass filter is

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(1.0\text{ k}\Omega)(0.005\ \mu\text{F})} = 31.8\text{ kHz}$$

The idealized Bode plot is shown with the red line on the semilog graph in Figure 19–6. The approximate actual response curve is shown with the blue line. Notice first that the horizontal scale is logarithmic and the vertical scale is linear. The frequency is on the logarithmic scale, and the filter output in decibels is on the vertical.

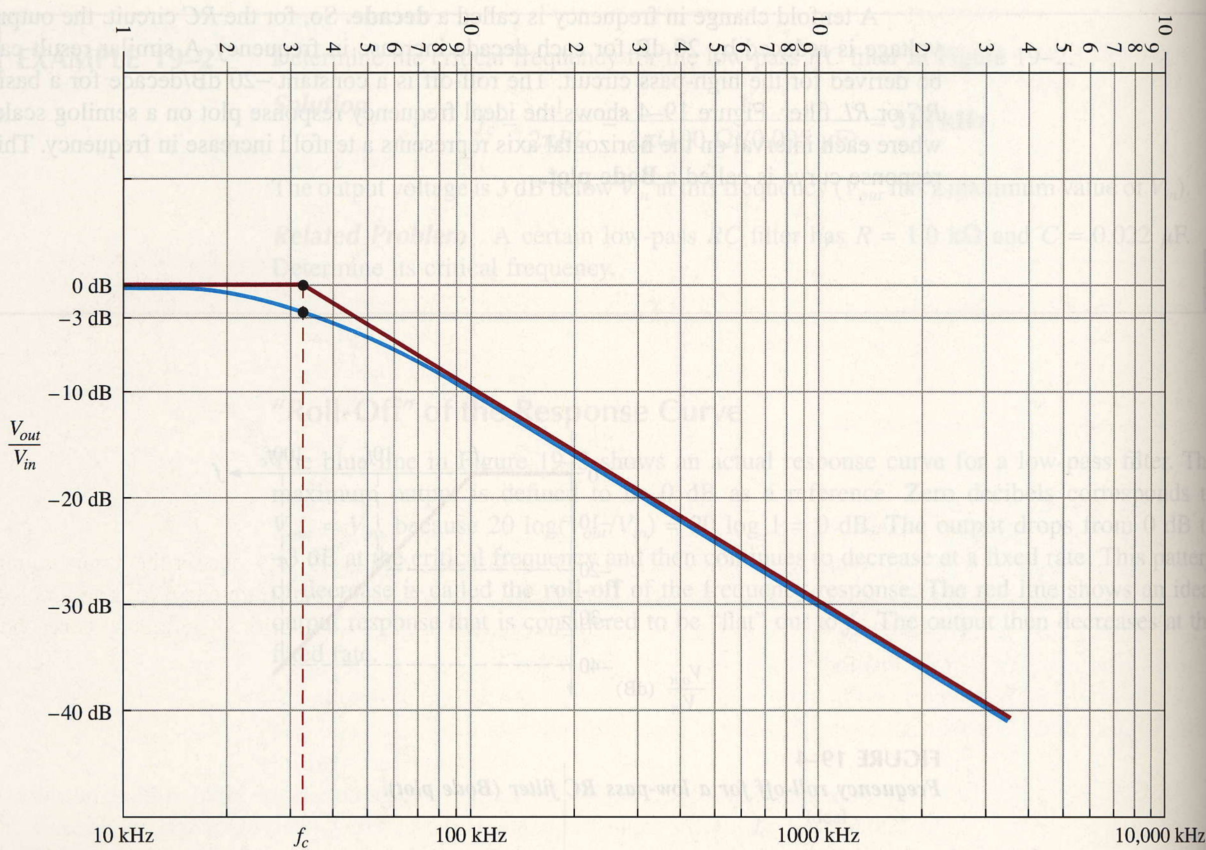


FIGURE 19–6

Bode plot for Figure 19–5. The red line represents the ideal response curve and the blue line represents the actual response.

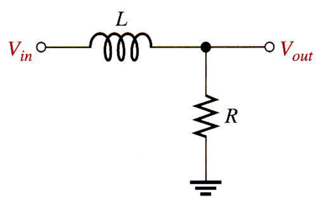
The output is flat below f_c (31.8 kHz). As the frequency is increased above f_c, the output drops at a −20 dB/decade rate. Thus, for the ideal curve, every time the frequency is increased by ten, the output is reduced by 20 dB. A slight variation from this occurs in actual practice. The output is actually at −3 dB rather than 0 dB at the critical frequency.

Related Problem What happens to the critical frequency and roll-off rate if C is reduced to 0.001 μF in Figure 19–5?

RL Low-Pass Filter

A basic *RL* low-pass filter is shown in Figure 19–7. Notice that the output voltage is taken across the resistor.

FIGURE 19–7
***RL* low-pass filter.**



When the input is dc (0 Hz), the output voltage ideally equals the input voltage because X_L is a short (if R_W is neglected). As the input frequency is increased, X_L increases and, as a result, V_{out} gradually decreases until the critical frequency is reached. At this point, $X_L = R$ and the frequency is

$$2\pi f_c L = R$$

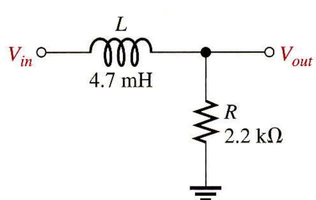
$$f_c = \frac{R}{2\pi L}$$

$$f_c = \frac{1}{2\pi(L/R)} \qquad \textbf{(19–4)}$$

Just as in the *RC* low-pass filter, $V_{out} = 0.707V_{in}$ and, thus, the output voltage is –3 dB below the input voltage at the critical frequency.

EXAMPLE 19–4 Make a Bode plot for the filter in Figure 19–8 for three decades of frequency. Use semilog graph paper.

FIGURE 19–8



Solution The critical frequency for this low-pass filter is

$$f_c = \frac{1}{2\pi(L/R)} = \frac{1}{2\pi(4.7\text{ mH}/2.2\text{ k}\Omega)} = 74.5\text{ kHz}$$

The idealized Bode plot is shown with the red line on the semilog graph in Figure 19–9. The approximate actual response curve is shown with the blue line. Notice first that the horizontal scale is logarithmic and the vertical scale is linear. The frequency is on the logarithmic scale, and the filter output in decibels is on the vertical.

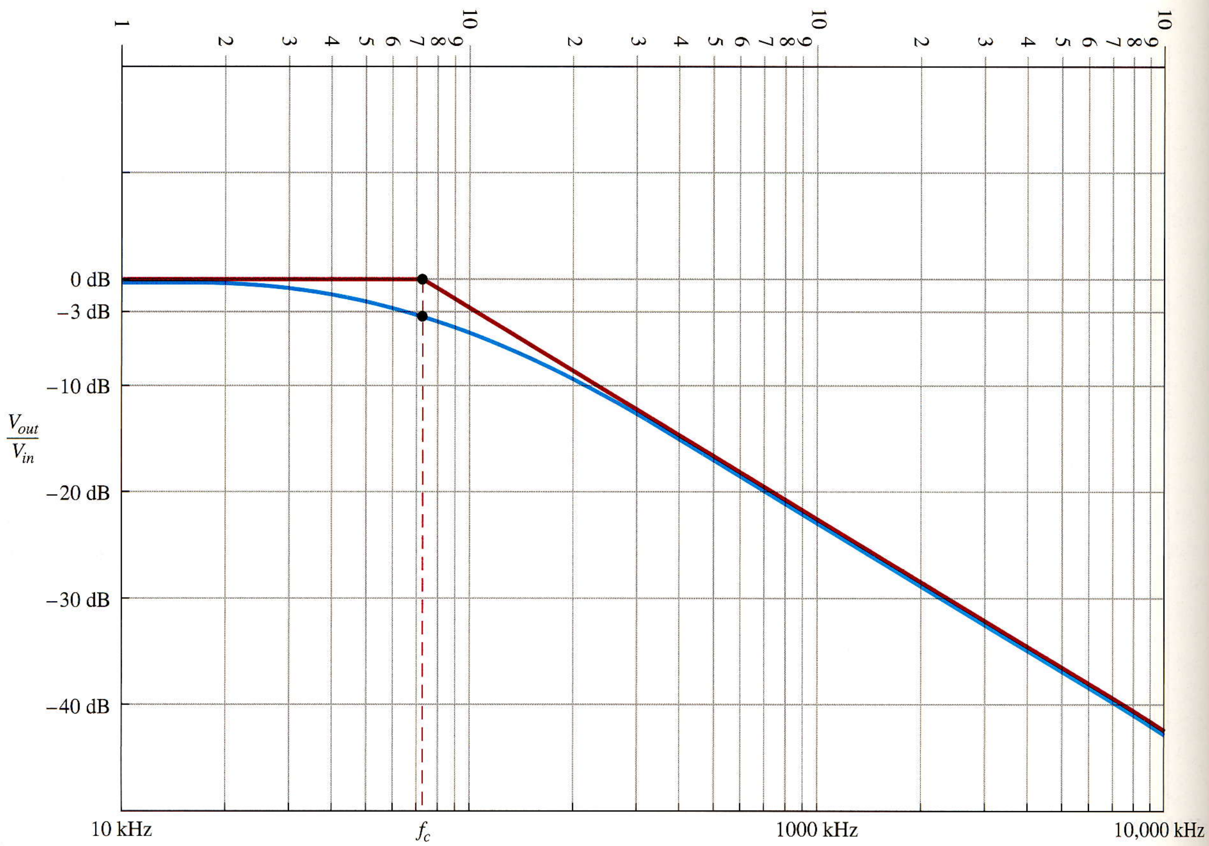


FIGURE 19–9

Bode plot for Figure 19–8. The red line is the ideal response curve and the blue line is the actual response curve.

The output is flat below f_c (74.5 kHz). As the frequency is increased above f_c, the output drops at a −20 dB/decade rate. Thus, for the ideal curve, every time the frequency is increased by ten, the output is reduced by 20 dB. A slight variation from this occurs in actual practice. The output is actually at −3 dB rather than 0 dB at the critical frequency.

Related Problem What happens to the critical frequency and roll-off rate if L is reduced to 1 mH in Figure 19–8?

Phase Shift in a Low-Pass Filter

The *RC* low-pass filter acts as a lag network. Recall from Chapter 16 that the phase shift from input to output is expressed as

$$\phi = -90^\circ + \tan^{-1}\left(\frac{X_C}{R}\right)$$

At the critical frequency, $X_C = R$ and, therefore, $\phi = -45^\circ$. As the input frequency is reduced, ϕ decreases and approaches 0° when the frequency approaches zero. Figure 19–10, illustrates this phase characteristic.

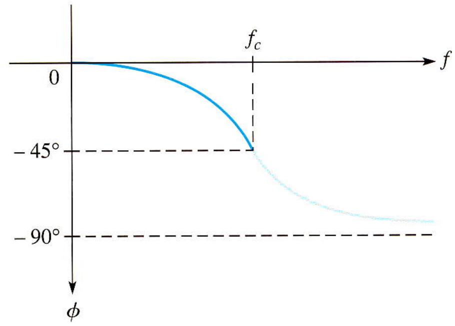


FIGURE 19–10
Phase characteristic of a low-pass filter.

The *RL* low-pass filter also acts as a lag network. Recall from Chapter 17 that the phase shift is expressed as

$$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$$

As in the *RC* filter, the phase shift from input to output is –45° at the critical frequency and decreases for frequencies below f_c.

SECTION 19–1 REVIEW

1. In a certain low-pass filter, $f_c = 2.5$ kHz. What is its passband?
2. In a certain low-pass filter, $R = 100\ \Omega$ and $X_C = 2\ \Omega$ at a frequency, f_1. Determine $\mathbf{V}_{out}$ at f_1 when $\mathbf{V}_{in} = 5\angle 0°$ V rms.
3. $V_{out} = 400$ mV, and $V_{in} = 1.2$ V. Express the ratio V_{out}/V_{in} in dB.

19–2 ■ HIGH-PASS FILTERS

A high-pass filter allows signals with higher frequencies to pass from input to output while rejecting lower frequencies.

After completing this section, you should be able to

- **Analyze the operation of *RC* and *RL* high-pass filters**
  - Determine the critical frequency of a high-pass filter
  - Explain the difference between actual and ideal response curves
  - Generate a Bode plot for a high-pass filter
  - Discuss phase shift in a high-pass filter

Figure 19–11 shows a block diagram and a general response curve for a high-pass filter. The frequency considered to be the lower end of the passband is called the *critical frequency.* Just as in the low-pass filter, it is the frequency at which the output is 70.7% of the maximum, as indicated in the figure.

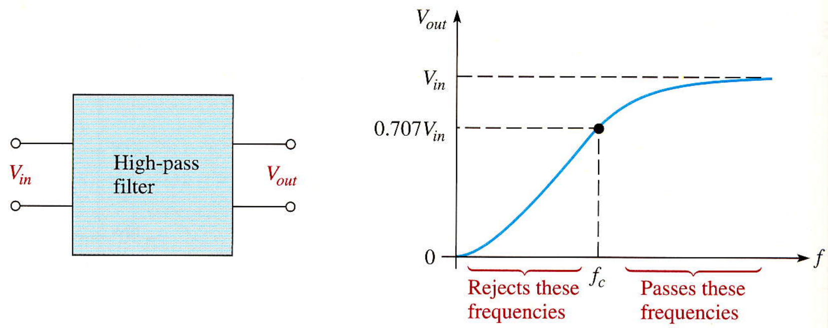
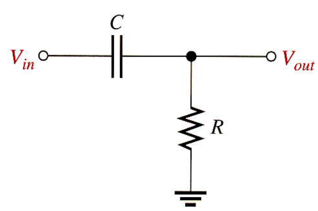


FIGURE 19–11
High-pass filter block diagram and response curve.

RC High-Pass Filter

A basic *RC* high-pass filter is shown in Figure 19–12. Notice that the output voltage is taken across the resistor.

FIGURE 19–12
RC high-pass filter.



When the input frequency is at its critical value, $X_C = R$ and the output voltage is $0.707V_{in}$, just as in the case of the low-pass filter. As the input frequency increases above f_c, X_C decreases and, as a result, the output voltage increases and approaches a value equal to V_{in}. The expression for the critical frequency of the high-pass filter is the same as for the low-pass filter.

$$f_c = \frac{1}{2\pi RC}$$

Below f_c, the output voltage decreases (rolls off) at a rate of −20 dB/decade. Figure 19–13 shows an actual and an ideal response curve for a high-pass filter.

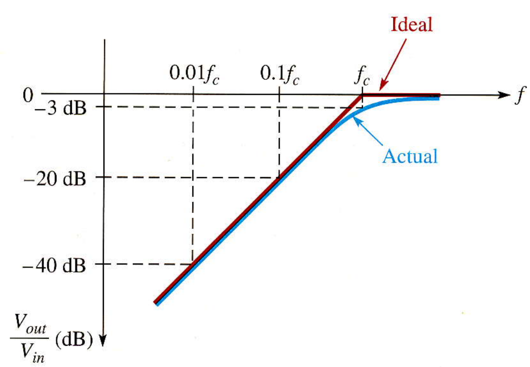
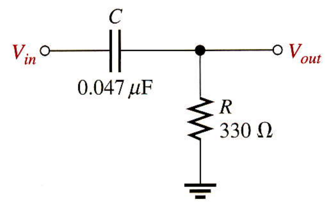


FIGURE 19–13
Actual and ideal response curves for a high-pass filter.

EXAMPLE 19–5

Make a Bode plot for the filter in Figure 19–14 for three decades of frequency. Use semilog graph paper.

FIGURE 19–14



Solution The critical frequency for this high-pass filter is

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(330\ \Omega)(0.047\ \mu\text{F})} = 10.3\ \text{kHz} \cong 10\ \text{kHz}$$

The idealized Bode plot is shown with the red line on the semilog graph in Figure 19–15. The approximate actual response curve is shown with the blue line. Notice first that the horizontal scale is logarithmic and the vertical scale is linear. The frequency is on the logarithmic scale, and the filter output in decibels is on the vertical.

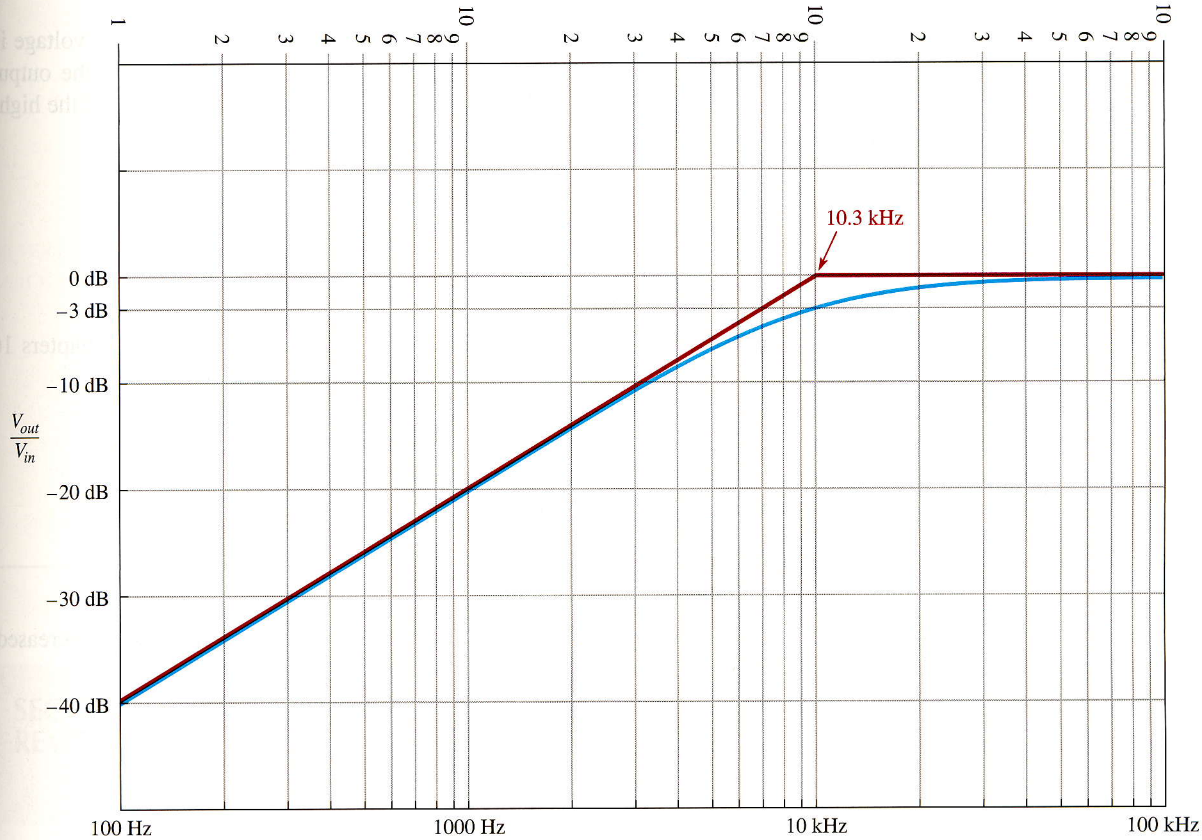


FIGURE 19–15
Bode plot for Figure 19–14. The red line is the ideal response curve and the blue line is the actual response curve.

The output is flat beyond f_c (approximately 10 kHz). As the frequency is reduced below f_c, the output drops at a −20 dB/decade rate. Thus, for the ideal curve, every time the frequency is reduced by ten, the output is reduced by 20 dB. A slight variation from this occurs in actual practice. The output is actually at −3 dB rather than 0 dB at the critical frequency.

Related Problem If the frequency for the high-pass filter is decreased to 10 Hz, what is the output to input ratio in decibels?

RL High-Pass Filter

A basic *RL* high-pass filter is shown in Figure 19–16. Notice that the output is taken across the inductor.

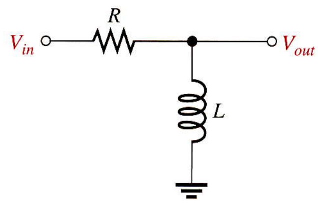


FIGURE 19–16
RL high-pass filter.

When the input frequency is at its critical value, $X_L = R$, and the output voltage is $0.707V_{in}$. As the frequency increases above f_c, X_L increases and, as a result, the output voltage increases until it equals V_{in}. The expression for the critical frequency of the high-pass filter is the same as for the low-pass filter.

$$f_c = \frac{1}{2\pi(L/R)}$$

Phase Shift in a High-Pass Filter

Both the *RC* and the *RL* high-pass filters act as lead networks. Recall from Chapters 16 and 17 that the phase shift from input to output for the *RC* lead network is

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

and for the *RL* lead network is

$$\phi = 90° - \tan^{-1}\left(\frac{X_L}{R}\right)$$

At the critical frequency, $X_L = R$ and, therefore, $\phi = 45°$. As the frequency is increased, ϕ decreases toward 0°, as shown in Figure 19–17.

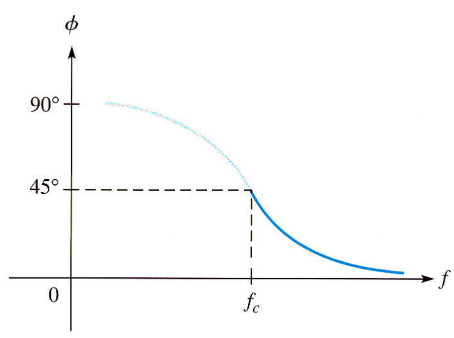


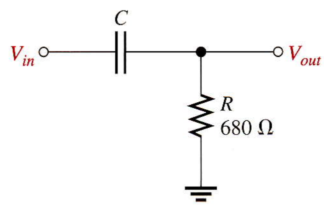
FIGURE 19–17
Phase characteristic of a high-pass filter.

EXAMPLE 19–6

(a) In Figure 19–18, find the value of C so that X_C is ten times less than R at an input frequency of 10 kHz.

(b) If a 5 V sine wave with a dc level of 10 V is applied, what are the output voltage magnitude and the phase shift?

FIGURE 19–18



Solution

(a) The value of C is determined as follows:

$$X_C = 0.1R = 0.1(680\ \Omega) = 68\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(10\text{ kHz})(68\ \Omega)} = \mathbf{0.234\ \mu F}$$

The nearest standard value of C is 0.22 μF.

(b) The magnitude of the sine wave output is determined as follows:

$$V_{out} = \left(\frac{R}{\sqrt{R_2 + X_C^2}}\right)V_{in} = \left(\frac{680\ \Omega}{\sqrt{(680\ \Omega)^2 + (68\ \Omega)^2}}\right)5\text{ V} = \mathbf{4.98\ V}$$

The phase shift is

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{68\ \Omega}{680\ \Omega}\right) = \mathbf{5.7°}$$

At $f =$ 10 kHz, which is a decade above the critical frequency, the sinusoidal output is almost equal to the input in magnitude, and the phase shift is very small. The 10 V dc level has been filtered out and does not appear at the output.

Related Problem Repeat parts (a) and (b) of the example if R is changed to 220 Ω.

SECTION 19–2 REVIEW

1. The input voltage of a high-pass filter is 1 V. What is V_{out} at the critical frequency?
2. In a certain high-pass filter, $\mathbf{V}_{in} = 10\angle 0°$ V, $R =$ 1.0 kΩ, and $X_L =$ 15 kΩ. Determine $\mathbf{V}_{out}$.

19–3 ■ BAND-PASS FILTERS

A band-pass filter allows a certain band of frequencies to pass and attenuates or rejects all frequencies below and above the passband.

After completing this section, you should be able to

- **Analyze the operation of band-pass filters**
  - Show how a band-pass filter is implemented with low-pass and high-pass filters
  - Define *bandwidth*

- Explain the series-resonant band-pass filter
- Explain the parallel-resonant band-pass filter
- Calculate the bandwidth and output voltage of a band-pass filter

Figure 19–19 shows a typical band-pass response curve.

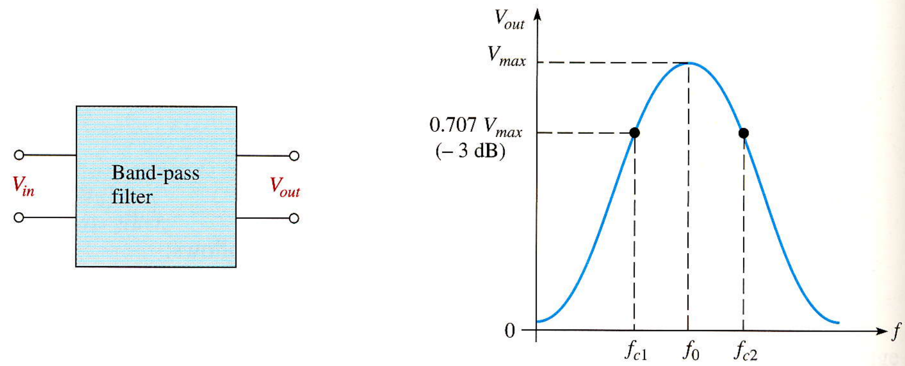


FIGURE 19–19
Typical band-pass response curve.

Low-Pass/High-Pass Filter

A combination of a low-pass and a high-pass filter can be used to form a **band-pass filter,** as illustrated in Figure 19–20. The loading effect of the second filter on the first must be taken into account.

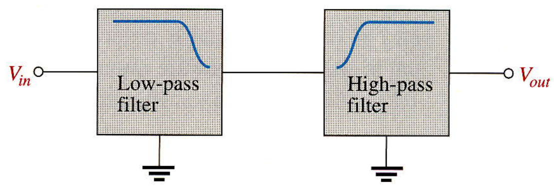


FIGURE 19–20
Low-pass and high-pass filters used to form a band-pass filter.

If the critical frequency, $f_{c(l)}$, of the low-pass filter is higher than the critical frequency, $f_{c(h)}$, of the high-pass filter, the responses overlap. Thus, all frequencies except those between $f_{c(h)}$ and $f_{c(l)}$ are eliminated, as shown in Figure 19–21.

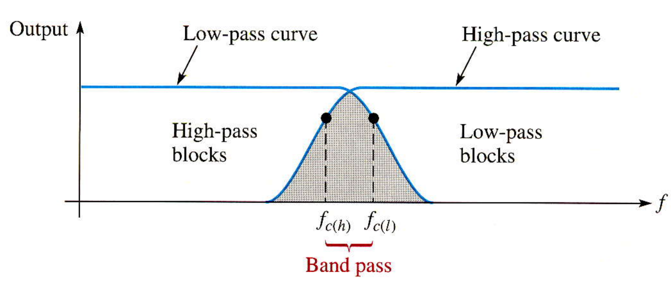


FIGURE 19–21
Overlapping response curves of a high-pass/low-pass filter.

The bandwidth of a band-pass filter is the range of frequencies for which the current, and therefore the output voltage, is equal to or greater than 70.7% of its value at the resonant frequency.

As you know, bandwidth is often abbreviated *BW.*

EXAMPLE 19–7

A high-pass filter with $f_c = 2$ kHz and a low-pass filter with $f_c = 2.5$ kHz are used to construct a band-pass filter. Assuming no loading effect, what is the bandwidth of the passband?

Solution $BW = f_{c(l)} - f_{c(h)} = 2.5\text{ kHz} - 2\text{ kHz} = \mathbf{500\ Hz}$

Related Problem If $f_{c(l)} = 9$ kHz and the bandwidth is 1.5 kHz, what is $f_{c(h)}$?

Series Resonant Band-Pass Filter

A type of series resonant band-pass filter is shown in Figure 19–22. As you learned in Chapter 18, a series resonant circuit has minimum impedance and maximum current at the resonant frequency, f_r. Thus, most of the input voltage is dropped across the resistor at the resonant frequency. Therefore, the output across R has a band-pass characteristic with a maximum output at the frequency of resonance. The resonant frequency is called the **center frequency, f_0.** The bandwidth is determined by the quality factor, Q, of the circuit and the resonant frequency, as was discussed in Chapter 18. Recall that $Q = X_L/R$.

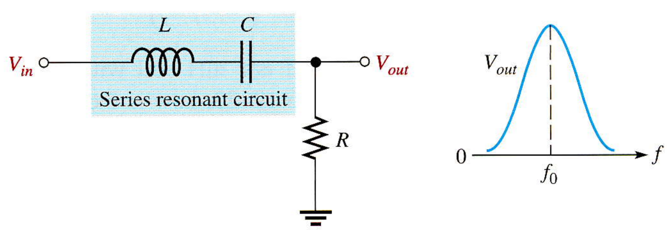


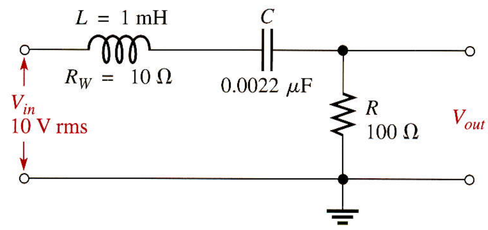
FIGURE 19–22
Series resonant band-pass filter.

A higher value of Q results in a smaller bandwidth. A lower value of Q causes a larger bandwidth. A formula for the bandwidth of a resonant circuit in terms of Q is stated in the following equation:

$$BW = \frac{f_0}{Q} \tag{19–5}$$

EXAMPLE 19–8 Determine the output voltage magnitude at the center frequency (f_0) and the bandwidth for the filter in Figure 19–23.

FIGURE 19–23



Solution At f_0, the impedance of the resonant circuit is equal to the winding resistance, R_W. By the voltage-divider formula,

$$V_{out} = \left(\frac{R}{R + R_W}\right)V_{in} = \left(\frac{100\ \Omega}{110\ \Omega}\right)10\ \text{V} = \mathbf{9.09\ V}$$

The center frequency is

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1\ \text{mH})(0.0022\ \mu\text{F})}} = 107\ \text{kHz}$$

At f_0, the inductive reactance is

$$X_L = 2\pi fL = 2\pi(107\ \text{kHz})(1\ \text{mH}) = 672\ \Omega$$

and the total resistance is

$$R_{tot} = R + R_W = 100\ \Omega + 10\ \Omega = 110\ \Omega$$

Therefore, the circuit Q is

$$Q = \frac{X_L}{R_{tot}} = \frac{672\ \Omega}{110\ \Omega} = 6.11$$

The bandwidth is

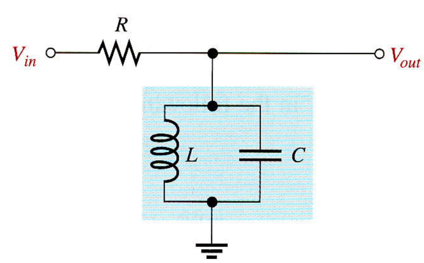
$$BW = \frac{f_0}{Q} = \frac{107\ \text{kHz}}{6.11} = \mathbf{17.5\ kHz}$$

Related Problem If a 1 mH coil with a winding resistance of 18 Ω replaces the existing coil in Figure 19–23, how is the bandwidth affected?

Parallel Resonant Band-Pass Filter

A type of band-pass filter using a parallel resonant circuit is shown in Figure 19–24. Recall that a parallel resonant circuit has maximum impedance at resonance. The circuit in Figure 19–24 acts as a voltage divider. At resonance, the impedance of the tank is much greater than the resistance. Thus, most of the input voltage is across the tank, producing a maximum output voltage at the resonant (center) frequency.

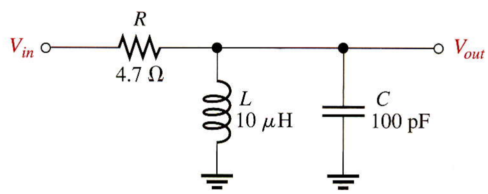
FIGURE 19–24
Parallel resonant band-pass filter.



For frequencies above or below resonance, the tank impedance drops off, and more of the input voltage is across R. As a result, the output voltage across the tank drops off, creating a band-pass characteristic.

EXAMPLE 19–9 What is the center frequency of the filter in Figure 19–25? Assume $R_W = 0\ \Omega$.

FIGURE 19–25



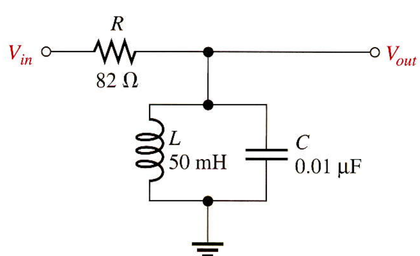
Solution The center frequency of the filter is its resonant frequency.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10\ \mu\text{H})(100\ \text{pF})}} = \mathbf{5.03\ MHz}$$

Related Problem Determine f_0 in Figure 19–25 if C is changed to 1000 pF.

EXAMPLE 19–10 Determine the center frequency and bandwidth for the band-pass filter in Figure 19–26 if the inductor has a winding resistance of 15 Ω.

FIGURE 19–26



Solution Recall from Chapter 18 (Eq. 18–15) that the resonant (center) frequency of a nonideal tank circuit is

$$f_0 = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}} = \frac{\sqrt{1 - (15\ \Omega)^2(0.01\ \mu\text{F})/50\ \text{mH}}}{2\pi\sqrt{(50\ \text{mH})(0.01\ \mu\text{F}}} = \mathbf{7.12\ kHz}$$

The Q of the coil at resonance is

$$Q = \frac{X_L}{R_W} = \frac{2\pi f_0 L}{R_W} = \frac{2\pi(7.12\ \text{kHz})(50\ \text{mH})}{15\ \Omega} = 149$$

The bandwidth of the filter is

$$BW = \frac{f_0}{Q} = \frac{7.12\ \text{kHz}}{149} = \mathbf{47.8\ Hz}$$

Note that since $Q > 10$, the simpler formula (Eq. 18–6) could have been used to calculate f_0.

Related Problem Knowing the value of Q, recalculate f_0 using the simpler formula.

SECTION 19–3 REVIEW

1. For a band-pass filter, $f_{c(h)} = 29.8$ kHz and $f_{c(l)} = 30.2$ kHz. What is the bandwidth?
2. A parallel resonant band-pass filter has the following values: $R_W = 15\ \Omega$, $L = 50\ \mu$H, and $C = 470$ pF. Determine the approximate center frequency.

19–4 ■ BAND-STOP FILTERS

A band-stop filter is essentially the opposite of a band-pass filter in terms of the responses. A band-stop filter allows all frequencies to pass except those lying within a certain stopband.

After completing this section, you should be able to

- **Analyze the operation of band-stop filters**
  - Show how a band-stop filter is implemented with low-pass and high-pass filters
  - Explain the series-resonant band-stop filter
  - Explain the parallel-resonant band-stop filter
  - Calculate the bandwidth and output voltage of a band-stop filter

Figure 19–27 shows a general band-stop response curve.

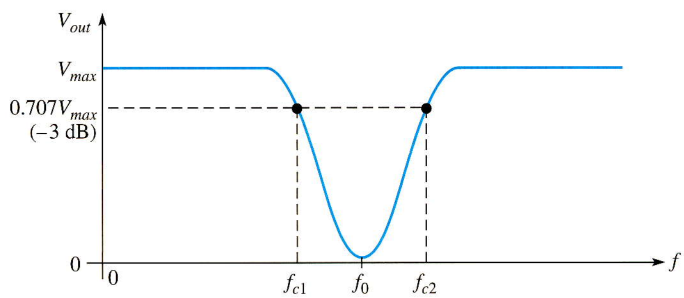


FIGURE 19–27
General band-stop response curve.

Low-Pass/High-Pass Filter

A **band-stop filter** can be formed from a low-pass and a high-pass filter, as shown in Figure 19–28.

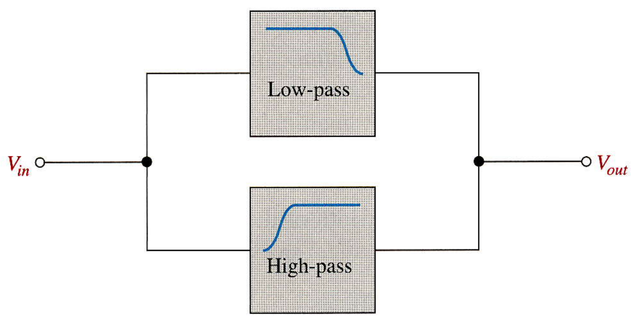
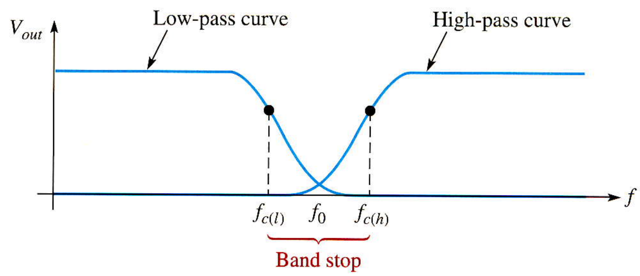


FIGURE 19–28
Low-pass and high-pass filters used to form a band-stop filter.

If the low-pass critical frequency, $f_{c(l)}$, is set lower than the high-pass critical frequency, $f_{c(h)}$, a band-stop characteristic is formed as illustrated in Figure 19–29.

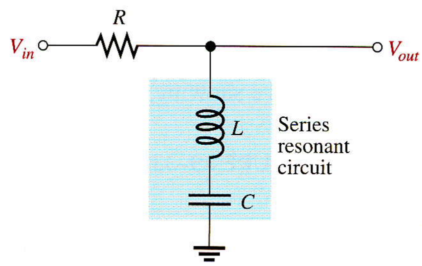
FIGURE 19–29
Band-stop response curve.



Series Resonant Band-Stop Filter

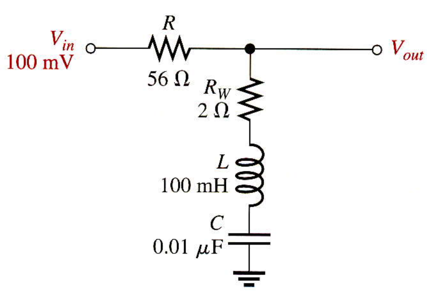
A series resonant circuit used in a band-stop configuration is shown in Figure 19–30. Basically, it works as follows: At the resonant frequency, the impedance is minimum, and therefore the output voltage is minimum. Most of the input voltage is dropped across R. At frequencies above and below resonance, the impedance increases, causing more voltage across the output.

FIGURE 19–30
Series resonant band-stop filter.



EXAMPLE 19–11 Find the output voltage magnitude at f_0 and the bandwidth in Figure 19–31.

FIGURE 19–31



Solution Since $X_L = X_C$ at resonance, the output voltage is

$$V_{out} = \left(\frac{R_W}{R + R_W}\right)V_{in} = \left(\frac{2\ \Omega}{58\ \Omega}\right)100\text{ mV} = \mathbf{3.45\ mV}$$

To determine the bandwidth, calculate the center frequency and Q of the coil.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(100\text{ mH})(0.01\ \mu\text{F})}} = 5.03\text{ kHz}$$

$$Q = \frac{X_L}{R} = \frac{2\pi fL}{R} = \frac{2\pi(5.03\text{ kHz})(100\text{ mH})}{58\ \Omega} = \frac{3.16\text{ k}\Omega}{58\ \Omega} = 54.5$$

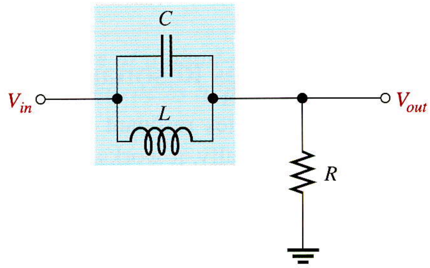
$$BW = \frac{f_0}{Q} = \frac{5.03\text{ kHz}}{54.5} = \mathbf{92.3\ Hz}$$

Related Problem Assume $R_W = 10\ \Omega$ in Figure 19–31. Determine V_{out} and the bandwidth.

Parallel Resonant Band-Stop Filter

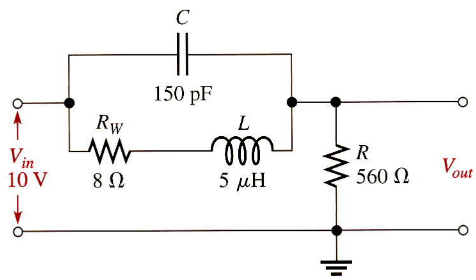
A parallel resonant circuit used in a band-stop configuration is shown in Figure 19–32. At the resonant frequency, the tank impedance is maximum, and so most of the input voltage appears across it. Very little voltage is across R at resonance. As the tank impedance decreases above and below resonance, the output voltage increases.

FIGURE 19–32
Parallel resonant band-stop filter.



EXAMPLE 19–12 Find the center frequency of the filter in Figure 19–33. Sketch the output response curve showing the minimum and maximum voltages.

FIGURE 19–33



Solution The center frequency is

$$f_0 = \frac{\sqrt{1 - R_W^2 C/L}}{2\pi\sqrt{LC}} = \frac{\sqrt{1 - (8\ \Omega)^2(150\ \text{pF})/5\ \mu\text{H}}}{2\pi\sqrt{(5\ \mu\text{H})(150\ \text{pF})}} = \mathbf{5.79\ MHz}$$

At the center (resonant) frequency,

$$X_L = 2\pi f_0 L = 2\pi(5.79\ \text{MHz})(5\ \mu\text{H}) = 182\ \Omega$$

$$Q = \frac{X_L}{R_W} = \frac{182\ \Omega}{8\ \Omega} = 22.8$$

$$Z_r = R_W(Q^2 + 1) = 8\ \Omega(22.8^2 + 1) = 4.17\ \text{k}\Omega \quad \text{(purely resistive)}$$

Next, use the voltage-divider formula to find the minimum output voltage magnitude.

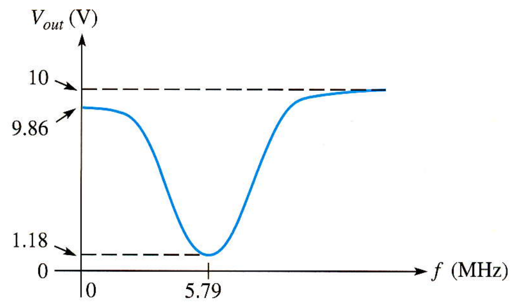
$$V_{out(min)} = \left(\frac{R}{R + Z_r}\right)V_{in} = \left(\frac{560\ \Omega}{4.73\ \text{k}\Omega}\right)10\ \text{V} = 1.18\ \text{V}$$

At zero frequency, the impedance of the tank is R_W because $X_C = \infty$ and $X_L = 0\ \Omega$. Therefore, the maximum output voltage below resonance is

$$V_{out(max)} = \left(\frac{R}{R + R_W}\right)V_{in} = \left(\frac{560\ \Omega}{568\ \Omega}\right)10\ \text{V} = 9.86\ \text{V}$$

As the frequency increases much higher than f_0, X_C approaches 0 Ω, and V_{out} approaches V_{in} (10 V). Figure 19–34 shows the response curve.

FIGURE 19–34

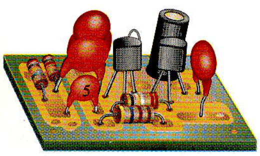


Related Problem What is the minimum output voltage if $R = 1.0\ k\Omega$ in Figure 19–33?

SECTION 19–4 REVIEW

1. How does a band-stop filter differ from a band-pass filter?
2. Name three basic ways to construct a band-stop filter.

19–5 ■ TECHnology Theory Into Practice



In this TECH TIP, you will plot the frequency responses of two types of filters based on a series of oscilloscope measurements and identify the type of filter in each case.

The filters are contained in sealed modules as shown in Figure 19–35. You are concerned only with determining the filter response characteristics and not the types of internal components.

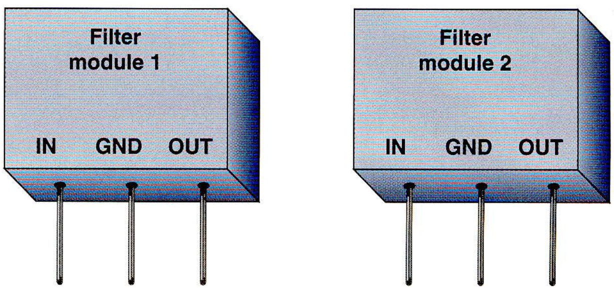


FIGURE 19–35
Filter modules.

Filter Measurement and Analysis

- Refer to Figure 19–36. Based on the series of four oscilloscope measurements, create a Bode plot for the filter under test, specify applicable frequencies, and identify the type of filter.
- Refer to Figure 19–37. Based on the series of six oscilloscope measurements, create a Bode plot for the filter under test, specify applicable frequencies, and identify the type of filter.

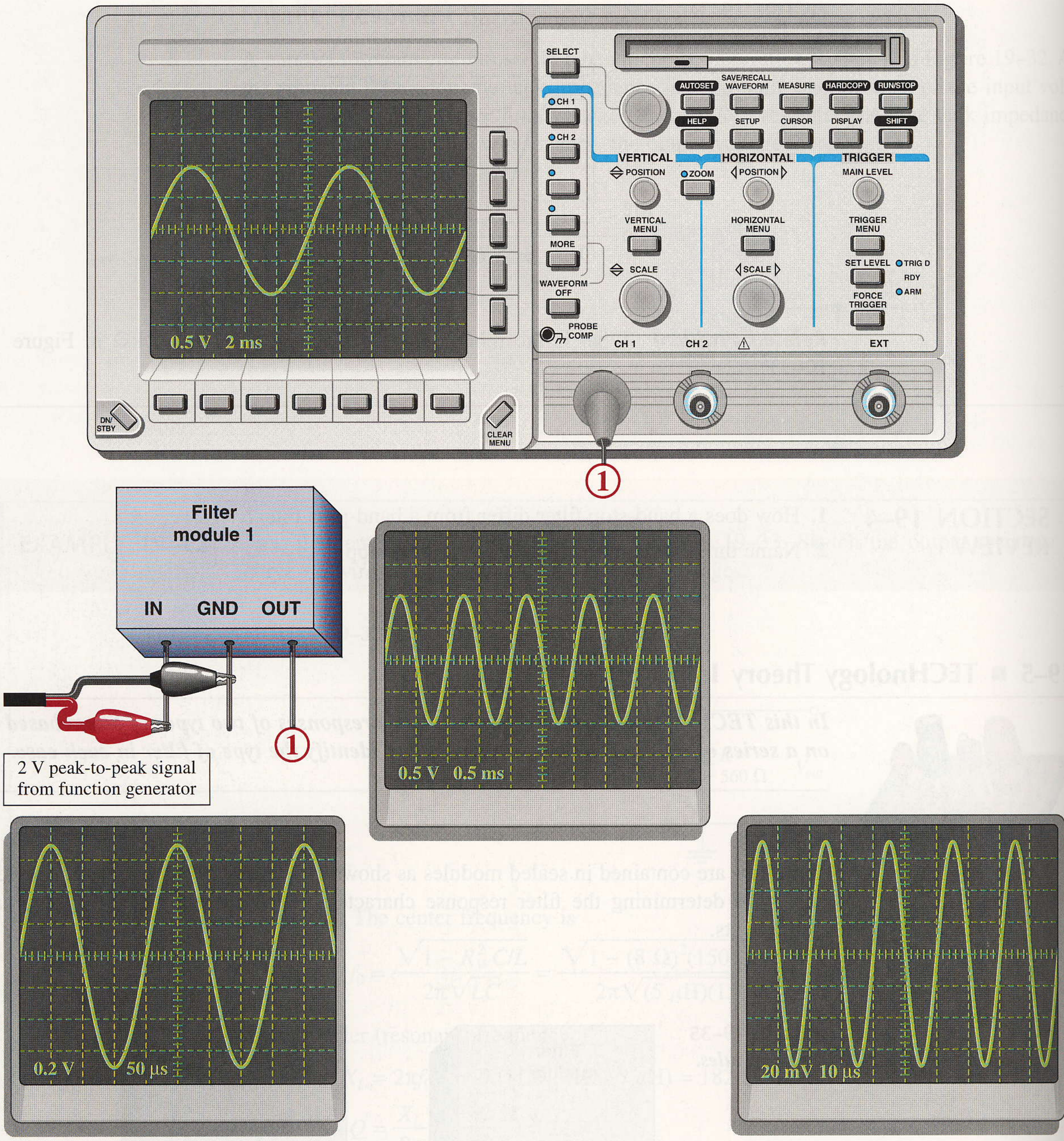

0.5 V 2 ms
1
Filter module 1
IN GND OUT
2 V peak-to-peak signal from function generator
0.5 V 0.5 ms
0.2 V 50 µs
20 mV 10 µs

FIGURE 19–36

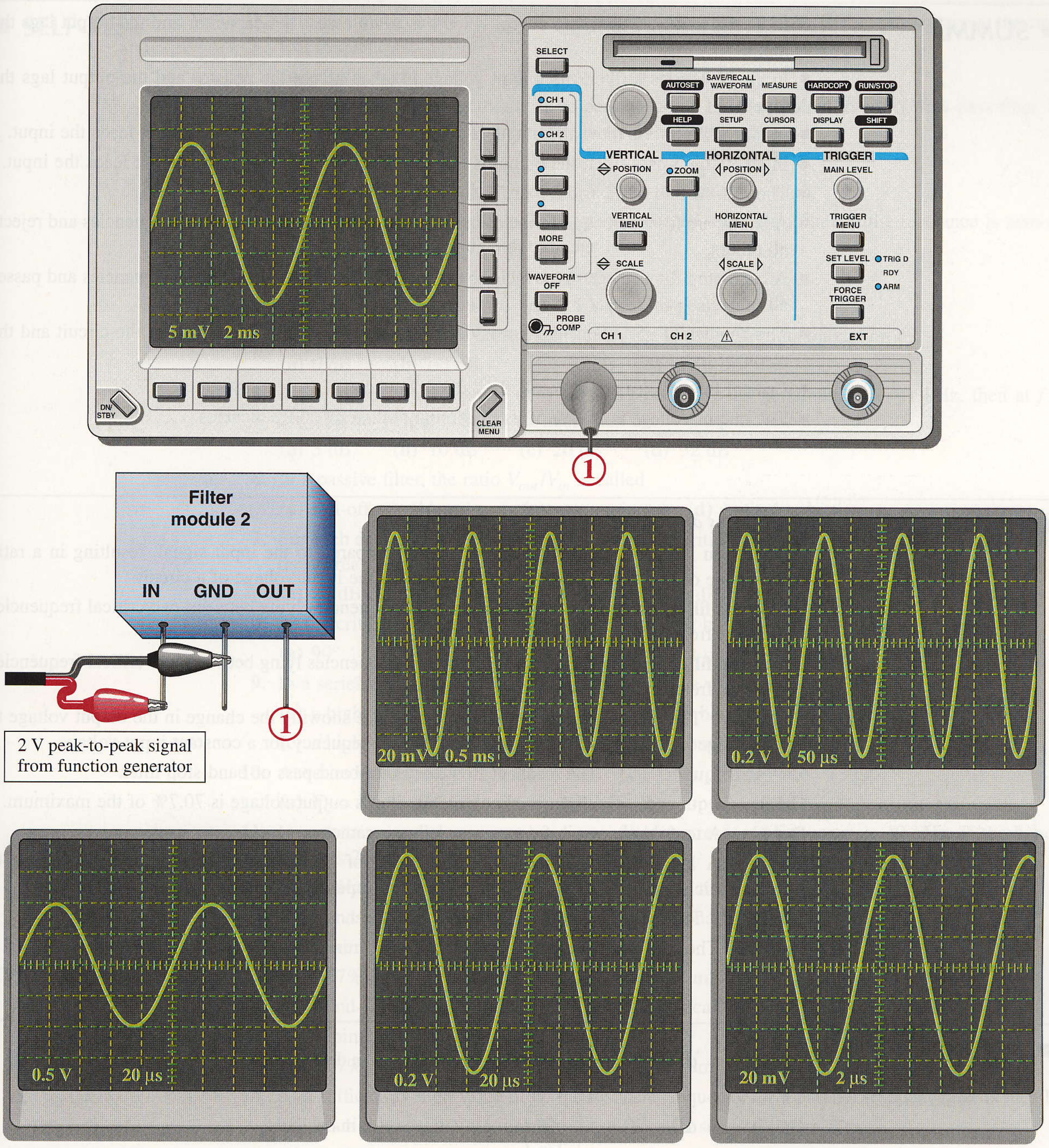


FIGURE 19–37

SECTION 19–5 REVIEW

1. Explain how the waveforms in Figure 19–36 indicate the type of filter.
2. Explain how the waveforms in Figure 19–37 indicate the type of filter.

■ SUMMARY

- In an RC low-pass filter, the output voltage is taken across the capacitor and the output lags the input.
- In an RL low-pass filter, the output voltage is taken across the resistor and the output lags the input.
- In an RC high-pass filter, the output is taken across the resistor and the output leads the input.
- In an RL high-pass filter, the output is taken across the inductor and the output leads the input.
- The roll-off rate of a basic RC or RL filter is −20 dB per decade.
- A band-pass filter passes frequencies between the lower and upper critical frequencies and rejects all others.
- A band-stop filter rejects frequencies between its lower and upper critical frequencies and passes all others.
- The bandwidth of a resonant filter is determined by the quality factor (Q) of the circuit and the resonant frequency.
- Critical frequencies are also called −3 dB frequencies.
- The output voltage is 70.7% of its maximum at the critical frequencies.

■ GLOSSARY

These terms are also in the end-of-book glossary.

Attenuation A reduction of the output signal compared to the input signal, resulting in a ratio with a value of less than 1 for the output voltage to the input voltage of a circuit.

Band-pass filter A filter that passes a range of frequencies lying between two critical frequencies and rejects frequencies above and below that range.

Band-stop filter A filter that rejects a range of frequencies lying between two critical frequencies and passes frequencies above and below that range.

Bode plot The graph of a filter's frequency response showing the change in the output voltage to input voltage ratio expressed in dB as a function of frequency for a constant input voltage.

Center frequency (f_0) The resonant frequency of a band-pass or band-stop filter.

Critical frequency The frequency at which a filter's output voltage is 70.7% of the maximum.

Decade A tenfold change in frequency or other parameter.

Decibel A logarithmic measurement of the ratio of one power to another or one voltage to another, which can be used to express the input-to-output relationship of a filter.

Passband The range of frequencies passed by a filter.

Roll-off The rate of decrease of a filter's frequency response.

■ FORMULAS

(19–1)	$\text{dB} = 10\log\left(\frac{P_{out}}{P_{in}}\right)$	Power ratio in decibels
(19–2)	$\text{dB} = 20\log\left(\frac{V_{out}}{V_{in}}\right)$	Voltage ratio in decibels
(19–3)	$f_c = \frac{1}{2\pi RC}$	Critical frequency
(19–4)	$f_c = \frac{1}{2\pi(L/R)}$	Critical frequency
(19–5)	$BW = \frac{f_0}{Q}$	Bandwidth

■ SELF-TEST

1. The maximum output voltage of a certain low-pass filter is 10 V. The output voltage at the critical frequency is
   (a) 10 V **(b)** 0 V **(c)** 7.07 V **(d)** 1.414 V
2. A sinusoidal voltage with a peak-to-peak value of 15 V is applied to an *RC* low-pass filter. If the reactance at the input frequency is zero, the output voltage is
   (a) 15 V peak-to-peak **(b)** zero
   (c) 10.6 V peak-to-peak **(d)** 7.5 V peak-to-peak
3. The same signal in Question 2 is applied to an *RC* high-pass filter. If the reactance is zero at the input frequency, the output voltage is
   (a) 15 V peak-to-peak **(b)** zero
   (c) 10.6 V peak-to-peak **(d)** 7.5 V peak-to-peak
4. At the critical frequency, the output of a filter is down from its maximum by
   (a) 0 dB **(b)** −3 dB **(c)** −20 dB **(d)** −6 dB
5. If the output of a low-pass *RC* filter is 12 dB below its maximum at $f = 1$ kHz, then at $f =$ 10 kHz, the output is below its maximum by
   (a) 3 dB **(b)** 10 dB **(c)** 20 dB **(d)** 32 dB
6. In a passive filter, the ratio V_{out}/V_{in} is called
   (a) roll-off **(b)** gain **(c)** attenuation **(d)** critical reduction
7. For each decade increase in frequency above the critical frequency, the output of a low-pass filter decreases by
   (a) 20 dB **(b)** 3 dB **(c)** 10 dB **(d)** 0 dB
8. At the critical frequency, the phase shift through a high-pass filter is
   (a) 90° **(b)** 0° **(c)** 45° **(d)** dependent on the reactance
9. In a series resonant band-pass filter, a higher value of Q results in
   (a) a higher resonant frequency **(b)** a smaller bandwidth
   (c) a higher impedance **(d)** a larger bandwidth
10. At series resonance,
    (a) $X_C = X_L$ **(b)** $X_C > X_L$ **(c)** $X_C < X_L$
11. In a certain parallel resonant band-pass filter the resonant frequency is 10 kHz. If the bandwidth is 2 kHz, the lower critical frequency is
    (a) 5 kHz **(b)** 12 kHz **(c)** 9 kHz **(d)** not determinable
12. In a band-pass filter, the output voltage at the resonant frequency is
    (a) minimum **(b)** maximum
    (c) 70.7% of maximum **(d)** 70.7% of minimum
13. In a band-stop filter, the output voltage at the critical frequencies is
    (a) minimum **(b)** maximum
    (c) 70.7% of maximum **(d)** 70.7% of minimum
14. At a sufficiently high value of Q, the resonant frequency for a parallel resonant filter is ideally
    (a) much greater than the resonant frequency of a series resonant filter
    (b) much less than the resonant frequency of a series resonant filter
    (c) equal to the resonant frequency of a series resonant filter

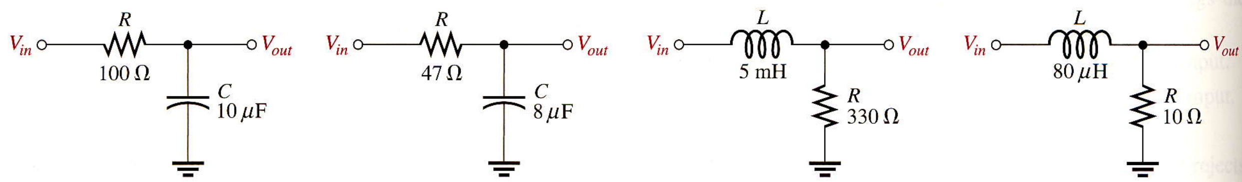
■ PROBLEMS

More difficult problems are indicated by an asterisk (*).

SECTION 19–1 Low-Pass Filters

1. In a certain low-pass filter, $X_C = 500\ \Omega$ and $R = 2.2\ \text{k}\Omega$. What is the output voltage ($\mathbf{V}_{out}$) when the input is 10 V rms?
2. A certain low-pass filter has a critical frequency of 3 kHz. Determine which of the following frequencies are passed and which are rejected:
   (a) 100 Hz **(b)** 1 kHz **(c)** 2 kHz **(d)** 3 kHz **(e)** 5 kHz

3. Determine the output voltage ($\mathbf{V}_{out}$) of each filter in Figure 19–38 at the specified frequency when $V_{in} = 10$ V.



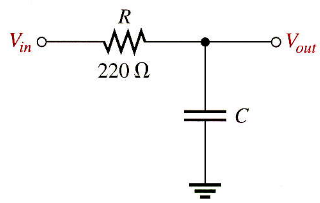
(a) $f = 60$ Hz (b) $f = 400$ Hz (c) $f = 1$ kHz (d) $f = 2$ kHz

FIGURE 19–38

4. What is f_c for each filter in Figure 19–38? Determine the output voltage at f_c in each case when $V_{in} = 5$ V.
5. For the filter in Figure 19–39, calculate the value of C required for each of the following critical frequencies:

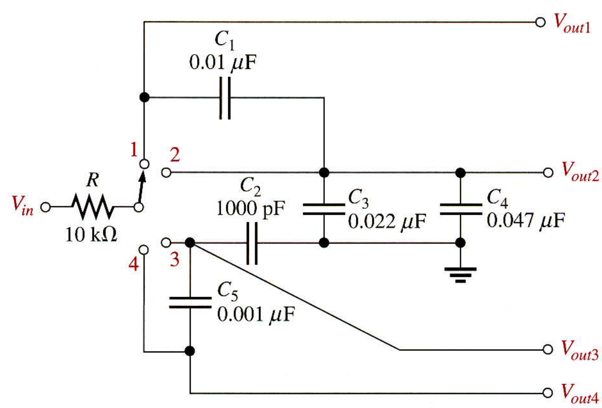
   (a) 60 Hz **(b)** 500 Hz **(c)** 1 kHz **(d)** 5 kHz

FIGURE 19–39



6. Determine the critical frequency for each switch position on the switched filter network of Figure 19–40.

FIGURE 19–40



7. Sketch a Bode plot for each part of Problem 5.
8. For each following case, express the voltage ratio in dB:

   (a) $V_{in} = 1$ V, $V_{out} = 1$ V **(b)** $V_{in} = 5$ V, $V_{out} = 3$ V
   (c) $V_{in} = 10$ V, $V_{out} = 7.07$ V **(d)** $V_{in} = 25$ V, $V_{out} = 5$ V
9. The input voltage to a low-pass RC filter is 8 V rms. Find the output voltage at the following dB levels:

   (a) −1 dB **(b)** −3 dB **(c)** −6 dB **(d)** −20 dB
10. For a basic RC low-pass filter, find the output voltage in dB relative to a 0 dB input for the following frequencies ($f_c = 1$ kHz):

    (a) 10 kHz **(b)** 100 kHz **(c)** 1 MHz

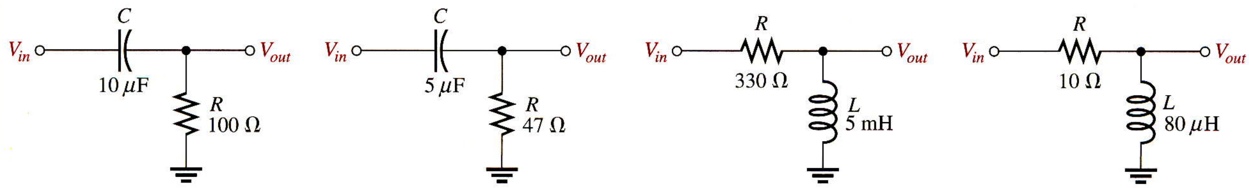
SECTION 19–2 High-Pass Filters

11. In a high-pass filter, $X_C = 500\ \Omega$ and $R = 2.2\ \text{k}\Omega$. What is the output voltage ($\mathbf{V}_{out}$) when $V_{in} = 10$ V rms?

12. A high-pass filter has a critical frequency of 50 Hz. Determine which of the following frequencies are passed and which are rejected:

(a) 1 Hz **(b)** 20 Hz **(c)** 50 Hz **(d)** 60 Hz **(e)** 30 kHz

13. Determine the output voltage of each filter in Figure 19–41 at the specified frequency when $V_{in} = 10$ V.



(a) f = 60 Hz (b) f = 400 Hz (c) f = 1 kHz (d) f = 2 kHz

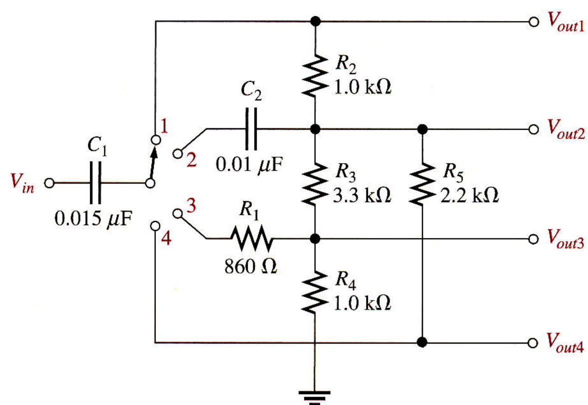
FIGURE 19–41

14. What is f_c for each filter in Figure 19–41? Determine the output voltage at f_c in each case ($V_{in} = 10$ V).

15. Sketch the Bode plot for each filter in Figure 19–41.

***16.** Determine f_c for each switch position in Figure 19–42.

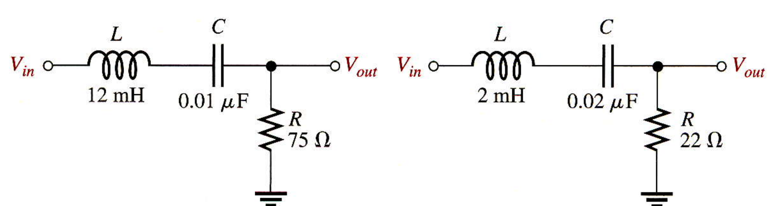
FIGURE 19–42



SECTION 19–3 Band-Pass Filters

17. Determine the center frequency for each filter in Figure 19–43.

FIGURE 19–43



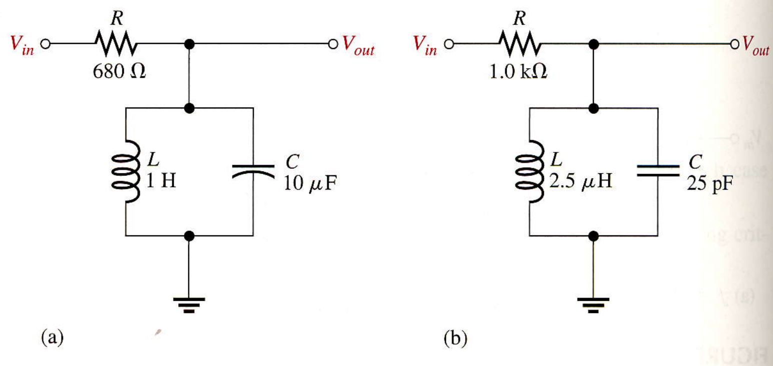
(a) (b)

18. Assuming that the coils in Figure 19–43 have a winding resistance of 10 Ω, find the bandwidth for each filter.

19. What are the upper and lower critical frequencies for each filter in Figure 19–43? Assume the response is symmetrical about f_0.

20. For each filter in Figure 19–44, find the center frequency of the passband. Neglect R_W.

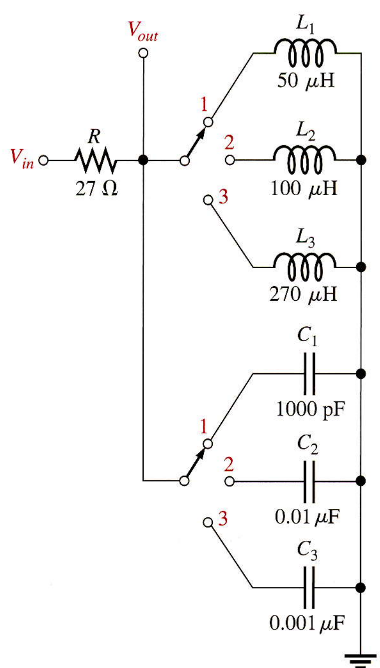
FIGURE 19–44



21. If the coils in Figure 19–44 have a winding resistance of 4 Ω, what is the output voltage at resonance when $V_{in} = 120$ V?

***22.** Determine the separation of center frequencies for all switch positions in Figure 19–45. Do any of the responses overlap? Assume $R_W = 0$ Ω for each coil.

FIGURE 19–45

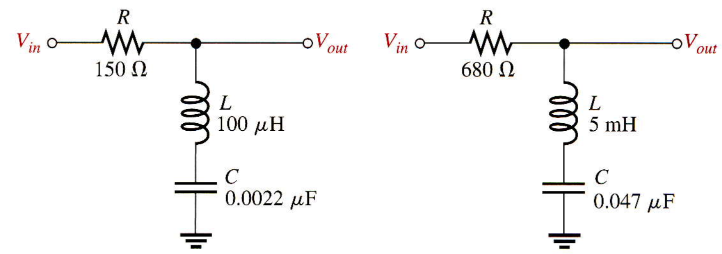


***23.** Design a band-pass filter using a parallel resonant circuit to meet all of the following specifications: $BW = 500$ Hz; $Q = 40$; and $I_{C(max)} = 20$ mA, $V_{C(max)} = 2.5$ V.

SECTION 19–4 Band-Stop Filters

24. Determine the center frequency for each filter in Figure 19–46.

FIGURE 19–46

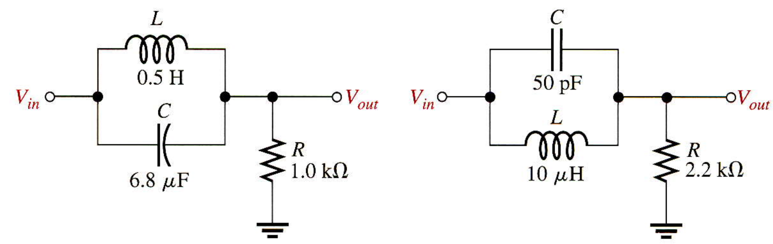


(a) (b)

25. For each filter in Figure 19–47, find the center frequency of the stopband.

26. If the coils in Figure 19–47 have a winding resistance of 8 Ω, what is the output voltage at resonance when $V_{in} = 50$ V?

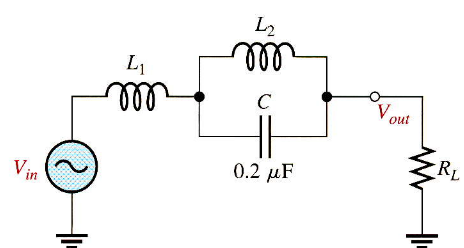
FIGURE 19–47



(a) (b)

***27.** Determine the values of L_1 and L_2 in Figure 19–48 to pass a signal with a frequency of 1200 kHz and stop (reject) a signal with a frequency of 456 kHz.

FIGURE 19–48



EWB Troubleshooting and Analysis

These problems require your EWB compact disk.

28. Open file PRO19-28.EWB and determine if there is a fault. If so, find the fault.

29. Open file PRO19-29.EWB and determine if there is a fault. If so, find the fault.

30. Open file PRO19-30.EWB and determine if there is a fault. If so, find the fault.

31. Open file PRO19-31.EWB and determine if there is a fault. If so, find the fault.

32. Open file PRO19-32.EWB and determine if there is a fault. If so, find the fault.

33. Open file PRO19-33.EWB and determine if there is a fault. If so, find the fault.

34. Open file PRO19-34.EWB and determine the center frequency of the circuit.

35. Open file PRO19-35.EWB and determine the bandwidth of the circuit.

ANSWERS TO SECTION REVIEWS

Section 19–1

1. The passband is 0 Hz to 2.5 kHz.
2. $\mathbf{V}_{out} = 100\angle -88.9°$ mV rms
3. $20 \log(V_{out}/V_{in}) = -9.54$ dB

Section 19–2

1. $V_{out} = 0.707$ V
2. $\mathbf{V}_{out} = 9.98\angle 3.81°$ V

Section 19–3

1. $BW = 30.2 \text{ kHz} - 29.8 \text{ kHz} = 400$ Hz
2. $f_0 \cong 1.04$ MHz

Section 19–4

1. A band-stop filter rejects, rather than passes, a certain band of frequencies.
2. High-pass/low-pass combination, series resonant circuit, and parallel resonant circuit

Section 19–5

1. The waveforms indicate that the output amplitude decreases with an increase in frequency as in a low-pass filter.
2. The waveforms indicate that the output amplitude is maximum at 10 kHz and drops off above and below as in a band-pass filter.

ANSWERS TO RELATED PROBLEMS FOR EXAMPLES

19–1 −1.41 dB
19–2 7.23 kHz
19–3 f_c increases to 159 kHz. Roll-off rate remains −20 dB/decade.
19–4 f_c increases to 350 kHz. Roll-off rate remains −20 dB/decade.
19–5 −60 dB
19–6 $C = 0.723\ \mu$F; $V_{out} = 4.98$ V; $\phi = 5.7°$
19–7 10.5 kHz
19–8 BW increases to 18.8 kHz.
19–9 1.59 MHz
19–10 7.12 kHz (no significant difference)
19–11 $V_{out} = 15.2$ mV; $BW = 105$ Hz
19–12 1.94 V

ANSWERS TO SELF-TEST

1. (c) **2.** (b) **3.** (a) **4.** (b) **5.** (d) **6.** (c) **7.** (a) **8.** (c)
9. (b) **10.** (a) **11.** (c) **12.** (b) **13.** (c) **14.** (c)