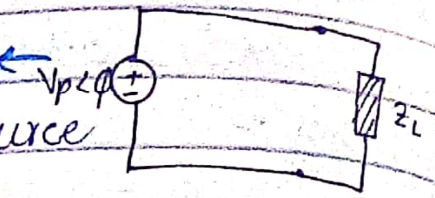
Chapter#12: Three-Phase Circuits

12.1 Introduction:-

Single phase two wire system, where V_p is rms magnitude of source voltage & ϕ is phase



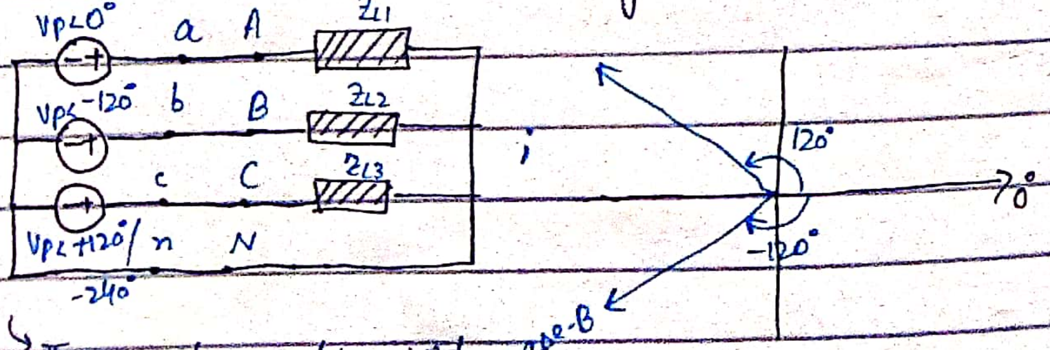
Poly-Phase:-

"Circuits or systems in which ac-sources operates at same frequency but different phases are known as poly-phase."

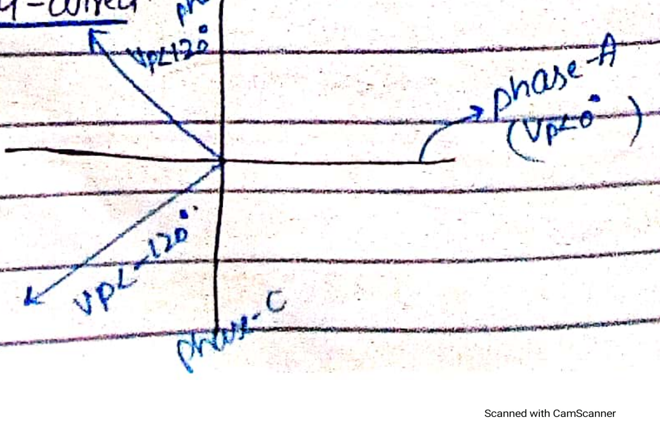
or: A Three-phase system is produced by generator consisting of 3-sources having same amplitude & frequency but out of phase with each other by 120°.

Importance of 3-phase System:-

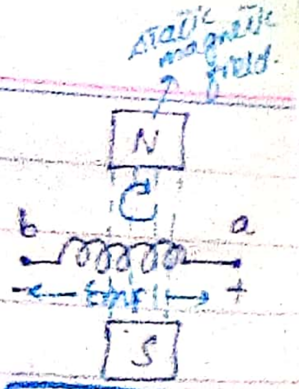
1) All electric power is generated & distributed in 3-phase; at operating freq. of 60Hz (or $\omega = 377$ rad/s) → $\omega = 2\pi f \Rightarrow \omega = 2\times3.14\times60$.
2) The inst. power in 3-phase can be const.
3) For same amount of power, 3-phase system is more economical than single-phase.



Three-phase 4-wired system.



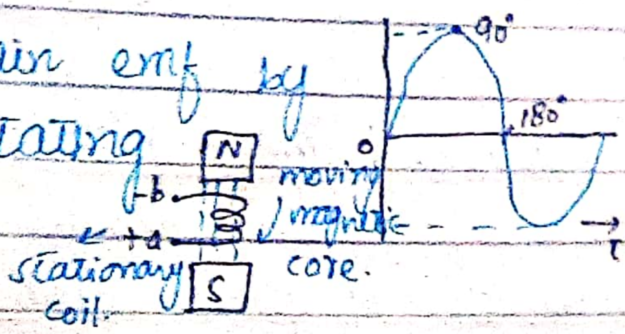
→ This emf is alternating in nature. We took static magnetic field & rotating coil / winding.



→ The axis of coil is ⊥ to each other. Alternating emf is generated at end of coil.

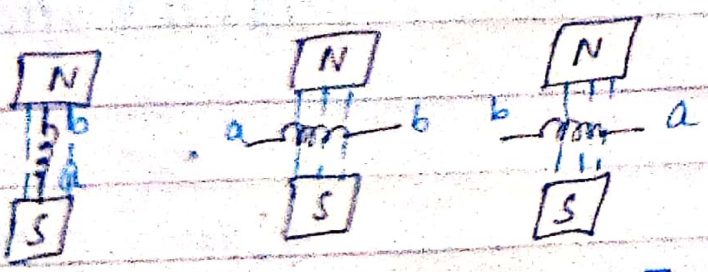
$\boxed{E_{ab} = \text{Emf}}$ b/w a & b terminals.

→ We can also obtain emf by stationary coil & rotating magnetic field.



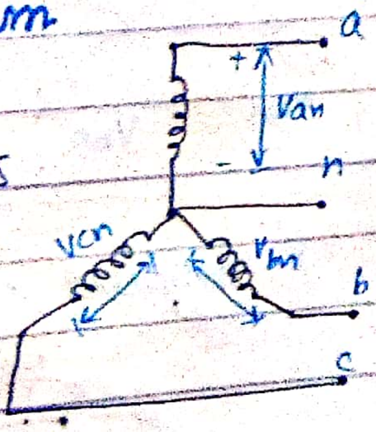
→ This is single-phase voltage & we use mostly for domestic / home appliances i.e:- lights, fans, bulbs etc. **but single phase has limitations.**

1) It can run only small domestic load, but for high-power electric load, we need 3-phase.

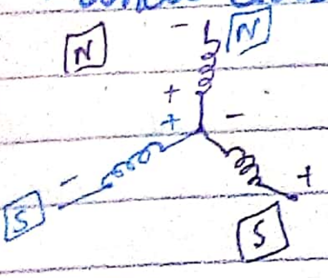


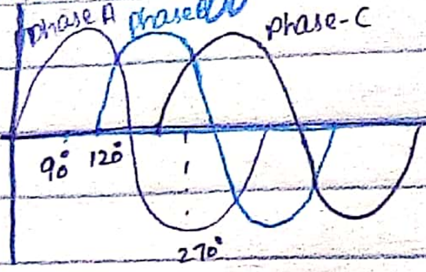
3-phase, 4-wire system

→ 3-phase emf can be generated through 3-set of coils. Each coil is displaced at 120° with each other. The 3-coil produces 3-phase emf with phase-diff. of 120° with each other.



→ What does 120° phase difference means?





Three-phase single wire → each sine wave is 120° out of phase.

12.2 Balanced Three-phase voltages:-

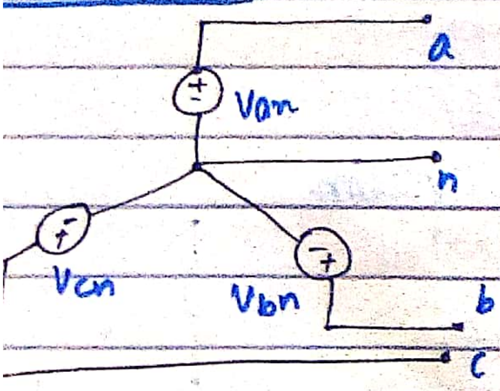
"Balanced 3-phase voltage are equal in magnitude & are out of phase with each other by 120°."

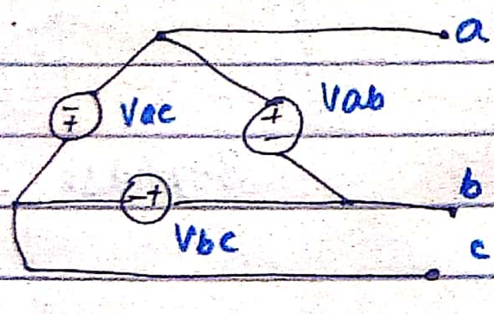
→ 3-phase voltages are often produced with a 3-phase AC generator (or alternator).

→ Generator consist of rotating magnet (rotor) surrounded by a stationary winding (stator)

→ Three separate windings or coils with terminals a-a', b-b', c-c' are physically placed at 120° apart around stator. (Fig: 12.4 → from book)

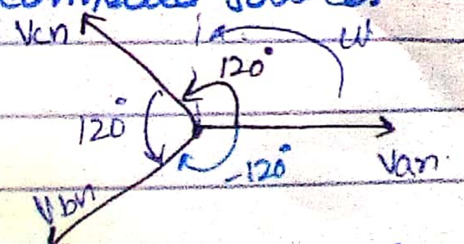
Connections-



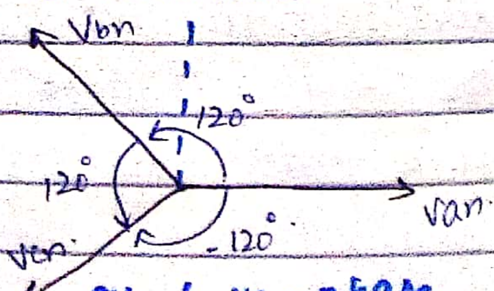


Δ-connected.

Y-connected sources.



abc / positive phase-sequence

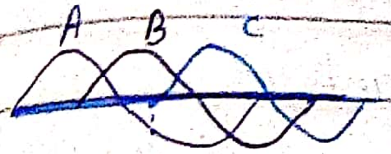


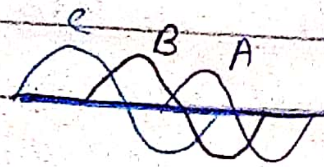
acb / -ve-phase sequence

Phase-Sequence:-

Phase sequence mean, that order/arrangement of all 3-phase reaches their max./peak-value. As mentioned in wave-form, the phase-B is lagging 120° by phase of A. Similarly, emf of phase-C is laging 120° by emf of phase-B.

→ "Order in which phase A, B, C reaches their max. values respectively. This order/arrangement is called **phase order/sequence**." A→B→C (ABC)





clock-wise (A→B→C)/ABC , anti-clockwise (C→B→A)/CBA

direction of alternator.

→ If we change direction of alternator, phase-sequence also changes.

→ "Blanced phase voltage are equal in magnitude & out of phase with each other by 120°." ($V_{an}+V_{bn}+V_{cn}=0$)

$V_A = e_A = E_m \sin(\omega t)$, → Phase sequence is order, in which voltage pass through their respective max. values.

$V_B = e_B = E_m \sin(\omega t - 120^\circ)$,

$V_C = e_C = E_m \sin(\omega t - 240^\circ)$,

→ Since, 3-phase are 120° out of phase, so possible combinations are;

$V_{an} + V_{bn} + V_{cn} = 0$ → voltage said to be balanced.

$|V_{an}| = |V_{bn}| = |V_{cn}|$

$V_{an} = V_P\angle 0^\circ$; $V_{bn} = V_P\angle -120^\circ$ ∵ V_P = rms. value of phase voltage.

$V_{cn} = V_P\angle -240^\circ$ / $V_P\angle +120^\circ$

→ ABC is called +ve-phase sequence. (RYB) → red, yellow, blue.

→ CBA " " -ve-phase sequence. (RBY) → red, blue, yellow.

- $V_{an} + V_{bn} + V_{cn} = 0$

$V_P\angle 0^\circ + V_P\angle -120^\circ + V_P\angle +120^\circ = 0$ (rect. → polar).

$V_P[1 - \cancel{0.5} - j\cancel{0.866} - \cancel{0.5} + j\cancel{0.866}] = 0$

$V_P[0] = 0.$

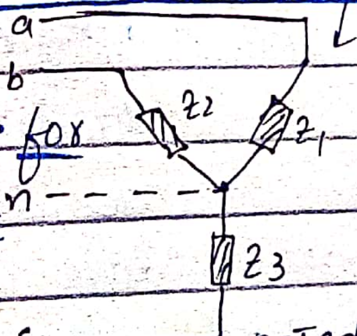
$\boxed{0 = 0}$

So, $\boxed{V_{an} + V_{bn} + V_{cn} = 0}$ **proved**

→ Balanced load:-

"A balanced load is one in which the phase impedences are equal in magnitude & in-phase". for a balanced wye-connected load,

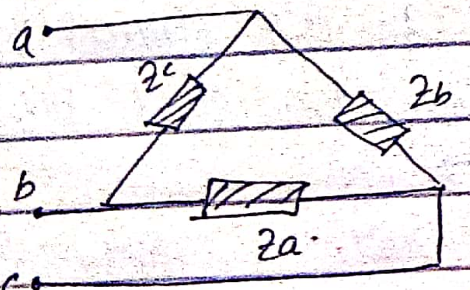
$Z_1 = Z_2 \pm Z_3 = Z_Y$



Y-connected load.

- Z_Y is load impedence per phase. for balanced delta-connected load.

$Z_a = Z_b \pm Z_c = Z_\Delta$



Δ-connected load

→ Z_Δ is load impedence ~~for~~ per-phase.

ch#9 {9.69}

- $Z_\Delta = 3Z_Y$ or
- $Z_Y = \frac{1}{3} Z_\Delta$

$$Z_\Delta = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1} = \frac{Z^2 + Z^2 + Z^2}{Z} = \frac{3Z^{\cancel{2}}}{\cancel{Z}}$$

$\boxed{Z_\Delta = 3Z_Y}$

→ As we know that, Y-connected load can be transformed into a Δ-connected load or vice versa

$Z_\Delta = 3Z_Y$ or $Z_Y = \frac{1}{3} Z_\Delta$.

→ Since both 3-phase source and the 3-phase load can either wye-or-delta connected, so we have four possible connections.

1. Y-Y connected, i.e. Y-connected load to Y-source
2. Y-Δ connected
3. Δ-Y connected
4. Δ-Δ connected.

→ A balanced delta-connected load is more common than a balanced wye-connected load.

→ This is due to ease, with which loads may be added or removed from each phase of a delta-connected load.

→ A Δ/Y cct. is said to be balanced if it has equal impedences in all three branches. ($Z_1 = Z_2 = Z_3 = Z_Y$.)

Y-Δ: $Z_\Delta = \frac{Z_1Z_2 + Z_3Z_2 + Z_3Z_1}{Z_1} = \frac{3Z_Y^2}{Z_Y} = 3Z_Y$.

$\boxed{Z_\Delta = 3Z_Y}$ or $\boxed{Z_Y = \frac{1}{3} Z_\Delta}$

Ex# 12.1:- Determine phase sequence of set of voltages: $V_{cn} = 200\cos(\omega t - 110^\circ)$.

∴ $V_{an} = 200\cos(\omega t + 10^\circ)$; $V_{bn} = 200\cos(\omega t - 230^\circ)$

Voltages in phasor-form are:-

$V_{an} = 200\angle 10^\circ$, $V_{bn} = 200\angle -230^\circ$, $V_{cn} = 200\angle -110^\circ$.

Van leads Vcn by 120° & Vcn leads Vbn by 120°-

acb sequence here.

PP# 12.1:- $V_{bn} = 220\angle 30° V$ & find V_{an} & V_{cn} assuming +ve (abc) sequence.

For abc-sequence; V_{an} leads V_{bn} by $120°$ & V_{bn} leads V_{cn} by $120°$. So,

$$V_{an} = 220\angle 30° + 120° = \boxed{220\angle 150° \, V}$$

$$V_{cn} = 220\angle 30° - 120° = \boxed{220\angle -90° \, V}$$

$$\boxed{V_{bn} = 220\angle 30° V}$$

12.3 Balanced Wye-Wye connections:-

"A balanced Y-Y system is a 3-phase system with a balanced Y-connected source & a balanced Y-connected load."

$$Z_Y = Z_s + Z_l + Z_L \text{ ——— } (12.9)$$

Z_Y = Total impedence per phase.

Z_s = source " (internal impedence of phase-winding of generator)

Z_l = Line " (Impedence of line joing phase of source with phase of load).

Z_L = Load ".

Z_n = impedence of neutral line.

→ (Theory from book pg # 507)

$Z_Y = Z_s + Z_l + Z_L$

when Z_s & Z_l are small compared to Z_L. So,

$$\boxed{Z_Y = Z_L}$$

$V_{an} = V_p\angle 0°$

$V_{bn} = V_p\angle -120°$

$V_{cn} = V_p\angle +120° \,/\, V_p\angle -240°$.

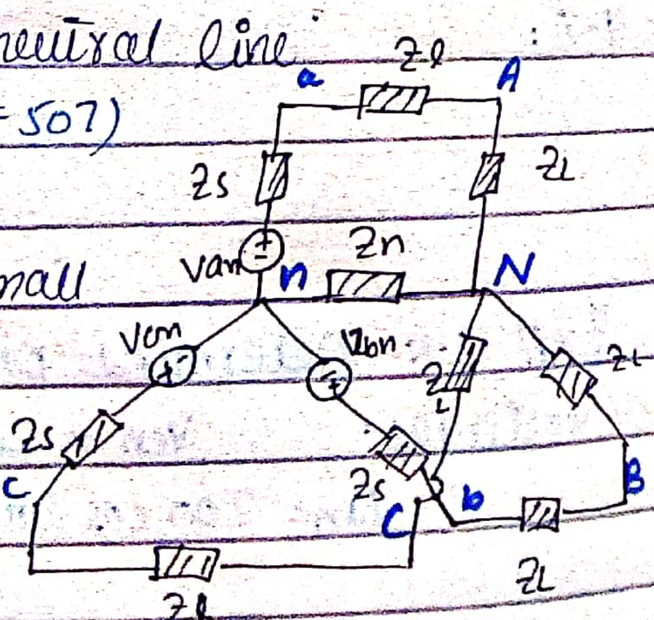


fig # 12.9

→ (Theory from book) pg # 507/508.

$V_{ab} = V_{an} + (-V_{bn})$

$= \boxed{V_p \angle 0^\circ - V_p \angle -120^\circ}$

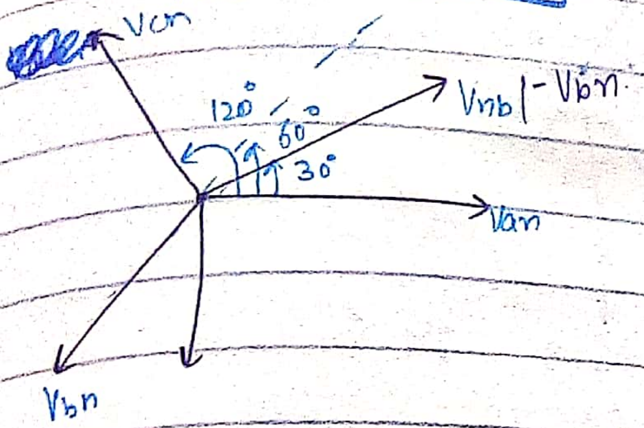
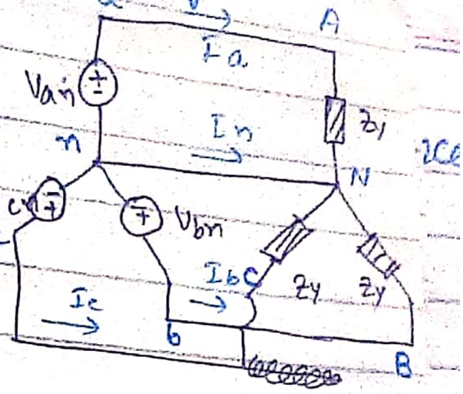


fig 12.10



→ **converting from Rectangular to polar:-**

$V_{ab} = V_p[\cos(0) + j\sin(0)] - V_p[\cos(-120^\circ) + j\sin(-120^\circ)]$

$V_{ab} = V_p(1+0) - V_p[(-0.5) + j(-0.866)]$

$V_{ab} = V_p[1 + 0.5 + j0.866]$.

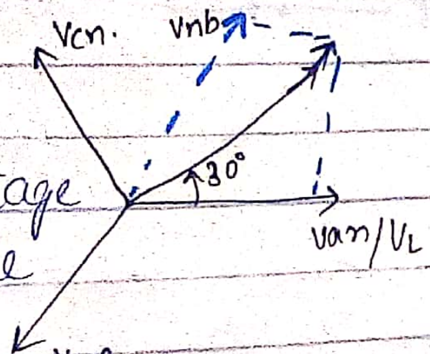
$V_{ab} = V_p(1.5 + j0.866)$

$V_{ab} = V_p[1.73 \angle 30^\circ] = \sqrt{3}\, V_p \angle 30^\circ$

or

$\boxed{V_{ab} = V_L = \sqrt{3}\, V_p}$

$\because V_p = V_{ph}$ = phase-voltages.



Thus, magnitude of line-voltage V_L is $\sqrt{3}$ time the magnitude of phase-voltage (V_p).

$\boxed{V_L = \sqrt{3}\, V_p}$

→ Similarly,

$V(\text{phase}) = V_p = |V_{an}| = |V_{bn}| = |V_{cn}|$

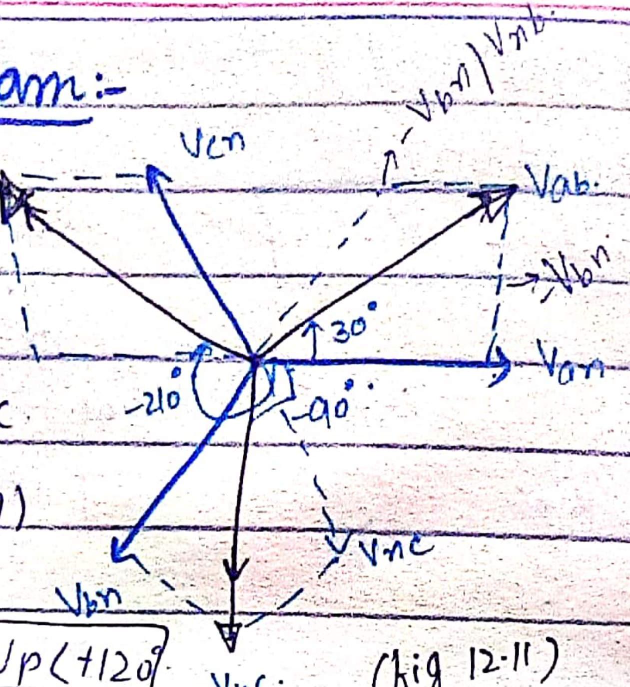
& $V(\text{line}) = V_L = |V_{ab}| = |V_{bc}| = |V_{ca}|$

where; $V_{ab} = \sqrt{3}\, V_p \angle 30^\circ$ → 12.11(a)

$V_{bc} = \sqrt{3}\, V_p \angle -90^\circ$ → 12.11(b)

$V_{ca} = \sqrt{3}\, V_p \angle -210^\circ$ → 12.11(c)

Vector diagram:-



$V_{bc} = V_{bn} + V_{nc}$

$V_{bc} = V_{bn} + (-V_{cn})$

$= V_{bn} - V_{cn}$

$\boxed{V_{bc} = V_p\angle -120^\circ - V_p\angle +120^\circ}$ (fig 12.11)

or $V_{bc} = V_p\angle -120^\circ - V_p\angle +120^\circ$. (phasor → rect.)

$V_{bc} = V_p[\cos(-120^\circ) + j\sin(-120^\circ)] - V_p(\cos(120^\circ) + j\sin(120^\circ)]$.

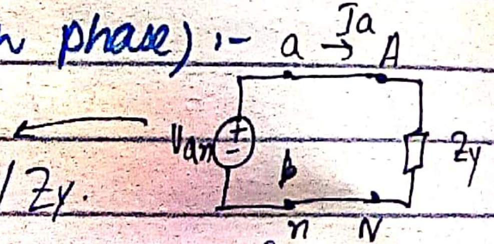
$V_{bc} = V_p[-0.5 - j0.866] - V_p[-0.5 + j0.866]$.

$V_{bc} = \cancel{-0.5V_p} - jV_p 0.866 + \cancel{0.5V_p} - jV_p 0.866$

$V_{bc} = -jV_p 1.732 = \sqrt{3}V_p\angle -90^\circ$ $\qquad -j = \angle -90^\circ$.

$V_{bc} = \boxed{V_L = \sqrt{3}V_p}$ $\qquad 1.73 = \sqrt{3}$.

from fig 12.10 :- (KVL on each phase) :-



$V_{an} = I_a Z_Y$

Line current $= I_a = V_{an}/Z_Y$.

&, $I_b = \frac{V_{bn}}{Z_Y} = \frac{V_{an}\angle -120^\circ}{Z_Y}$.

So, $I_b = I_a\angle -120^\circ$ — (12.15)

& $I_c = \frac{V_{cn}}{Z_Y} = \frac{V_{an}\angle -240^\circ}{Z_Y} = I_a\angle -240^\circ$

We can say, line current add up to zero:-

$I_a + I_b + I_c = 0$ — 12.16

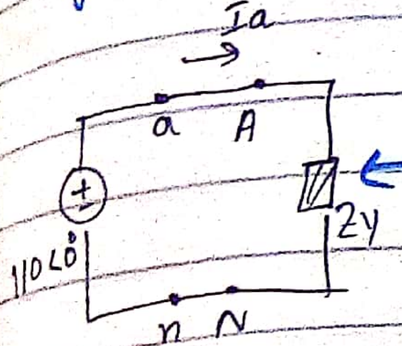
So, $I_n = -(I_a + I_b + I_c)$ — 12.17

$V_{nN} = Z_n I_n = 0$. → voltage across neutral wire =0

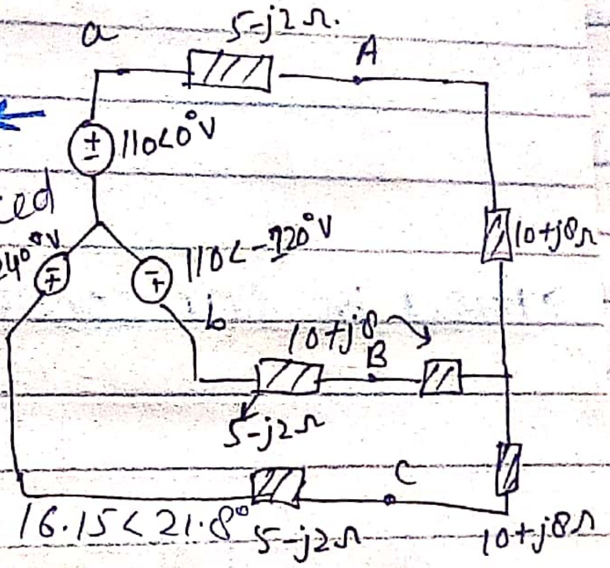
Thus, neutral wire can be removed without affecting system.

→ In fact, in long distance, power transmission conductor in multiples of three are used with earth itself acting as neutral conductor.

Ex# 12.2:- Calculate line current in 3-wire Y-Y system of given fig:-



← 3-phase balanced so can be replaced by 1-phase equivalent ckt ←



$Z_Y = (5-j2)+(10+j8) = 15+j6 = 16.15\angle 21.8°$

as: $I_a = \dfrac{V_{an}}{Z_Y}$

$I_a = \dfrac{110\angle 0°}{16.15\angle 21.8°} = \boxed{6.81\angle -21.8°\,A}$

& $I_b = I_a\angle -120° = \boxed{6.81\angle -141.8°\,A}$

$I_c = I_a\angle -240° = \boxed{6.81\angle 98.2°\,A}$

PP# 12.2:-

a) $V_{ab} = V_{an} - V_{bn} = 120\angle 30° - 120\angle -90° = 207.8\angle 60°\,V$ (polar)

V_{ab} leads $30°$ to V_{an} & has mag. $\sqrt{3}$ time to V_{an}.

So,

$V_{ab} = \sqrt{3}(120)\angle(30+30°) = 207.8\angle 60°\,V$

abc sequence ⇒ $V_{bc} = 207.8\angle -60°\,V$

$V_{ca} = 207.8\angle \pm 180°\,V$

b) $I_a = \dfrac{V_{an}}{Z_Y}$

$Z_y = (0.4+j0.3)+(24+j19)+(0.6+j0.7)$

$Z_y = 25+j20 = 32\angle 38.66^\circ$

So,

$$I_a = \frac{120\angle 30^\circ}{32\angle 38.66^\circ} = 3.75\angle -8.66^\circ A$$

$$I_b = I_a\angle -120^\circ = 3.75\angle -128.66^\circ A$$

$$I_c = I_a\angle -240^\circ = 3.75\angle 111.34^\circ A$$

→ Power in star connection:-

Total active power $= P = 3\times(\text{phase-power})$

$= 3\times(V_{ph}\times I_{ph}\cos\phi)$

ϕ = angle b/w phase voltage & phase current

As we know:-

in Y-Y → $V_L = \sqrt{3}\,V_{ph}$ & $I_{ph} = I_L$

$V_{ph} = \frac{V_L}{\sqrt{3}}$

So,

$$P = 3\times\left[\frac{V_L}{\sqrt{3}}\times I_L\cos\phi\right] \text{ or } = \sqrt{3}\cdot\cancel{\sqrt{3}}\left[\frac{V_L}{\cancel{\sqrt{3}}}\times I_L\cdot\cos\phi\right]$$

$\boxed{P = \sqrt{3}\,[V_L I_L\cos\phi]}$ = active/ reactive power.

Similarly;

$\boxed{Q = \text{reactive power} = \sqrt{3}\,V_L I_L \sin\phi}$

$\boxed{S = \text{apparent power} = \sqrt{3}\,V_L I_L}$

12.5 Balanced Delta-Delta connection:-

"A balanced Δ-Δ system is one in which both the balance source & balanced load are Δ-connected."

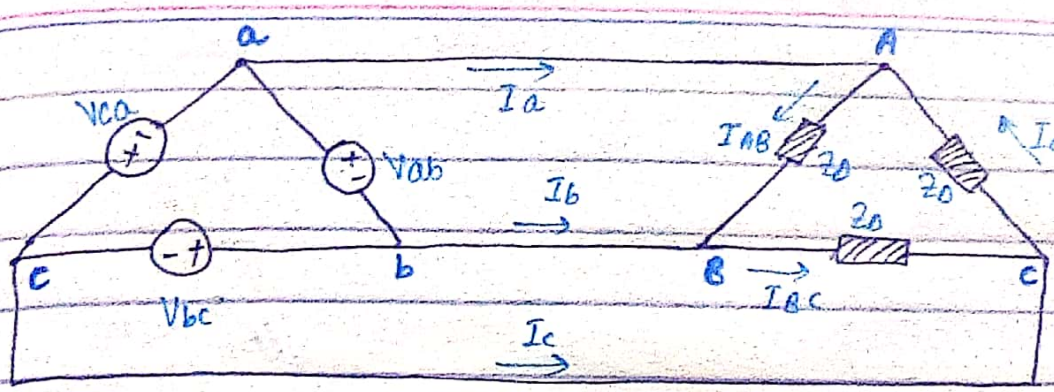


Fig 12.17 (A balanced Δ-Δ Connection)

→ Assuming a positive sequence, the phase voltage for a delta-connected source are;

$$V_{ab} = V_p\angle 0^\circ,\ V_{bc} = V_p\angle -120^\circ,\ V_{ca} = V_p\angle +120^\circ \Big] \rightarrow 12.29$$

* Line voltages are same as phase voltages.

→ Assuming their is no line impedence - phase voltage of Δ-connected source are equal to voltage across impedence.

$$V_{ab} = V_{AB},\quad V_{bc} = V_{BC},\quad V_{ca} = V_{CA}$$

Hence, phase current:

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{V_{ab}}{Z_\Delta}$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta} = \frac{V_{bc}}{Z_\Delta}$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta} = \frac{V_{ca}}{Z_\Delta}$$

line currents are obtained from phase currents by applying KCL at node A, B & C.

$$I_a = I_{AB} - I_{CA},\quad I_b = I_{BC} - I_{AB},\quad I_c = I_{CA} - I_{BC}$$

Since; $I_{CA} = I_{AB}\angle -240^\circ$

So, $I_a = I_{AB} - I_{AB}\angle -240^\circ$

or $I_a = I_{AB}(1 - 1\angle -240^\circ)$

$= I_{AB}(1 + 0.5 - j0.866)$

$\boxed{I_a = I_{AB}\sqrt{3}\angle -30^\circ}$ → Mag. of I_L is $\sqrt{3}$ times of I_{ph}.

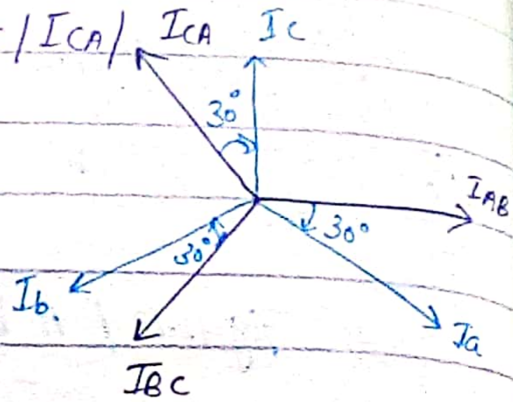
$$\boxed{I_L = \sqrt{3} \cdot I_{ph}}$$

$I_{ph} = I_p = |I_{AB}| = |I_{BC}| = |I_{CA}|$

$I_L = |I_a| = |I_b| = |I_c|$

$I_L = \sqrt{3}\, I_p$

or $\boxed{I_p = \dfrac{I_L}{\sqrt{3}}}$



~~Ex#12.3:-~~

~~Balanced abc Y connected source with~~ Van=100<10°V ~~is connected to Δ~~

Ex#12.4:-

Balanced Δ-connected load has $Z = 20 - j15\,\Omega$ is connected to Δ-connected +ve sequence generator having $V_{ab} = 330\angle 0°$ V; calculate I_{ph} of load & line currents.

Solution:-

$Z_\Delta = 20 - j15 = 25\angle -36.86°$

Here; $V_{ab} = V_{AB}$; So; I_{ph}:-

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{330\angle 0°}{25\angle -36.86}$$

$$\boxed{I_{AB} = 13.2\angle 36.87° \text{ A}}$$

$I_{BC} = I_{AB}\angle -120° = \boxed{13.2\angle -83.13° \text{ A}}$

$I_{CA} = I_{AB}\angle -240° / +120° = \boxed{13.2\angle 156.87° \text{ A}}$

for Delta-load; I_L always lags corresponding I_{ph} by 30°. & has mag. $\sqrt{3}$ times I_{ph}. So,

line currents:- $I_a = I_{AB} \cdot \sqrt{3}\angle -30° = (13.2\angle 36.87°)(\sqrt{3}\angle -30°)$

$$\boxed{I_a = 22.86\angle 6.87° \text{ A}}$$

$$\boxed{I_b = I_a\angle -120° = 22.86\angle -113.3° \text{ A}}$$

$$\boxed{I_c = I_a\angle +120° = 22.86\angle 126.7° \text{ A}}$$

PP#12.4:- +ve- seq. balance ~~load~~ Δ-connected sources supplies a balanced Δ-connected load. If impedence per phase of load is $18+j12\,\Omega$, $I_a = 9.609\angle 35°A$, find I_{AB} & V_{AB}.

$$Z_\Delta = 18+j12\,\Omega = 21.63\angle 33.69°$$

$$I_a = 9.609\angle 35°A \;=\; I_{AB}\,(\sqrt{3}\angle -30)$$

$$I_{AB} = \frac{9.609\angle 35°}{\sqrt{3}\angle -30°} = \boxed{5.547\angle 65°A}$$

$$V_{AB} = I_{AB} \times Z_\Delta$$

$$V_{AB} = (5.547\angle 65°)(21.63\angle 33.69°)$$

$$V_{AB} = 119.99\angle 98.69°V$$

or $\boxed{V_{AB} = 120\angle 98.69°V.}$

12.4 Balanced Y-Δ connection:-

"A balanced Y-Δ system consists of a balanced Y-connected source feeding balanced Δ-connected load"

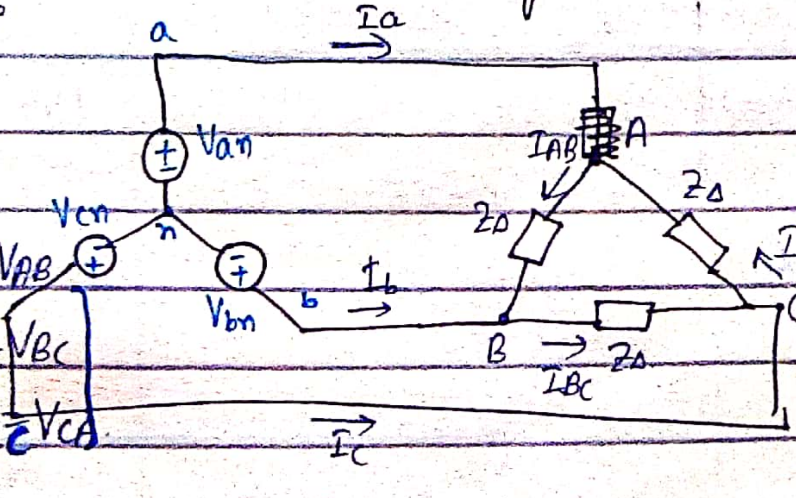
→ This is most practical 3-phase system used.

$V_{phase} = V_{an} = V_p\angle 0°$

$V_{bn} = V_p\angle -120°$,

$V_{cn} = V_p\angle +120°$

as
$$\begin{cases} V_{ab} = \sqrt{3}\,V_p\angle 30° = V_{AB} \\ V_{bc} = \sqrt{3}\,V_p\angle -90° = V_{BC} \\ V_{ca} = \sqrt{3}\,V_p\angle -210° = V_{CA} \end{cases}$$



$$I_{AB} = \frac{V_{AB}}{Z_\Delta} \rightarrow I_{BC} = I_{AB}\angle -120° = \frac{V_{BC}}{Z_\Delta},\quad I_{CA} = \frac{V_{CA}}{Z_\Delta} = I_{AB}\angle +120°$$

$$\boxed{I_p = \frac{1}{\sqrt{3}} I_L} \rightarrow \Delta.$$

$\boxed{12.6}$ Balanced Δ-Y connection:- → rare.

↳ "consists of a balanced Δ-connected source feeding a balanced Y-connected load."

$$Z_\Delta = \frac{1}{3} Z_Y$$

$\boxed{\text{Table\#12.1}}$

Summary of Phase & Line Current/voltages for balanced 3-phase system.

(+ve-phase sequence assumed) ↳ abc

Connections	Phase voltage/current	Line voltage/current
Y-Y	$V_{an} = V_p \angle 0°$ $V_{bn} = V_p \angle -120°$ $V_{cn} = V_p \angle +120°$ & $I_{phase} = I_{line}$.	$V_{ab} = \sqrt{3} V_p \angle 30°$ $V_{bc} = \sqrt{3} V_p \angle -90°$ / $V_{ab} \angle -120°$ $V_{ca} = V_{ab} \angle +120°$. $I_a = V_{an}/Z_Y$, $I_b = I_a \angle -120°$ $I_c = I_a \angle +120°$
Y-Δ	$V_{an} = V_p \angle 0°$, $V_{bn} = V_p \angle -120°$ $V_{cn} = V_p \angle +120°$. $I_{AB} = V_{AB}/Z_\Delta$ $I_{BC} = V_{BC}/Z_\Delta$ $I_{CA} = V_{CA}/Z_\Delta$	$V_{ab} = V_{AB} = \sqrt{3} V_p \angle 30°$ $V_{bc} = V_{BC} = V_{ab} \angle -120°$ $V_{ca} = V_{CA} = V_{ab} \angle +120°$] line & phase voltages are same. $I_a = I_{AB}\sqrt{3} \angle -30°$, $I_b = I_a \angle -120°$. $I_c = I_a \angle +120°$.
Δ-Δ	$V_{ab} = V_p \angle 0°$, $V_{bc} = V_p \angle -120°$ $V_{ca} = V_p \angle +120°$. $I_{AB} = V_{ab}/Z_\Delta$ $I_{BC} = V_{bc}/Z_\Delta$ $I_{CA} = V_{ca}/Z_\Delta$	line voltages = Phase voltages $(V_L = V_p)$ $I_a = I_{AB}\sqrt{3} \angle -30°$ $I_b = I_a \angle -120°$ $I_c = I_a \angle +120°$
Δ-Y	$V_{ab} = V_p \angle 0°$, $V_{bc} = V_p \angle -120°$ $V_{ca} = V_p \angle +120°$. phase current = line current	$I_a = \dfrac{V_p/\sqrt{3} \angle -30°}{Z_Y}$ or $I_a = 0.577 V_p \angle -30° / Z_Y$ $I_b = I_a \angle -120°$ $I_c = I_a \angle +120°$

Ex#12.3 :-

A balanced abc-sequence Y-connected source with $V_{an} = 100\angle 10^\circ$ V is connected to Δ connected balanced load $(8+4j)\Omega$ per phase; calculate phase & line currents.

Solution:-

$$Z_\Delta = (8+4j)\Omega = 8.944\angle 26.56^\circ \Omega$$

$$V_{an} = V_{ph} = 100\angle 10^\circ \; ; \; V_{ab} = V_L = V_{an}\sqrt{3}\angle 30^\circ$$

$$\boxed{V_{ab} = 173.2\angle 40^\circ \text{ V}} = V_{AB}$$

phase currents:- $I_{AB} = \dfrac{V_{AB}}{Z_\Delta} = \dfrac{173.2\angle 40^\circ}{8.944\angle 26.56^\circ}$

$$I_{AB} = \boxed{19.36\angle 13.44^\circ A}$$

$$I_{BC} = I_{AB}\angle -120^\circ = \boxed{19.36\angle -106.56^\circ A}$$

$$I_{CA} = I_{AB}\angle +120^\circ = \boxed{19.36\angle 133.44^\circ A}$$

line currents:- $I_a = I_{AB}\sqrt{3}\angle -30^\circ = 33.53\angle -16.57^\circ A$

$$I_b = I_a\angle -120^\circ = 33.53\angle -136.57^\circ A$$

$$I_c = I_a\angle +120^\circ = 33.53\angle 103.43^\circ A$$

PP#12.3 :- $V_{AB} = 120\angle -20^\circ$ V , $Z_\Delta = 20\angle 40^\circ \Omega$ - $I_{ph} = ?, I_L = ?$

phase currents:-

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{120\angle -20^\circ}{20\angle 40^\circ} = 6\angle -60^\circ A$$

$$I_{BC} = I_{AB}\angle -120^\circ = 6\angle -180^\circ \; , \; I_{CA} = I_{AB}\angle +120^\circ = 6\angle 60^\circ A$$

line currents:-

$$I_a = I_{AB}\sqrt{3}\angle -30^\circ = (6)(\sqrt{3})\angle -30^\circ - 60^\circ = 10.39\angle -90^\circ A$$

$$I_b = I_a\angle -120^\circ = 10.39\angle -210^\circ A$$

$$I_c = I_a\angle +120^\circ = 10.39\angle 30^\circ A$$

Ex#12.5:-

A balanced Y-connected load with phase impedance of $40+j25\,\Omega$ is supplied by a balanced +ve-sequence Δ-connected source with $V_L = 210\,V$. Phase current = ?

$\overline{V_{ab}}$

$Z_Y = 40+j25\,\Omega = 47.1699 \angle 32°$

$V_{ab} = 210 \angle 0°\,V$

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} \angle -30° = 121.24 \angle -30°\,V$$

line currents:- $I_a = \dfrac{V_{an}}{Z_Y} = \dfrac{121.24\angle -30°}{47.17\angle 32°} = 2.57\angle -62°A$

$I_b = I_a \angle -120° = 2.57\angle -182°A$

$I_c = I_a \angle +120° = 2.57\angle 58°A$

same as phase currents

PP#12.5:- $V_{ab} = 440\angle 15°$, $Z_Y = 12+j15\,\Omega$, line currents?

$Z_Y = 12+j15\,\Omega = 19.21\angle 51.34°\,\Omega$

$V_{ab} = 440\angle 15°\,V$

$$V_{an} = \frac{V_{ab}}{\sqrt{3}}\angle -30° = \frac{440}{\sqrt{3}}\angle -30+15 = 254.03\angle -15°\,V$$

$$I_a = \frac{V_{an}}{Z_Y} = \frac{254.03\angle -15°}{19.21\angle 51.34°} = \boxed{13.224\angle -66.34°A}$$

$I_b = I_a\angle -120° = \boxed{13.224\angle -186.34°A}$

$I_c = I_a\angle +120° = \boxed{13.224\angle 53.66°A}$

Ex#12.7:- $P = 5.6kW$; $V_L = 220\,V$, $I_L = 18.2A$.

p.f = ?

$S = \sqrt{3}V_L I_L = (\sqrt{3})(220)(18.2) = 6935.13\,VA$

$P = S\cos\theta = 5600\,W$

$$P.f = \cos\theta = \frac{P}{S} = \frac{5600}{6935.13} = \boxed{0.8075}$$

PP# 12.7:- $P = 30\,kW$; $p.f = \cos\theta = 0.85$ lagging,

$V_L = 550\,V$; $I_L = ?$

$$P = S\cos\theta \Rightarrow S = \frac{P}{\cos\theta} = \frac{30{,}000}{0.85} = 35294.117\,VA$$

$$S = \sqrt{3}\,V_L I_L \Rightarrow I_L = \frac{35294.117}{(\sqrt{3})(550)}$$

$$\boxed{I_L = 37.05\,A.}$$

Ex# 12.9:- $Z_A = 15\,\Omega$, $Z_B = 10 + j5\,\Omega$, $Z_C = 6 - j8\,\Omega$,

balanced volts = 100V.

$$I_a = \frac{100\angle 0^\circ}{15} = \boxed{6.67\angle 0^\circ A}$$

$$I_b = \frac{100\angle 120^\circ}{10 + j5} = \boxed{8.94\angle 93.44^\circ A}, \quad I_c = \frac{100\angle -120^\circ}{6 - j8}$$

$$\boxed{I_C = 10\angle -66.87^\circ A.}$$

neutral-line current:-

$$I = -(I_a + I_b + I_c) = 10.06\angle 178.4^\circ A.$$