

Example 11.1

Given that

$$v(t) = 120 \cos(377t + 45^\circ) \text{ V}$$

$$i(t) = 10 \cos(377t - 10^\circ) \text{ A}$$

Find instantaneous power and average power by the passive linear network.

Solution:

Instantaneous power =  $P = Vi$

$$P = Vi = 120 \cos(377t + 45^\circ) 10 \cos(377t - 10^\circ)$$

$$P = 1200 (\cos(377t + 45^\circ)) (\cos(377t - 10^\circ))$$

$$P = \frac{1200}{2} [2 \cos(377t + 45^\circ) \cos(377t - 10^\circ)]$$

$$P = 600 [\cos(377t + 45^\circ + 377t - 10^\circ) + \cos(377t + 45^\circ - 377t + 10^\circ)]$$

$$P = 600 [\cos(754t + 35^\circ) + \cos 55^\circ]$$

$$P = 344.2 + 600 \cos(754t + 35^\circ) \text{ W}$$

Average power =  $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$$P = \frac{1}{2} (120)(10) \cos(45^\circ - 10^\circ)$$

$$P = 344.2 \text{ W}$$

Practice 11.1

Calculate the instantaneous power and average power absorbed by the passive linear network

$$v(t) = 330 \cos(10t + 20^\circ) \text{ V}$$

$$i(t) = 33 \sin(10t + 60^\circ) \text{ A} = 33 \cos(10t + 60^\circ - 90^\circ) = 33 \cos(10t - 30^\circ)$$

Solution:

Instantaneous power =  $P = Vi$

$$P = Vi = 330 \cos(10t + 20^\circ) 33 \sin(10t + 60^\circ)$$

$$P = 10890 \cos(10t + 20^\circ) \sin(10t + 60^\circ)$$

$$P = \frac{10890}{2} [2 \cos(10t + 20^\circ) \sin(10t + 60^\circ)]$$

$$P = 5445 [\sin(10t + 20^\circ + 10t + 60^\circ) - \sin(10t + 20^\circ - 10t - 60^\circ)]$$

$$P = 5445 [\sin(20t + 80^\circ) - \sin(-40^\circ)]$$

$$P = 5445 [\sin(20t + 80^\circ) + \sin 40^\circ]$$

$$P = 3499.97 + 5445 \sin(20t + 80^\circ)$$

OR

$$P = 3499.97 + 5445 \cos(20t - 10^\circ) \text{ W}$$

Average power =  $P = \frac{1}{2} (330)(33) \cos(20^\circ - 30^\circ)$

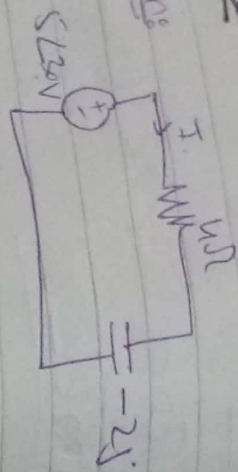
$$P = 3499.97 \text{ W}$$





Example 11.3

For the circuit given. Find the average power and the average power absorbed by the resistor.



Solutions:

$$Z = 1 - 2j \Omega$$

$$V = 5 \angle 30^\circ \text{ V}$$

$$I = \frac{V}{Z} = \frac{5 \angle 30^\circ}{1 - 2j}$$

$$I = 1.011 \angle 56.56^\circ \text{ A}$$

i) Average power =  $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$$P = \frac{1}{2} (5) (1.011) \cos(30 - 56.56)$$

$$P = 2.482 \text{ W}$$

Average power =  $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

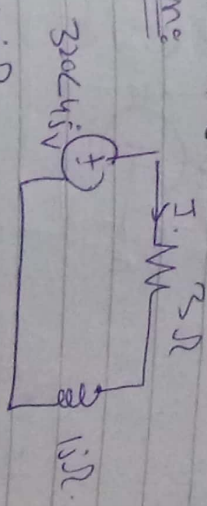
$$P = \frac{1}{2} (2.22) (1.011) \cos(-33.44 - 56.56)$$

$$P = 0 \text{ W}$$

ii)  $P = \frac{1}{2} (1.011) (1.011) \cos(56.56 - 56.56)$

Practice 11.3

In the given circuit, calculate the average power and inductor. Find the average power supplied by the voltage source.



Solutions:

$$Z = 3 + j\Omega$$

$$V = 320 \angle 45^\circ$$

$$I = \frac{V}{Z} = \frac{320 \angle 45^\circ}{3 + j}$$

$$I = 101.192 \angle 26.56^\circ \text{ A}$$

$$P_R = I^2 R = 303.57 \angle 26.56^\circ \quad V = 101.192 \angle 116.56^\circ$$

i)  $P = \frac{1}{2} (303.57) (101.192) \cos(26.56 - 26.56)$

$$P = 15359.4 \text{ W}$$

$$P = \frac{1}{2} (101.192) (101.192) \cos(116.56 - 26.56)$$

$$P = 0 \text{ W}$$

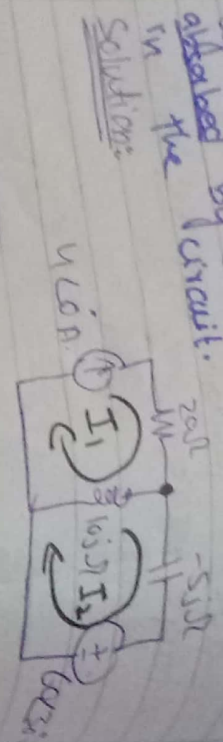
ii)  $P = \frac{1}{2} (320) (101.192) \cos(45 - 26.56)$

$$P = 15359.4 \text{ W}$$

$$-V + 20I_1 + (I_1 - I_2)10j \Rightarrow V = 20I_1 + 10j(I_1 - I_2)$$

$$P_{10j} = \frac{1}{2} (184.87) (4) \cos(6.20 - 0^\circ)$$

Example 11.4  
Determine the average power generated by each source and average power absorbed by each passive element in the circuit.



$$I_1 = 4\angle 0^\circ \text{ A}$$

$$-5jI_2 + 60\angle 30^\circ + 10j(I_2 - 4\angle 0^\circ) = 0$$

$$-5jI_2 + 60\angle 30^\circ + 10jI_2 - 40j = 0$$

$$5jI_2 = -[52.91\angle -10.89^\circ]$$

$$I_2 = 10.58\angle -79.10^\circ \text{ A}$$

$$I_1 - I_2 = 10.58\angle -79.10^\circ$$

$$P_g = \frac{1}{2} (60) (10.58) \cos(30 - 79.10)$$

$$P_{20\Omega} = 207.81 \text{ W}$$

$$P_{5j} = \frac{1}{2} (52.9) (10.58) \cos(10.9 - 79.10)$$

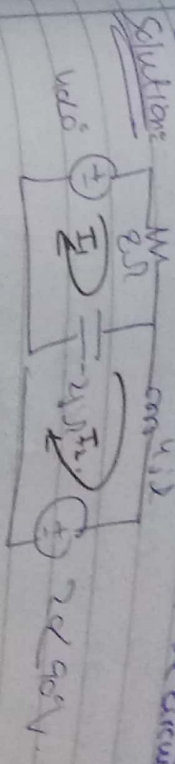
$$P_{-5j} = 0 \text{ W}$$

$$P_{10j} = \frac{1}{2} (105.7) (10.58) \cos(10.9 + 79.10)$$

$$P_{60\angle 20^\circ} = 0 \text{ W}$$

Practice 11.4

Calculate the average power absorbed by each of the five elements in the circuit.



$$-40\angle 0^\circ + 8I_1 - 2j(I_1 - I_2) = 0$$

$$8I_1 - 2jI_1 + 2jI_2 = 40\angle 0^\circ$$

$$(8 - 2j)I_1 + 2jI_2 = 40\angle 0^\circ \quad (1)$$

$$4jI_2 + 20\angle 90^\circ - 2j(I_2 - I_1) = 0$$

$$4jI_2 + 20\angle 90^\circ - 2jI_2 + 2jI_1 = 0$$

$$2jI_1 + 2jI_2 = -20\angle 90^\circ \quad (2)$$

$$I_2 = -360 - 80j - 13 - 4j = -373 - 84j$$

$$I_1 = -7 + 6 + 160j - 3 + 4j = 5\angle 53.13^\circ$$

$$I_1 - I_2 = 16 + 8j = 17.82\angle 26.56^\circ$$

$$P_{20\Omega} = \frac{1}{2} (40) (5) \cos(10 - 53.13^\circ)$$

$$P_{20\Omega} = 60 \text{ W}$$

$$P_{20\angle 90^\circ} = \frac{1}{2} (20) (13.60) \cos(90^\circ + 10.89)$$

$$P_{20\angle 90^\circ} = +100 \text{ W}$$

$$P_R = \frac{1}{2} (40) (5) \cos(53.13 - 53.13)$$

$$P_R = 100 \text{ W}$$

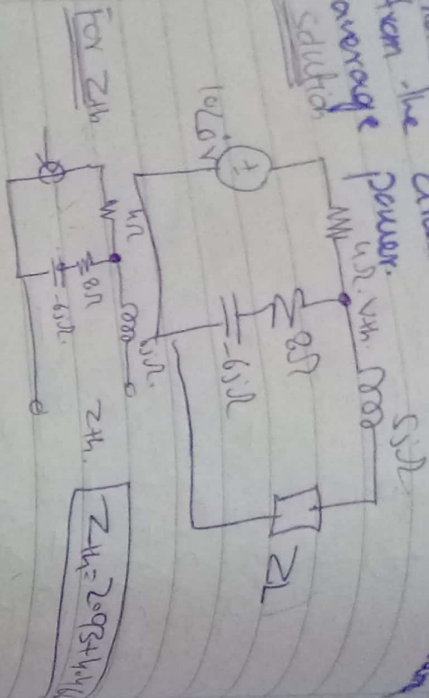
$$P_{20\angle 90^\circ} = \frac{1}{2} (35.71) (17.80) \cos(53.13 - 26.56)$$



$$Z_L = Z_{th}$$

Example 11.5

Determine the load impedance  $Z_L$  that maximizes the average power drawn from the circuit. What is the maximum average power solution?



For  $Z_{th}$

$$V_{th} = \frac{8 - 6j}{(8 - 6j) + (4 + 8j)} \times 10 \angle 0^\circ$$

$$V_{th} = 7.045 \angle -10.30^\circ$$

$$Z_L = Z_{th}^* = 2.093 - 4.0466j \Omega$$

$$P_{max} = \frac{|V_{th}|^2}{8R_{th}}$$

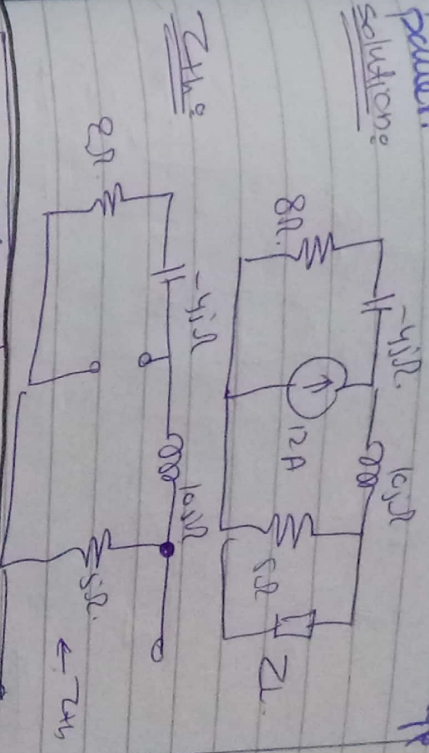
$$P_{max} = \frac{|7.045 \angle -10.30^\circ|^2}{8(2.093)}$$

$$P_{max} = 2.367 \text{ W}$$

$$P = \frac{|V_{th}|^2}{8R_{th}}$$

Practice 11.5

For the circuit. Find load impedance  $Z_L$  that absorbs maximum average power. Calculate that maximum average power.



For  $Z_{th}$

$$Z_{th} = 13 + 6j \Omega$$

$$Z_L = 13 - 6j \Omega$$

~~Wrong track~~

$$8jI - 4jI + 10I + 5I \neq 0$$

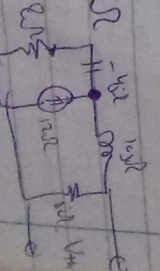
$$Z_{th} = 3 \parallel (19 - 4j + 8)$$

$$Z_{th} = 3 \parallel (27 - 4j) = 0.7183 \parallel 27$$

$$Z_L = 3 \parallel (27 - 4j)$$

$$V = IZ = (12)(8 - 4j)$$

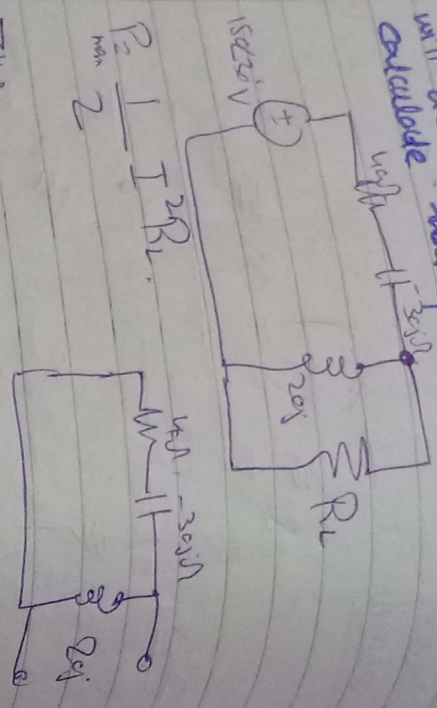
$$V = 96 - 48j = 107.53 \angle -26.56^\circ$$



$R_L = Z_{Th}$

Example 11.6

In the circuit, find the value of  $R_L$  that will absorb the maximum average power. Calculate that power.



$Z_{Th} = 9.411 + j22.035 \Omega$   
 $Z_L = 9.411 - j22.035 \Omega$

$R_L = 9.411 \Omega = 24.25$

$I = \frac{V_{Th}}{Z_{Th} + R_L} = \frac{120 \angle 76.4134^\circ}{40.411 \angle 33.58^\circ}$

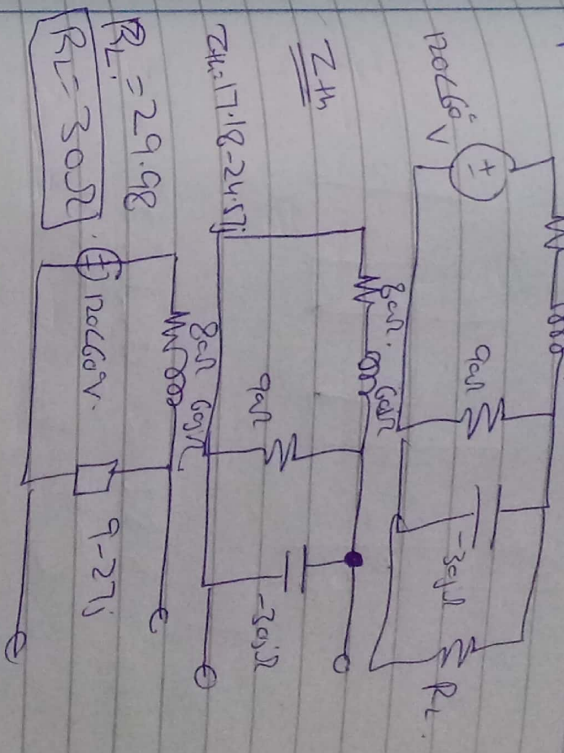
$I = 1.8002 \angle 100.414^\circ$

$P_{max} = \frac{1}{2} (I)^2 (R_L)$

$P_{max} = 39.29$

Practice 11.6

In figure, the resistor  $R_L$  is adjusted until it absorbs the maximum average power. Calculate  $R_L$  and maximum average power absorbed by it.



$R_L = 29.98$

$R_L = 30 \Omega$

$V_{Th} = \frac{9 - j27j}{80 + 60j + 9 - j27j} \times 120 \angle 60^\circ$

$V_{Th} = 35.98 \angle -31.90^\circ$

$I = \frac{V_{Th}}{Z_{Th} + R_L} = \frac{35.98 \angle -31.90^\circ}{60.6763 \angle -4.399^\circ}$

$P_{max} = \frac{1}{2} I^2 R_L$

$P_{max} = 6.86 W$

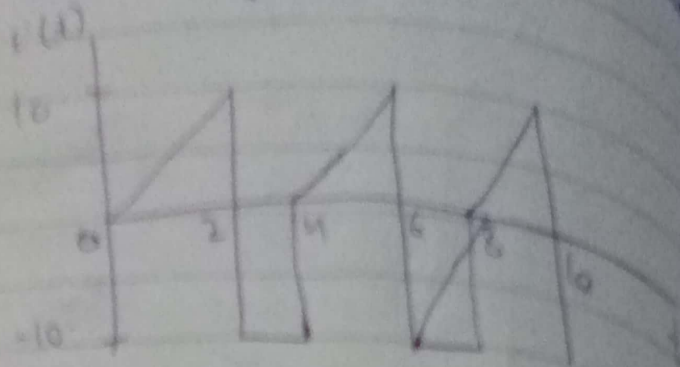


Date: \_\_\_\_\_

Day: \_\_\_\_\_

## Example 11.7

Determine the rms value of current waveform. If the current is passed through a  $2\Omega$  resistor. Find the power absorbed by the resistor.



$$T = 4$$

$$i(t) = \begin{cases} 5t, & 0 \leq t < 2 \\ -10, & 2 \leq t < 4 \end{cases}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{4} \left[ \int_0^2 (5t)^2 dt + \int_2^4 (-10)^2 dt \right]}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{4} \left[ \frac{25t^3}{3} \Big|_0^2 + (-100)t \Big|_2^4 \right]}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{4} \left[ \frac{25(2)^3}{3} - \frac{5(0)}{3} + 200 \right]}$$

$$I_{\text{rms}} = 8.16 \text{ A}$$

$$P = I^2 R = 8.16^2 \times 2 = 133.17 \text{ W}$$

Example 11.9

A series connected load draws a current  $i(t) = 4 \cos(100\pi t + \theta) \text{ A}$ . When the applied voltage is  $v(t) = 120 \cos(100\pi t - 20^\circ) \text{ V}$ . Find the apparent power and the power factor of the load. Determine the element value of the load. The series connected load value.

Solution:

The apparent power is  $S = V_{\text{rms}} I_{\text{rms}}$

$$S = \frac{120}{\sqrt{2}} \cdot \frac{4}{\sqrt{2}}$$

$$S = 240 \text{ VA}$$

Power factor is

$$PF = \cos(\theta_v - \theta_i) = \cos(-20^\circ - 10^\circ) = \cos(-30^\circ)$$

$$Z = \frac{V}{I} = \frac{120 \angle -20^\circ}{4 \angle 10^\circ} = 30 \angle -30^\circ$$

$$Z = 25.98 - 15j \Omega$$

$$X_c = \frac{-1}{\omega C}$$

$$-15 = \frac{-1}{\omega C}$$

$$C_2 = \frac{1}{1500}$$

$$C_2 = \frac{1}{15 \times 100\pi}$$

Practice 11.9

Obtain the power factor and apparent power of load whose impedance is  $Z = 60 + 40j \Omega$ . When the applied voltage is  $v(t) = 320 \cos(377t + 16^\circ) \text{ V}$ .

Solution

$$\text{Apparent Power} = S = V_{\text{rms}} I_{\text{rms}}$$

$$S = V_{\text{rms}} I_{\text{rms}}$$

$$I = \frac{V}{Z} = \frac{320 \angle 16^\circ}{72.11 \angle 33.69^\circ} = 4.43 \angle -17.69^\circ$$

$$S = \frac{320}{\sqrt{2}} \cdot \frac{4.43}{\sqrt{2}}$$

$$S = 708.8 \text{ VA}$$

$$PF = \cos(\theta_v - \theta_i)$$

$$PF = \cos(16^\circ + 17.69^\circ)$$

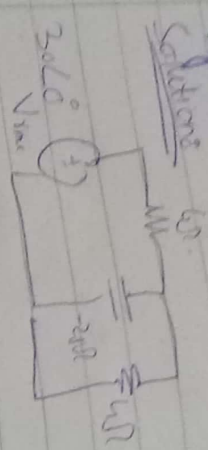
$$PF = 0.8321 \text{ lagging}$$



$\rightarrow$  leading  
 $\leftarrow$  lagging

Example 11.16

Determine the power factor of the entire circuit as seen by the source. Calculate the average power delivered by the source.



$Z = (1 || 2) + 4 = 7 \angle -13.24^\circ$

$Z = 6.8 - 1.6j$

$R = 6.85 \Omega$

$I = \frac{V}{Z} = \frac{30 \angle 0^\circ}{7 \angle -13.24^\circ}$

$I = 4.286 \angle 13.24^\circ \text{ A}$

$\text{P.f.} = \cos(\theta_v - \theta_i) = \cos(0 - 13.24)$

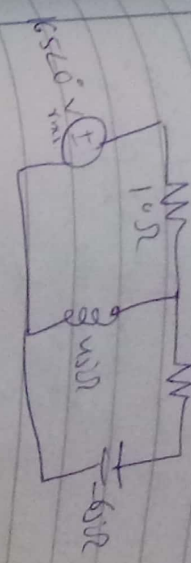
$\text{P.f.} = 0.9734$  leading

$P = I_{\text{rms}}^2 R$

$P = (4.286)^2 (6.8)$

Practice 11.10

Calculate the power factor of the entire circuit as seen by the source. What is the average power supplied by the source?



$Z = 10 || 4 + 1 = 8 - 6j$

$Z = 12.088 \angle -26.65^\circ \Omega$

$Z = 12.0882 + 9.4179j$

$I = \frac{V}{Z} = \frac{48 \angle 0^\circ}{12.088 \angle -26.65^\circ}$

$I = 3.961 \angle 26.65^\circ \text{ A}$

$\text{P.f.} = \cos(0 + 26.65)$

$\text{P.f.} = 0.9857$

$\text{P.f.} = 0.9359$  lagging

$P = I_{\text{rms}}^2 R$

$P = 2004.96 \text{ W}$

Example 11.11

The voltage across the load is  $V(t) = 60 \cos(\omega t - 10^\circ)$  V and the current through the element in the direction of the voltage drop is  $i(t) = 1.5 \cos(\omega t + 50^\circ)$  A. Find a) Complex and apparent power, b) real and reactive power, c) power factor and load impedance.

Complex power  $S = V_{rms} I_{rms}$

$$S = \frac{60 \angle -10^\circ}{\sqrt{2}} \frac{1.5 \angle -50^\circ}{\sqrt{2}}$$

$$S = 22.5 - 38.97j$$

$$S = 45 \angle -60^\circ$$

$$S = |8| = 45 \text{ VA}$$

$$P = 22.5 \text{ W}$$

$$Q = -38.97 \text{ VAR}$$

$$\text{Pf} = \cos(-10 - 50^\circ)$$

$$\text{Pf} = 0.5 \text{ leading}$$

$$Z = \frac{V}{I} = 40 \angle -60^\circ \Omega$$

Practice 11.11

For a load  $V_{rms} = 110 \angle 85^\circ$  V,  $I_{rms} = 0.4 \angle 15^\circ$  A. Determine a) the complex and apparent power, b) real and reactive power, c) power factor and load impedance.

Solutions:

$$S = V_{rms} I_{rms}$$

$$S = 110 \angle 85^\circ \times 0.4 \angle -15^\circ$$

$$S = 15.04 + 41.34j \text{ VA}$$

$$S = 44 \angle 70^\circ \text{ VA}$$

$$|S| = 44 \text{ VA}$$

$$P = 15.04 \text{ W}$$

$$Q = 41.34 \text{ VAR}$$

$$\text{Pf} = \cos(85 - 15)$$

$$\text{Pf} = 0.34 \text{ lagging}$$

$$Z = \frac{V}{I} = 275 \angle 70^\circ \Omega$$



Example 11.12

A load Z draws 12kVA at a power factor of 0.856 lagging from a 120V rms sinusoidal source. Calculate  
 a) average end delivered to load  
 b) peak current  
 c) load impedance

Solutions

$S = 12000 \text{ VA}$   
 $Pf = 0.856$   
 $V_{rms} = 120 \text{ V rms}$

$P = S \cos \phi$   
 $S = 12000$   
 $P = (12000)(0.856)$

$P = 10272 \text{ W}$

$I = 85.6 \text{ A}$   
 $I_{rms} = 100 \text{ A} \angle -31.13^\circ$

$I_{rms} = \frac{I_P}{\sqrt{2}}$   
 $I_P = 141.04$

$Z = \frac{V_{rms}}{I_{rms}}$

$Z = 1.02 \angle 31.13^\circ$

$Q = S \sin \phi$

$Q = 8560 \text{ VAR}$

$Q = (12000) \sin(\cos^{-1}(0.856))$

$Q_2 = 12 \times 10^3 \times 0.516$

$Q = 6.204 \text{ kVAR}$

$S = P + jQ$

$S = 10.272 + j6.204 \text{ kVA}$

$S = V_{rms} I_{rms}^*$

Practise 11.12

A sinusoidal source supplies 100kVAR reactive power to load  $Z = 25 \angle -75^\circ \Omega$ . Determine  
 a) power factor  
 b) apparent power delivered to load  
 c) rms voltage.

Solutions

$Q = 100 \text{ kVAR}$

$Z = 25 \angle -75^\circ \Omega$

$P = S \cos \phi$   
 $Q = S \sin \phi$

$Pf = \cos \phi$   
 $Pf = \cos(-75^\circ)$

$Pf = 0.25$  leading

$\cos \phi = 0.25$

$\phi = 75^\circ$

$S = \frac{Q}{\sin \phi} = 103.52 \text{ kVA}$

$Z = \frac{V_{rms}}{I_{rms}}$

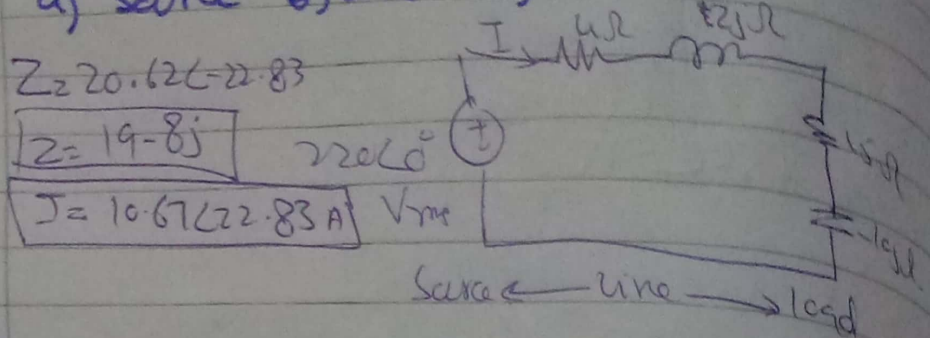
$V_{rms} = Z I_{rms}$

$I_{rms} = \frac{P}{V_{rms}}$   
 $V_{rms} = \sqrt{R} Z^* = 88.514 \angle 81.3092 \text{ V}$

?

Example 11.13

A load being fed by a voltage source through a transmission line. The impedance of the line is represented by the  $(4 + j2)\Omega$  and a return path. Find the real and reactive power absorbed by  
 a) source b) line c) load.



a)  $S = VI^* = (220 \angle 0^\circ)(10.67 \angle -22.83^\circ)$

$S = 2163.5 - 910.8j$  VA

$P = 2163.5$  W

$Q = 910.8$  VAR leading.

b)  $V = (2 + j4) I$

$V = 47.72 \angle 49.4^\circ$

$S = VI^*$

$S = 455.04 + 227.7j$  VA  
 P                      Q

c)  $V = (15 - 10j) I$

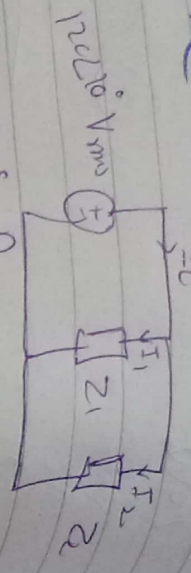
$V = 192.38 \angle -10.87^\circ$

$S = VI^* = 1708 - 1139j$  VA



Example 11.14

$Z_1 = 60 - j30 \Omega$   $Z_2 = 40 \angle 45^\circ \Omega$ . Calculate  
 a) Apparent Power  
 b) Total Power  
 c) reactive Power,  
 d)  $P_f$  supplied by the source, & seen by the source.



$I_1 = \frac{V}{Z_1} = 2 \angle 40^\circ \text{ A}$   
 $I_2 = \frac{V}{Z_2} = 3 \angle -35^\circ \text{ A}$

$S_1 = \frac{V_{rms}^2}{Z_1}$   $S_2 = \frac{V_{rms}^2}{Z_2}$   
 $S_1 = 2271.0 \text{ VA} \text{ (PQVA)}$   $S_2 = 254.62 \text{ VA} \text{ (PQVA)}$

$S_T = S_1 + S_2$   
 $S_T = 1462.0 \angle 134.6^\circ \text{ VA}$   
 $S_T = 181.6 \text{ VA}$   
 $P_f = 0.96$  lagging.

Practice 11.14

Two loads connected in parallel are respectively  $2 \text{ kW}$  at a  $P_f$  of  $0.75$  leading and  $1 \text{ kW}$  at a  $P_f$  of  $0.95$  lagging. Calculate the  $P_f$  of the loads. Find complex power supplied by the source.

Solution:  
 $P_1 = 2 \text{ kW}$   
 $P_f = 0.75$   
 $P_2 = 1 \text{ kW}$   
 $P_f = 0.95$

$\phi_1 = \cos^{-1}(0.75)$   $\phi_2 = \cos^{-1}(0.95)$   
 $Q_1 = 41.4 \text{ VAR}$   $Q_2 = 18.19 \text{ VAR}$

$S = S_1 + S_2 = (P_1 + Q_1j) + (P_2 + Q_2j)$

$S_1 = \frac{P}{\cos \phi_1}$   $S_2 = \frac{P}{\cos \phi_2}$

$S_1 = 2.671 \text{ kVA}$   $S_2 = 1.2105 \text{ kVA}$

$Q_1 = -S_1 \sin \phi_1$   $Q_2 = S_2 \sin \phi_2$   
 $Q_1 = -17.64 \text{ VAR}$   $Q_2 = 13.14 \text{ VAR}$

$S = S_1 + S_2$   
 $S = 6.44 \text{ kVA}$

$P_f = \frac{P_1 + P_2}{S} = \frac{3 \text{ kW}}{6.44 \text{ kVA}} = 0.4658$   
 $P_f = 0.9972$  leading

Example 11-15

When connected to a 120V (rms) source, a load absorbs 400W of real power. A load factor of 0.8. Find the value of the capacitor in microfarads to correct the pf to 0.95.

$P = 400W$

$P = S \cos \theta$   
 $400 = S \cos 0.8$

$S = \frac{400}{0.8} = 500VA$   
 $S_1 = 500VA$

$P = S_2 \cos \theta_2 \Rightarrow S_2 = \frac{P}{\cos \theta_2} = \frac{400}{0.95}$

$S_2 = 421.05VA$

$P = 0.16 S_2 \Rightarrow S_2 = \frac{P}{0.16} = \frac{400}{0.16}$

$S_2 = 2500VA$

$Q_1 = S_1 \sin \theta_1$   
 $Q_1 = 500 \sin 0.6$

$Q_2 = S_2 \sin \theta_2$   
 $Q_2 = 2500 \sin 0.95$

$Q_c = 2500 \sin 0.95 - 500 \sin 0.6$

$Q_c = 1684.91$

$Q_c = 1684.91$

Practice 11-15

Find the value of  $\mu$  needed to correct the power factor of the load to 0.95. Assume a 110V (rms) source.

$P = 600W$   
 $P = S \cos \theta$   
 $600 = S \cos 0.8$

$S = \frac{600}{0.8} = 750VA$   
 $S_1 = 750VA$

$P = S_2 \cos \theta_2 \Rightarrow S_2 = \frac{P}{\cos \theta_2} = \frac{600}{0.95}$

$S_2 = 631.58VA$

$Q_1 = 750 \sin 0.6$

$Q_2 = S_2 \sin \theta_2$   
 $Q_2 = 631.58 \sin 0.95$

$Q_c = 750 \sin 0.6 - 631.58 \sin 0.95$

$Q_c = 222.59VAR$

$P = 600W$

$P = 600W$

$S_1 = 750VA$

$S_2 = 631.58VA$