SEQUENCE

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Sequence: *A list of numbers having specific relation between the consecutive terms is generally called a sequence.*

e.g. 1, 3, 5, 7,..... (next term to a term is obtained by adding 2 with it)

& 2, 6, 18, 54,.....(next term to a term is obtained by multiplying 3 with it)

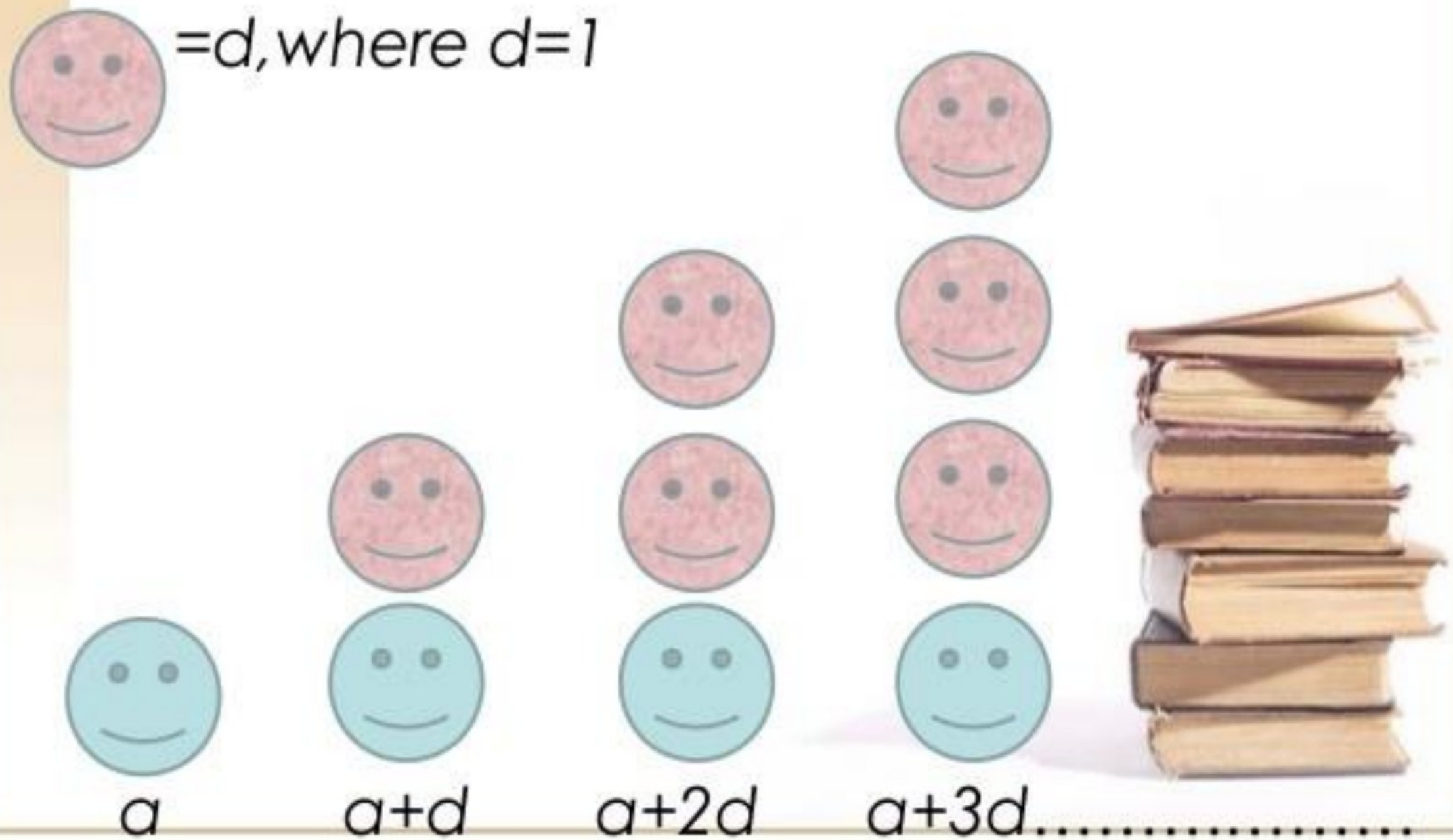
Arithmetic Progression: *If various terms of a sequence are formed by adding a fixed number to the previous term or the difference between two successive terms is a fixed number, then the sequence is called AP.*

e.g.1) 2, 4, 6, 8, ... the sequence of even numbers is an example of AP

2) 5, 10, 15, 20, 25.....

In this each term is obtained by adding 5 to the preceding term except first term.

Illustrative example for A.P.



The general form of an Arithmetic Progression is

$a , a + d , a + 2d , a + 3d \ldots\ldots\ldots\ldots\ldots, a + (n-1)d$

Where 'a' is first term and

'd' is called common difference.

Common Difference - *The fixed number which is obtained by subtracting any term of AP from its previous term.*

If we take
First term of an AP as a
and Common Difference
as d,
Then,
nth term of that AP will be

$$A_n = a + (n-1)d$$

3, 7, 11, 15, 19 ... $d = 4$ $a = 3$

Notice in this sequence that if we find the difference between any term and the term before it we always get 4. 4 is then called the common difference and is denoted with the letter d.

To get to the next term in the sequence we would add 4 so a recursive formula for this sequence is:

$$a_n = a_{n-1} + 4$$

The first term in the sequence would be a_1 which is sometimes just written as a.

+4 +4 +4 +4

3, 7, 11, 15, 19 ...

$d = 4$ $a = 3$

Each time you want another term in the sequence you'd add d. This would mean the second term was the first term plus d. The third term is the first term plus d plus d (added twice). The fourth term is the first term plus d plus d plus d (added three times). So you can see to get the nth term we'd take the first term and add d (n - 1) times.

$$a_n = a + (n-1)d$$

Try this to get the 5th term.

$$a_5 = 3 + (5-1)4 = 3 + 16 = 19$$

Let's see an example!!

Let a=2, d=2, n=12,find A_n

$A_n=a+(n-1)d$

$=2+(12-1)2$

$=2+(11)2$

$=2+22$

Therefore, $\mathbf{A_n=24}$

Hence solved.

To check that a given term is in A.P. or not.

2, 6, 10, 14....

Here first term a = 2,

find differences in the next terms

$a_2 - a_1 = 6 - 2 = 4$

$a_3 - a_2 = 10 - 6 = 4$

$a_4 - a_3 = 14 - 10 = 4$

Since the differences are common.

Hence the given terms are in A.P.

Problem : Find the value of k for which the given series is in A.P. 4, k –1 , 12

Solution : Given A.P. is 4, k –1 , 12.....

If series is A.P. then the differences will be common.

$$d_1 = d_1$$

$$a_2 - a_1 = a_3 - a_2$$

$$k - 1 - 4 = 12 - (k - 1)$$

$$k - 5 = 12 - k + 1$$

$$k + k = 12 + 1 + 5$$

The **sum of n terms,** we find as,

$$Sum = n \times [(first\ term + last\ term) / 2]$$

Now *last term* will be $= a + (n-1)\,d$

Therefore,

$$Sum(S_n) = n \times [\{a + a + (n-1)\,d\} / 2]$$

$$= n/2\ \ \mathbf{[2a + (n-1)d]}$$

DERIVATION

The sum to n terms is given by:

$$S_n = a + (a + d) + (a + 2d) + \ldots + (a + (n - 1)d) \longrightarrow (1)$$

If we write this out backwards, we get:

$$S_n = (a + (n - 1)d) + (a + (n - 2)d) + \ldots + a \longrightarrow (2)$$

Now let's add (1) and (2):

$$2S_n = [2a + (n - 1)d] + [2a + (n - 1)d] + \ldots$$
$$\ldots\ldots\ldots + [2a + (n - 1)d]$$

So, $\mathbf{S_n = n/2\,[2a + (n - 1)d]}$

Problem . Find number of terms of A.P. 100, 105, 110, 115,,.....500

Solution.

First term is $a = 100$, $a_n = 500$

Common difference is $d = 105 - 100 = 5$

nth term is $a_n = a + (n-1)d$

$$500 = 100 + (n-1)5$$

$$500 - 100 = 5(n - 1)$$

$$400 = 5(n - 1)$$

$$5(n - 1) = 400$$

$$5(n - 1) = 400$$

$$n - 1 = 400/5$$

$$n - 1 = 80$$

$$n = 80 + 1$$

$$n = 81$$

Hence the no. of terms are 81.

Problem . Find the sum of 30 terms of given A.P. ,12 , 20 , 28 , 36.....

Solution : Given A.P. is 12 , 20, 28 , 36

Its first term is $a = 12$

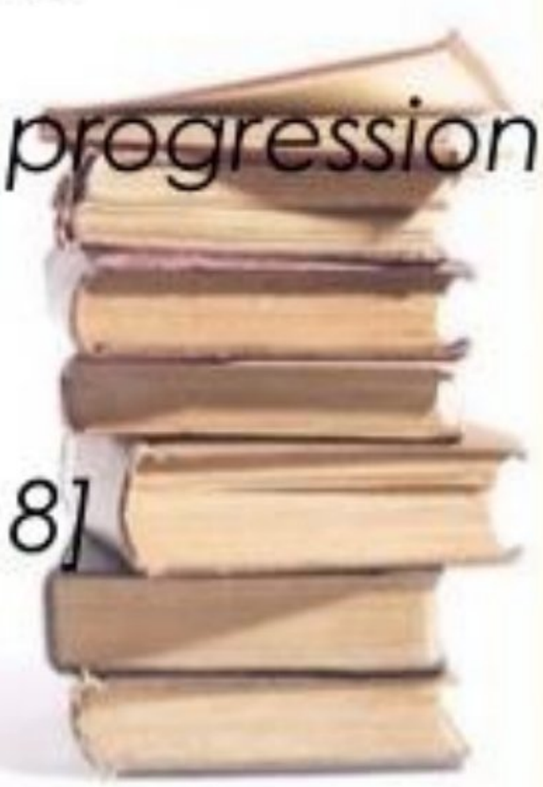
Common difference is $d = 20 - 12 = 8$

The sum to n terms of an arithmetic progression

$$S_n = n/2\ [\ 2a + (n - 1)d\ ]$$

$$= \tfrac{1}{2} \times 30\ [\ 2 \times 12 + (30-1) \times 8]$$

$$= 15\ [\ 24 + 29 \times 8]$$



$= 15[24 + 232]$

$= 15 \times 246$

$= 3690$

THE SUM OF TERMS IS 3690

Problem . Find the sum of terms in given A.P.

2 , 4 , 6 , 8 , ... 200

Solution: Its first term is $a = 2$

Common difference is $d = 4 - 2 = 2$

nth term is $a_n = a + (n-1)d$

$$200 = 2 + (n-1)2$$

$$200 - 2 = 2(n - 1)$$

$$2(n - 1) = 198$$

$$n - 1 = 99, \quad n = 100$$

The sum to n terms of an arithmetic progression

$S_n = n/2[\ 2a + (n - 1)d\ ]$

$S_{100} = 100/2\ [\ 2x\ 2 + (100-1)x\ 2]$

$= 50\ [\ 4 + 198]$

$= 50[202]$

$= 10100$

Arithmetic Mean

- The terms between a_1 and a_n of an arithmetic sequence are called ***arithmetic means*** of a_1 and a_n. Thus, the arithmetic means between a_1 and a_5 are a_2, a_3 and a_4
- The arithmetic mean or the *"mean"* between two numbers is sometimes called the *average* of two numbers.

Sample Problem

1. Find four arithmetic means between 8 and -7.

Answer: Since we must insert four numbers between 8 and -7, there are six numbers in the arithmetic sequence. Thus, $a_1 = 8$ and $a_6 = -7$, we can solve for d using the formula $a_n = a_1 + (n-1)d$.

$$-7 = 8 + (6-1)d$$

$$d = -3$$

Hence,

$$a_2 = a_1 + d = 8 - 3 = 5$$

$$a_3 = a_2 + d = 5 - 3 = 2$$

$$a_4 = a_3 + d = 2 - 3 = -1$$

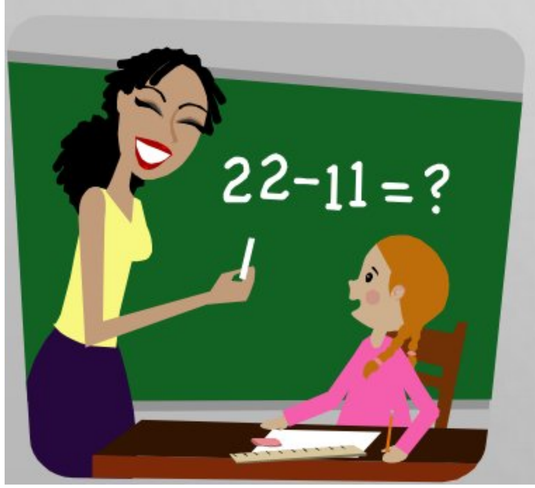
$$a_5 = a_4 + d = -1 - 3 = -4$$

Therefore, the four arithmetic means between 8 and -7 are 5, 2, -1, and -4.

TEST YOURSELF

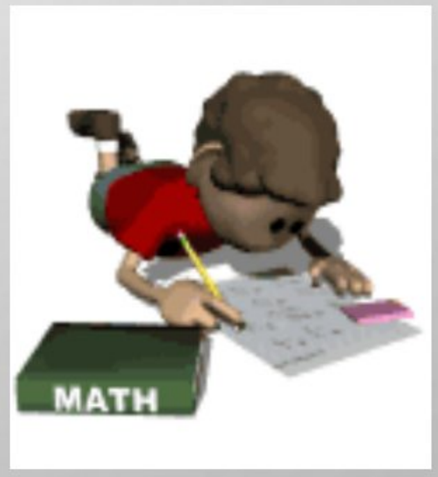
1. Insert seven arithmetic means between 3 and 23.
2. Insert four arithmetic means between 8 and 18.
3. Insert six arithmetic means between 16 and 2.
4. Insert five arithmetic means between 0 and -12.
5. Insert 5 arithmetic means between 7 and 70.

ACTIVITIES



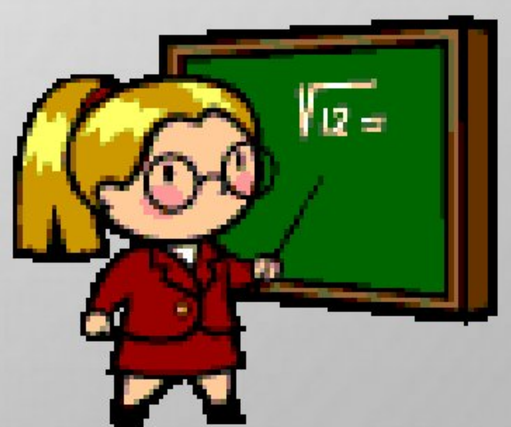
I. DETERMINE WHICH OF THE FOLLOWING SEQUENCES ARE IN A.P. FOR THOSE THAT ARE IN A.P, GIVE THE COMMON DIFFERENCE, AND THE NEXT THREE TERMS OF THE SEQUENCE.

1. 0.1, 0.01, 0.001…
2. 40, 42, 44, 46…
3. 5, 8, 11, 14…
4. 1/3, 1/4, 1/6, 1/12…
5. 1.2, 1.8, 2.4…
6. -11, -7, -3, 1…
7. X+2, 2X+1, 3X…
8. 1/3, 1, 5/3..
9. 5/3, 15/4, 5…
10. √2, √3, √4, √5…



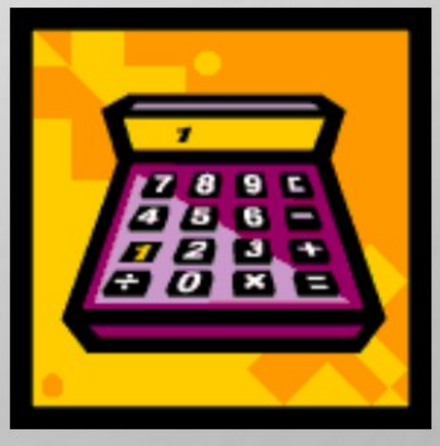
II. GIVEN THE FIRST TERM (A), AND THE COMMON DIFFERENCE (D) OF AN A.P, FIND THE NEXT 5 TERMS.

1. A = 2/5 D = 1/10
2. A = 1.5 D = 0.3
3. A = 3 D = -5
4. A = -3 D = 2
5. A = X+4 D = X-2



III. FIND THE COMMON DIFFERENCE AND INSERT FOUR ARITHMETIC MEANS BETWEEN THE GIVEN NUMBERS.

1. 9 AND 24
2. -25 AND 3
3. 4 AND 179
4. 50.1 AND 50. 7
5. A AND A+12
6. X + 2 AND X + 10



IV. PROBLEM SOLVING. FIND THE VARIABLE BEING ASKED.

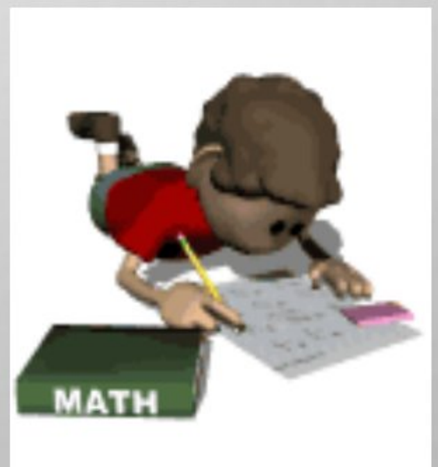
1. IF $5X - 3$, $X + 2$ AND $3X - 11$ FORM AN A.P, FIND X AND T_{21}.

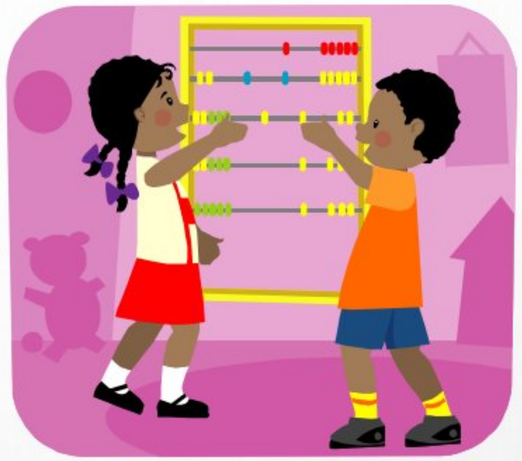
2. IF THE FIRST TERM IS -4, AND THE COMMON DIFFERENCE IS 3, WHAT TERM IS 116?

3. THE NINTH TERM OF AN A.P IS 15, AND THE 17^{TH} TERM IS 27, FIND THE A AND D.

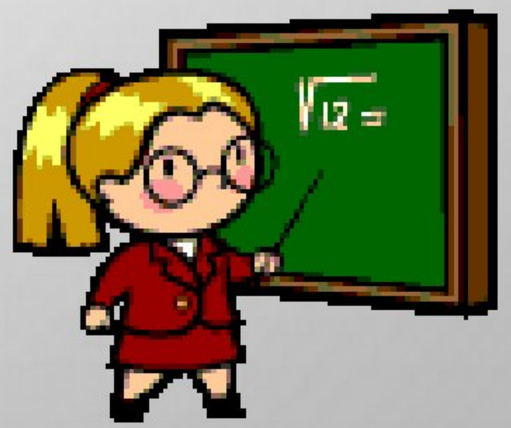
4. THE THIRD TERM OF AN A.P IS 9 AND ITS 7^{TH} TERM IS 49, WHAT IS THE 11^{TH} TERM?

5. A CARPENTER MADE A LADDER WITH 16 RUNGS. THE BOTTOM RUNG IS 70 CM. IF EACH SUCCEEDING RUNG IS 1 CM SHORTER THAN THE PRECEDING, HOW LONG IS THE TOP MOST RUNG?





ANSWERS



TEST I ANSWERS:

1. NOT A.P
2. A.P, D = 2, NEXT 3 TERMS = 48, 50, 52
3. A.P, D = 3, NEXT 3 TERMS = 17, 20, 23
4. A.P, D = -1/12, NEXT 3 TERMS = 0, -1/12, -1/6
5. A.P, D = 0.6, NEXT 3 TERMS = 3, 3.6, 4.2
6. A.P, D = 4, NEXT 3 TERMS = 5, 9, 13
7. A.P, D = X-1, NEXT 3 TERMS = 4X-1, 5X-2, 6X-3
8. A.P, D = 2/3, NEXT 3 TERMS = 2 1/3, 3, 3 2/3
9. NOT A.P
10. NOT A.P

TEST II ANSWERS:

1. 1/2, 3/5, 7/10, 4/5, 9/10
2. 1.8, 2.1, 2.4, 2.7, 3
3. -2, -7, -12, -17, -22
4. -1, 1, 3, 5, 7
5. 2X+2, 3X, 4X-2, 5X-4, 6X-6

TEST III ANSWERS:

1. D = 3, FOUR ARITHMETIC MEANS = 12, 15, 18, 21
2. D = 28/5, FOUR ARITHMETIC MEANS = -19 2/5, -13 4/5, -8 1/5, -2 3/5
3. D = 35, FOUR ARITHMETIC MEANS = 39, 74, 109, 144
4. D = 0.12, FOUR ARITHMETIC MEANS = 50.22, 50.34, 50.46, 50.58
5. D = 12/5, FOUR ARITHMETIC MEANS = A + 12/5, A + 24/5, A + 36/5, A + 48/5
6. D = 8/5, FOUR ARITHMETIC MEANS = X + 18/5, X + 26/5, X + 34/5, X + 42/5

TEST IV ANSWERS:

1. $X = 3, T_{21} = -128$
2. $N = 41^{ST}$ TERM
3. $D = 3/2, A = 3$
4. 11^{TH} TERM = 89
5. $T_{16} = 55$ CM

THE END . . .