Applications of Quadratic Equations

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We can now use factoring to solve quadratic equations that arise in application problems.

Solving an Applied Problem

Step 1: **Read** the problem carefully. What information is given? What are you asked to find?

Step 2: **Assign a variable** to represent the unknown value. Use a sketch, diagram, or table, as needed. If necessary, express any other unknown values in terms of the variable.

Step 3: **Write an equation,** using the variable expression(s).

Step 4: **Solve** the equation.

Step 5: **State the answer.** Label it appropriately. Does it seem reasonable?

Step 6: **Check** the answer in the words of the original problem.

In solving applied problems ***always check solutions*** against physical facts and discard any answers that are not appropriate.

Solve problems involving geometric figures.

Solving an Area Problem

A right triangle has one leg that is 3 m shorter than the other leg. The triangle has area of 54 m^2. Find the lengths of the legs.

Solution:

Let x = the length of the one leg.

Then $x - 3$ = the length of the shorter leg.

$$A = \frac{1}{2}bh$$

$$2 \cdot 54 = \left(\frac{1}{2}x(x-3)\right)2$$

$$108 - 108 = x^2 - 3x - 108$$

$$0 = x^2 - 3x - 108$$

$$0 = (x+9)(x-12)$$

$$0 - 9 = x + 9 - 9$$

$$x = -9$$

$$0 + 12 = x - 12 + 12$$

$$x = 12$$

Since − 9 doesn't make sense in this case the one leg of the triangle is 12 m and the other leg is 9 m in length.

Solve problems involving consecutive integers.

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Recall from **Section 2.4** that **consecutive integers** are next to each other on a number line, such as 5 and 6, or −11 and −10.

Consecutive odd integers are ***odd*** integers that are next to each other, such as 5 and 7, or −13 and −11.

Consecutive even integers are defined similarly; for example, 4 and 6 are consecutive ***even*** integers, as are −10 and −8.

PROBLEM-SOLVING HINT

if x represents the first integer, then for

- two consecutive integers, use x, $x + 1$;
- three consecutive integers, use x, $x + 1$; $x + 2$;
- two consecutive even or odd integers, use x, $x + 2$;
- three consecutive even or odd integers, use x, $x + 2$; $x + 4$.

Solving a Consecutive Integer Problem

The product of the first and third of three consecutive odd integers is 16 more than the middle integer. Find the integers.

Solution:

Let x = the first integer,
then $x + 2$ = the second integer,
and $x + 4$ = the third integer.

$$x(x+4)=(x+2)+16$$

$$x^2+4x-(x+18)=x+18-(x+18)$$

$$x^2+3x-18=0$$

$$(x+6)(x-3)=0$$

$$x-3+3=0+3$$

$$x=3$$

$$x+6-6=0-6$$

$$x=-6$$

Since −6 is an even integer, the first integer is 3, the second integer is 5, and the third integer is 7.

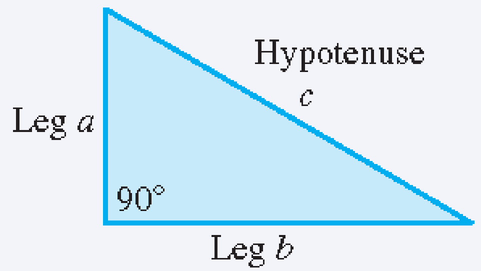
SOLVE PROBLEMS BY APPLYING THE PYTHAGOREAN THEOREM.

Solve problems by using the Pythagorean formula.

Pythagorean Formula

If a right triangle has longest side of length c and two other sides of lengths a and b, then

$$a^2 + b^2 = c^2.$$



The longest side, the **hypotenuse,** is opposite the right angle. The two shorter sides are the **legs** of the triangle.

PROBLEM-SOLVING HINT

In solving a problem involving the Pythagorean formula, be sure that the expressions for the sides are properly placed.

$$(\text{one leg})^2 + (\text{other leg})^2 = \text{hypotenuse}^2$$

Pythagorean formula can only be applied to right triangles.

Applying the Pythagorean Theorem

The hypotenuse of a right triangle is 3 in. longer than the longer leg. The shorter leg is 3 in. shorter than the longer leg. Find the lengths of the sides of the triangle.

Solution:

Let x = the length of the longer leg,
then $x + 3$ = the length of the hypotenuse,
and $x - 3$ = the length of the shorter leg.

$$(x+3)^2 = (x-3)^2 + x^2$$
$$x^2 + 6x + 9 = x^2 - 6x + 9 + x^2$$
$$(x^2+6x+9) - (x^2+6x+9) = 2x^2 - 6x + 9 - (x^2+6x+9)$$
$$0 = x^2 - 12x$$

$$0 = x(x-12)$$
$$0 = x$$

or

$$0 = x - 12$$
$$0 + 12 = x - 12 + 12$$
$$12 = x$$

Since 0 does not make sense in this case, the longer leg is 12 in., the shorter leg is 9 in., and the hypotenuse is 15 in.

SOLVE PROBLEMS BY USING GIVEN QUADRATIC MODELS.

Finding the Height of a Ball

The number *y* of impulses fired after a nerve has been stimulated is modeled by $y = -x^2 + 2x + 60$, where *x* is in milliseconds (ms) after the stimulation. When will 45 impulses occur? Do we get two solutions? Why is only one answer acceptable?

Solution:

$$45 - 45 = -x^2 + 2x + 60 - 45$$
$$0 = -x^2 + 2x + 15$$
$$0 = (-x + 5)(x + 3)$$

$$0 - 5 = -x + 5 - 5$$
$$-x = -5$$
$$x = 5$$

$$0 - 3 = x + 3 - 3$$
$$x = -3$$

After 5 ms; there are two solutions, −3 and 5; only one answer makes sense here, because a negative answer is not appropriate.

Modeling the Foreign-Born Population of the United States

Use the model, $y = 0.01048x^2 - 0.5400x + 15.43$, to find the foreign-born population of the United States in 1990. Give your answer to the nearest tenth of a million. How does it compare against the actual data from the table?

Year	Foreign-Born Population (millions)
1930	14.2
1940	11.6
1950	10.3
1960	9.7
1970	9.6
1980	14.1
1990	19.8
2000	28.4
2007	37.3

Solution:

When $x = 0$ represents 1930, then let $x = 60$ represents 1990.

$$y = 0.01048x^2 - 0.5400x + 15.43$$

$$y = 0.01048(60)^2 - 0.5400(60) + 15.43$$

$$y = 0.01048(3600) - 0.5400(60) + 15.43$$

$$y = 37.728 - 32.4 + 15.43$$

$$y = 20.758$$

20.8 million; The actual value is 19.8 million, so our answer using the model is somewhat high.