First Derivative test for Extrema

Let 'C' be a critical number for f, and suppose that f is continuous at 'C' and diff on on open interval I containing C. except possibly at C itself.

1) If f' is changes from + to − at C then $f(c)$ is local maximum of f

2) If f' is changed from − to +ve at c then $f(c)$ is local minimum of f.

3) If $f'(x) > 0$ or $f'(x) < 0$ for x in I except $x = c$ then $f(c)$ is not local extrema.

Question: Find the local extrema of the

$$f(x) = x^3 + x^2 - 5x - 5 = 0 \longrightarrow (1)$$

$$f'(x) = 3x^2 + 2x - 5 \longrightarrow (2)$$

For critical point's $f' = 0$ in (2)

$$3x^2 + 2x - 5 = 0$$

$$3x^2 - 3x + 5x - 5 = 0$$

$$3x(x-1) + 5(x-1) = 0$$

$$(x-1)(3x+5) = 0$$

$$x = 1 \quad , \quad x = -5/3$$

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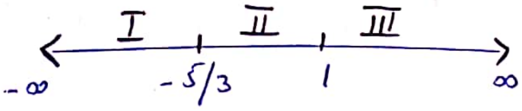


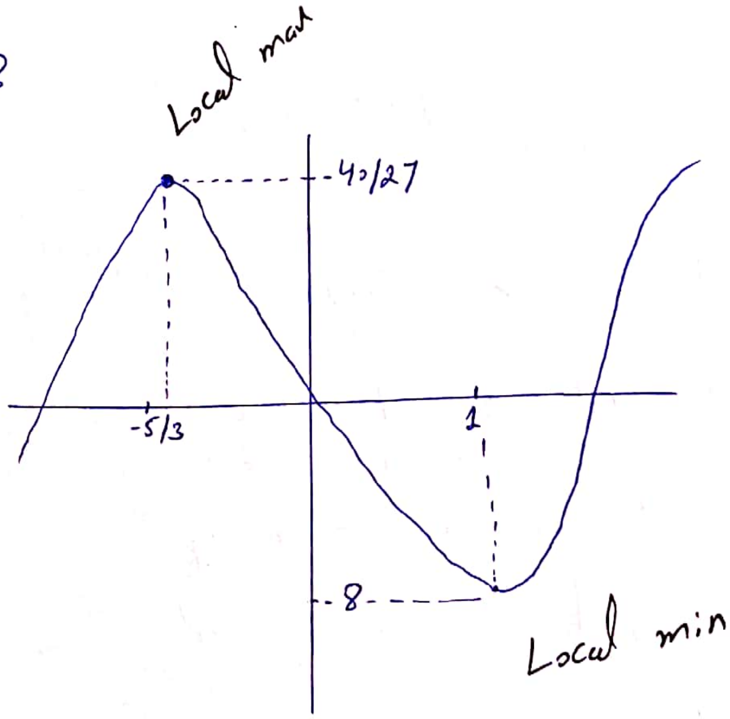
Intervals	$(-\infty, -5/3)$	$(-5/3, 1)$	$(1, \infty)$
Testing value k	-2	0	2
$f'(k)$	$f'(-2) = 3$	$f'(0) = -5$	$f'(2) = 11$
sign of $f'(k)$	$+$	$-$	$+$

So f has local maximum at $c = -5/3$ by 1^{st} derivative test.

$f(-\frac{5}{3}) = 40/27$

So f has local minimum at $c = 1$ by 1^{st} derivative test

$f(1) = -8$



Example: a) Find the interval on which F is increasing or decreasing
b) Find the local extrema c) Sketch the graph

$$F(x) = x^{2/3} \cdot (x-7)^2 + 2 \longrightarrow (1)$$

$$F'(x) = \frac{2}{3} x^{-1/3} (x-7)^2 + x^{2/3} \cdot 2(x-7) + 0$$

$$= 2(x-7)\left(\frac{1}{3} x^{-1/3}(x-7) + x^{2/3}\right)$$

$$F'(x) = 2(x-7)\left[\frac{1}{3} x^{-1/3+1} - \frac{7}{3} x^{-1/3} + x^{2/3}\right]$$

$$= 2(x-7)\left[\frac{1}{3} x^{2/3} - \frac{7}{3} x^{-1/3} + x^{2/3}\right]$$

$$= \frac{2}{3}(x-7)\left[x^{2/3} - 7x^{-1/3} + 3x^{2/3}\right]$$

$$= \frac{2}{3}(x-7)\left[4x^{2/3} - 7x^{-1/3}\right]$$

$$F'(x) = \frac{2}{3}(x-7)\left[4x^{2/3} - \frac{7}{x^{1/3}}\right]$$

$$= \frac{2}{3}(x-7)\left[\frac{4x - 7}{x^{1/3}}\right]$$

$$F'(x) = \frac{2(x-7)(4x-7)}{3x^{1/3}} \longrightarrow (2)$$

For critical number $F'(x) = 0$

$$2(x-7)(4x-7) = 0$$

$$x = 7, \quad x = 7/4$$

$F'(x)$ does not exist $3x^{1/3} = 0 \Rightarrow x = 0$

critical numbers $0, 7/4, 7$

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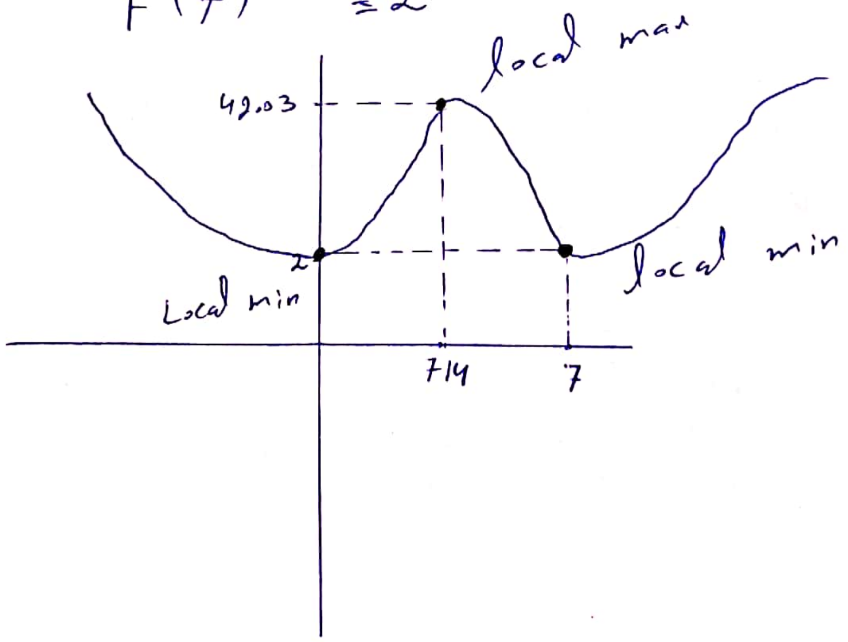
Intervals	$(-\infty, 0)$	$(0, 7/4)$	$(7/4, 7)$	$(7, \infty)$
Testing val k	-1	1	2	8
$f'(k)$	$f'(-1) = -58.6 < 0$	$f'(1) = 12 > 0$	$f'(2) = -2.7 < 0$	$f'(8) = 8.37 > 0$
Sign of $f'(k)$	$-$	$+$	$-$	$+$
Conclusion	f is dec on $(-\infty, 0]$	f is inc on $[0, 7/4]$	f is dec on $[7/4, 7]$	f is increasing on $[7, \infty)$

b)

1) At $c = 0$ f' changes from $-ve$ to $+ve$ on 0.
So f has local minimum at $0 \Rightarrow f(0) = 2$

2) At $c = 7/4$ f' changes from $+ve$ to $-ve$ at $7/4$, so f has local max at $7/4$
$\Rightarrow f(7/4) = 42.03$

3) At $c = 7$ f' changes from $-ve$ to $+ve$ at 7 so f has local min at 7.
$f(7) = 2$



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