Critical Numbers

An interior point 'c' of the domain of a function f is a critical number of f if either

$f'(c) = 0$ or $f'(c)$ does not exist

Example: Find the critical number of

$$f(x) = 2x^3 + x^2 - 20x + 4$$

$$f'(x) = 6x^2 + 2x - 20 \longrightarrow (1)$$

Critical number: $f'(x) = 0$ Put in (1)

$$6x^2 + 2x - 20 = 0$$

$$2(3x^2 + x - 10) = 0$$

$$3x^2 + x - 10 = 0$$

$$3x^2 + 6x - 5x - 10 = 0$$

$$3x(x+2) - 5(x+2) = 0$$

$$x + 2 = 0 \quad \text{or} \quad 3x - 5 = 0$$

$$x = -2 \quad \text{or} \quad x = 5/3$$

Critical numbers are $x = -2, x = \frac{5}{3}$

Example: Find the critical number of

$$f(x) = \sqrt{x^2 - 25}$$

$$f(x) = (x^2 - 25)^{1/2}$$

$$f'(x) = 1/2 (x^2 - 25)^{-1/2} (2x)$$

$$= \frac{2x}{2\sqrt{x^2 - 25}}$$

$$f'(x) = \frac{x}{\sqrt{x^2 - 25}} \longrightarrow (1)$$

Domain

$x^2 - 25 \geq 0$

$x \in (-\infty, -5] \cup [5, \infty)$

Put, $f'(x) = 0$ in (1)

$$\frac{x}{\sqrt{x^2-25}} = 0$$

$$x = 0$$

$$x^2 - 25 = 0$$

$$x^2 = 25$$

$$x = \pm 5$$

So C.N are $-5, 0, 5$

So G.N are $-5, 5$

Example: Find the critical numbers of

$$f(x) = \sin^2 x - \cos x$$

$$f'(x) = 2\sin x \cos x + \sin x$$

$$2\sin x \cos x + \sin x = 0$$

$$\sin x(2\cos x + 1) = 0$$

$$\sin x = 0 \quad \text{or} \quad 2\cos x + 1 = 0$$

$x = \sin^{-1}(0)$ $\qquad$ $\cos x = -1/2$

$x = 0$ $\qquad$ $x = \cos^{-1}(-1/2)$

$= 120^\circ = 2\pi/3$

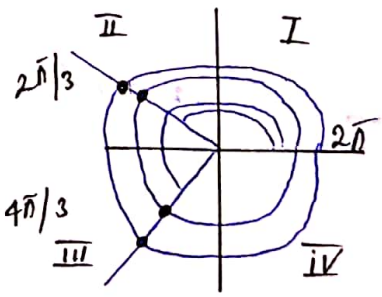
$= 240 = 4\pi/3$

Critical numbers:

$$x = 0 + n\pi$$

$$x = \frac{2\pi}{3} + 2n\pi$$

$$x = \frac{4\pi}{3} + 2n\pi$$



Example:

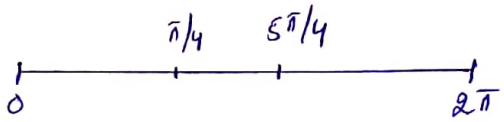
$f(x) = \sin x + \cos x \qquad [0, 2\pi]$

$f'(x) = \cos x - \sin x$

$0 = \cos x - \sin x$

$\cos x = \sin x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$



$(0, \pi/4), (\frac{\pi}{4}, \frac{5\pi}{4}), (\frac{5\pi}{4}, 2\pi)$

Increasing and Decreasing of a function by derivative:

Theorem: Let f be continuous on $[a,b]$ and differentiable on (a,b).

i) If $f'(x) > 0$, $x \in (a,b)$ then f is increasing on $[a,b]$

ii) If $f'(x) < 0$, $x \in (a,b)$ then f is decreasing on $[a,b]$

Question:

Find the intervals on which f is increasing decreasing. $f(x) = x^3 - x^2 - 40x + 8$

$f'(x) = 3x^2 - 2x - 40$

Critical number: $f'(0) = 0$

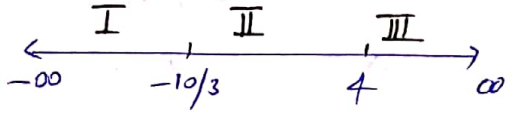
$\Rightarrow 3x^2 - 2x - 40 = 0$

$3x^2 - 12x + 10x - 40 = 0$

$3x(x-4) + 10(x-4) = 0$

$(x-4)(3x+10) = 0$

$x = 4 \quad , \quad x = -10/3$



Intervals	$(-\infty, -10/3)$	$(10/3, 4)$	$(4, \infty)$
Testing value	-4	0	5
f'	$f'(4) = 16 > 0$ +ve	$f'(0) = -40 < 0$ −ve	$f'(5) = 25$ +ve
conclusion	f is increasing on $(-\infty, -10/3]$	f is decreasing on $[-10/3, 4]$	f is increasing on $[4, \infty)$