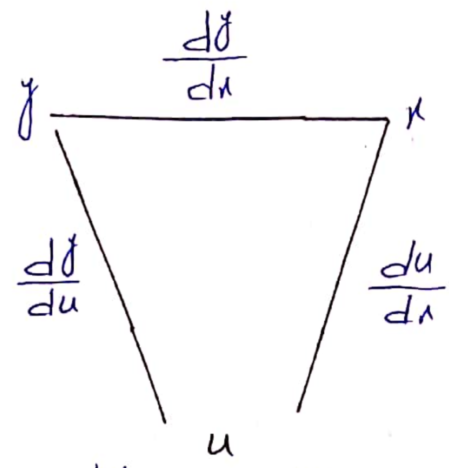
Chain - Rule



$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Q#1

$(x^2+x)^4$ w.r.t 'x'

Let $y = (x^2+x)^4$

$$y = (u)^4, \qquad u = x^2 + x$$

$$\frac{dy}{du} = 4u^3, \qquad \frac{du}{dx} = 2x+1$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 4u^3 \cdot (2x+1)$$

$$= 4(x^2+x)^3 (2x+1)$$

Q#2

$$y = \left(\frac{3+4x}{2-x}\right)^2 \quad \text{w.r.t 'x'}$$

$$y = (u)^2, \qquad u = \frac{3+4x}{2-x}$$

$$\frac{dy}{du} = 2u, \qquad \frac{du}{dx} = \frac{(2-x)(4)-(3+4x)(-1)}{(2-x)^2}$$

$$= \frac{8-4x+3+4x}{(2-x)^2}$$

$$= 11/(2-x)^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 2u \cdot \frac{11}{(2-x)^2}$$

$$= 2\left(\frac{3+4x}{2-x}\right) \cdot \left(\frac{11}{(2-x)^2}\right)$$

$$= \frac{22(3+4x)}{(2-x)^3}$$

Q#3

$y = v^3 + 2$, $v = 3u + 1$, $u = 9x + 1$, $\frac{dy}{dx} = ?$

$\frac{dy}{dv} = 3v^2$, $\frac{dv}{du} = 3$, $\frac{du}{dx} = 9$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= 3v^2 \cdot 3 \cdot 9$$

$$= 81 v^3$$

$$= 81(3u + 1)$$

$$= 81(3(9x+1) + 1)$$

$$= 81(27x + 3 + 1)$$

$$= 2187x + 243 + 81$$

$$= 2187x + 324$$

Q#4

$$y = (a + bx)^{1/2}$$

$y = (t)^{1/2}$ $(a+bx) = t$

$\frac{dy}{dt} = \frac{1}{2} t^{-1/2}$ $\frac{dt}{dx} = b$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$
$$= \frac{1}{2} t^{-1/2} \cdot b$$
$$= \frac{1}{2} (a+bx)^{-1/2} \cdot b$$
$$= \frac{b}{2\sqrt{a+bx}}$$

M-2

$$y = t^{1/2}, \quad t = (a+bx)$$
$$= \frac{1}{2} t^{-1/2} \frac{dt}{dx}$$
$$= \frac{1}{2} t^{-1/2} \frac{d}{dx}(a+bx)$$
$$= \frac{1}{2} (a+bx)^{-1/2} (0+b)$$
$$= \frac{b}{2\sqrt{a+bx}}$$

Q#5

$$y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$$

Let $y = (t)^2$

$$\frac{dy}{dx} = 2t \cdot \frac{dt}{dx}$$
$$= 2\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) \cdot \frac{d}{dx}\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)$$
$$= 2\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) \cdot \left(\frac{1}{2} x^{-1/2} + \left(-\frac{1}{2}\right) x^{-3/2}\right)$$
$$= 2\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) \cdot \left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x^3}}\right)$$

Mean Value Theorem

If f is continuous on closed interval $[a,b]$ and differentiable on (a,b), then there exist a number c in (a,b) such that:

$$f'(c) = \frac{f(b)-f(a)}{b-a} \Rightarrow f(b) - f(a) = f'(c)(b-a)$$

Question

If $f(x) = \frac{1}{4}x^2+1$, f is diff on $(-1,4)$ and and cont: on $[-1,4]$, Find a number c in $(-1,4)$ That satisfies the condition and conclusion of M.V.T.

f is cont and diff so

$$f(x) = \frac{1}{4}x^2 + 1$$

$$f'(x) = \frac{1}{4}(2x)$$

$$= \frac{x}{2}$$

$$f'(c) = \frac{c}{2}$$

$a = -1$, $b = 4$

$$f(a) = f(-1) = \frac{1}{4}(-1)^2 + 1$$

$$= \frac{1}{4} + 1 = 5/4$$

$$f(b) = f(4) = \frac{1}{4}(4)^2 + 1$$

$$= \frac{1}{4} \cdot 16 + 1$$

$$f(4) = 5$$

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

$$\frac{c}{2} = \frac{5 - \frac{5}{4}}{4-(-1)}$$

$$\frac{c}{2} = \frac{\frac{15}{4}}{5}$$

$$c = 3/2 \in (-1,4)$$