Derivative of Natural Logarithm function's

$F(x) = \ln x$

$\frac{d}{dx} \ln|x| = \frac{1}{x}$

$\frac{d}{dx}\left[\ln|F(x)| \right] = \frac{1}{f(x)} \cdot \frac{d}{dx}(f(x))$

Laws of Natural Log

If $p > 0$ and $q > 0$, then

i) $\ln(pq) = \ln p + \ln q$

ii) $\ln(p/q) = \ln p - \ln q$

iii) $\ln p^{r} = r \ln p$

For every rational number r.

Q#1:

Find y', if $y = \ln(2x+5)$

$$\frac{dy}{dx} = \frac{d}{dx}(\ln(2x+5))$$

$$= \frac{1}{2x+5} \frac{d}{dx}(2x+5)$$

$$= \frac{1}{2x+5}(2)$$

$$= \frac{2}{2x+5}$$

Q#2:

Find y', if $y = \ln(4x^2+3)^5$

$$y = 5\ln(4x^2+3)$$

$$\frac{dy}{dx} = 5 \cdot \frac{d}{dx} \ln(4x^2+3)$$

$$= 5 \cdot \frac{1}{4x^2+3}\left(\frac{d}{dx}(4x^2+3)\right)$$

$$= 5 \cdot \frac{1}{4x^2+3}(8x)$$

$$= \frac{40x}{4x^2+3}$$

Q#3:

Find $\frac{dy}{dx}$ if $y = \ln \sqrt[3]{\frac{x^2-3}{4x^2+5}}$

$$y = \ln\left(\frac{x^2-3}{4x^2+5}\right)^{1/3} = \frac{1}{3}\ln\left[\frac{x^2-3}{4x^2+5}\right]$$

$$y = \frac{1}{3}\left[\ln(x^2-3) - \ln(4x^2+5)\right]$$

$$y' = \frac{1}{3}\left[\frac{1}{x^2-3}\frac{d}{dx}(x^2-3) - \frac{1}{4x^2+5}\frac{d}{dx}(4x^2+5)\right] \quad \because \ln(P/Q) = \ln P - \ln Q$$

$$y' = \frac{1}{3}\left[\frac{2x}{x^2-3} - \frac{8x}{4x^2+5}\right]$$

Q#4

Use implicit diff. to find y' if y is a Function of x.

Assume $y = f(x)$

$$y^3 + \sin y + \ln\left(\frac{x}{y}\right) + \ln(y^2) = 3x^2+4$$

$$y^3 + \sin y + \ln x - \ln y - 2\ln y = 3x^2+4$$

$$3y^2\frac{dy}{dx} + \cos y\frac{dy}{dx} + \frac{1}{x}(1) - \frac{1}{y}\frac{dy}{dx} - 2\frac{1}{y}\frac{dy}{dx} = 6x$$

$$\frac{dy}{dx}\left[3y^2 + \cos y - \frac{1}{y} - \frac{2}{y}\right] = 6x - \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{6x - 1/x}{3y^2 + \cos y - 1/y - 2/y}$$

Derivative of Exponential fn:

Formula:

$$f(x) = e^x$$

$$F'(x) = e^x \, d/dx\,(x) = e^x$$

Q#1:

Find y' if $y = e^{\sqrt{4x^2+5}}$

$$y' = \frac{d}{dx} e^{(4x^2+5)^{\frac{1}{2}}}$$

$$= e^{\sqrt{4x^2+5}} \frac{d}{dx} (4x^2+5)^{1/2}$$

$$= e^{\sqrt{4x^2+5}} \frac{1}{2} (4x^2+5)^{-1/2} \frac{d}{dx} (4x^2+5)$$

$$= e^{\sqrt{4x^2+5}} \frac{8x}{2(4x^2+5)^{1/2}}$$

$$y' = \frac{8x \cdot e^{\sqrt{4x^2+5}}}{2\sqrt{4x^2+5}}$$

$$y' = \frac{4x\, e^{\sqrt{4x^2+5}}}{\sqrt{4x^2+5}}$$

Q#2:

Find y' if $y = e^{\sin e^{-3x}}$

$$y' = \frac{d}{dx} e^{\sin e^{-3x}}$$

$$= e^{\sin e^{-3x}} \frac{d}{dx} (\sin e^{-3x})$$

$$= e^{\sin e^{-3x}} \cos e^{-3x} \frac{d}{dx} (e^{-3x})$$

$$= e^{\sin e^{-3x}} \cos e^{-3x} (e^{-3x} (-3))$$

$-3e^{-3x} \cdot e^{\sin e^{-3x}} \cos e^{-3x}$

Q#3: Use Implisit diff to find y'

Assume $y = f(x)$, $1 + xy = e^{xy}$

$$\frac{d}{dx}(1+xy) = \frac{d}{dx}(e^{xy})$$

$$0 + (y + x\frac{dy}{dx}) = e^{xy}\left[\frac{d}{dx}(xy)\right]$$

$$y + xy' = e^{xy}[y + xy']$$

$$y + xy' = ye^{xy} + xy'e^{xy}$$

$$xy' - xy'e^{xy} = ye^{xy} - y$$

$$y'[x - xe^{xy}] = ye^{xy} - y$$

$$y' = \frac{ye^{xy} - y}{x - xe^{xy}}$$

Derivative of General Exponential Function:

$F(x) = a^x$, For every $a > 0$, For every real no. x.

$$\frac{d}{dx}\left[a^{f(x)}\right] = a^{f(x)} \cdot \ln a \cdot \frac{d}{dx} f(x)$$

Q#1: Find y', if $y = 2^{x^2}$

$$y' = 2^{x^2} \cdot \ln 2 \; d/dx\,(x^2)$$

$$= 2^{x^2} \cdot \ln 2\,(2x)$$

Q#2:

Find y' if $y = \pi^x$

$y' = \pi^x . \ln \pi \frac{d}{dx}(x)$

$= \pi^x \ln \pi (1)$

$= \pi^x \ln . \pi$

Q#3:

Find y', $y = (10^x + 10^{-x})^{10}$

$y' = 10(10^x + 10^{-x})^9 . \frac{d}{dx}(10^x + 10^{-x})$

$= 10(10^x + 10^{-x})^9 [10^x \ln 10 + 10^{-x} \ln 10(-1)]$

$= 10(10^x + 10^{-x})^9 [\ln 10 (10^x - 10^{-x})]$

Q#4:

Find y', $y = 5^{\sin^2(4x)}$

$y' = \frac{d}{dx}(5^{\sin^2(4x)})$

$= 5^{\sin^2(4x)} . \ln 5 . \frac{d}{dx}(\sin^2(4x))$

$= 5^{\sin^2(4x)} . \ln 5 . (2\sin 4x . \frac{d}{dx}(\sin 4x))$

$= 5^{\sin^2(4x)} . \ln 5 (2\sin 4x . \cos 4x . \frac{d}{dx}(4x))$

$= 5^{\sin^2(4x)} . \ln 5 (2\sin 4x . \cos 4x (4))$

$= 8 . 5^{\sin^2(4x)} . \ln 5 \sin 4x . \cos 4x$

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Q#5:

Find y', if $y = 5^{e^{\sin \ln x}}$

$$y' = \frac{d}{dx}\left[5^{e^{\sin \ln x}}\right]$$

$$= 5^{e^{\sin \ln x}} \cdot \ln 5 \cdot \frac{d}{dx}\left(e^{\sin \ln x}\right)$$

$$= 5^{e^{\sin \ln x}} \cdot \ln 5 \cdot e^{\sin \ln x} \frac{d}{dx}\left[\sin \ln x\right]$$

$$= 5^{e^{\sin \ln x}} \cdot \ln 5 \cdot e^{\sin \ln x} \cdot \cos \ln x \left(1/x\right)$$

Derivative of General Log-Fn's:

Formula:

$$\frac{d}{dx}\left(\log_a f(x)\right) = \frac{d}{dx}\left(\frac{\ln f(x)}{\ln a}\right) = \frac{1}{\ln a} \cdot \frac{1}{f(x)} \cdot f'(x)$$

Q#1:

Find $\frac{dy}{dx}$, if $y = \log_4 |x^2 + 5|$

$$y' = \frac{d}{dx}\left(\log_4 |x^2 + 5|\right)$$

$$= \frac{d}{dx}\left(\frac{\ln |x^2+5|}{\ln 4}\right)$$

$$= \frac{1}{\ln 4} \frac{d}{dx}\left(\ln |x^2 + 5|\right)$$

$$= \frac{1}{\ln 4} \cdot \frac{1}{x^2+5} \cdot d/dx \left(x^2 + 5\right)$$

$$= \frac{1}{\ln 4} \cdot \frac{1}{x^2+5}\left(2x\right)$$

Q#2

Find y', $y = \log\left|\frac{1-x^2}{2-5x^3}\right|$

$$= \log|1-x^2| - \log|2-5x^3|$$

$$y' = \frac{d}{dx}\left(\frac{\ln(1-x^2)}{\ln 10}\right) - \frac{d}{dx}\left(\frac{\ln(2-5x^3)}{\ln 10}\right)$$

$$= \frac{1}{\ln 10} \cdot \frac{1}{1-x^2} \frac{d}{dx}(1-x^2) - \frac{1}{\ln 10} \cdot \frac{1}{2-5x^3} \cdot \frac{d}{dx}(2-5x^3)$$

$$= \frac{1}{\ln 10} \cdot \frac{1}{1-x^2}(-2x) - \frac{1}{\ln 10} \cdot \frac{1}{2-5x^3}(-15x^2)$$

$$y' = \frac{-1}{\ln 10}\left[\frac{2x}{1-x^2} - \frac{15x^2}{2-5x^3}\right]$$

Derivative of Inverse Trigonometric function's

Q#1

Find y', $y = \sin^{-1}(\sqrt{x})$

$$y' = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{d}{dx}(x^{1/2})$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2}(x^{\frac{1}{2}-1})$$

$$= \frac{1}{2\sqrt{x} \cdot \sqrt{1-x}}$$

Formula's:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

Q#2

Find y', $y = \ln(\arctan x^2)$

$$y' = \frac{d}{dx}\left(\ln(\tan^{-1} x^2)\right)$$

$$= \frac{1}{\tan^{-1} x^2} \cdot \frac{d}{dx}(\tan^{-1} x^2)$$

$$= \frac{1}{\tan^{-1} x^2} \cdot \frac{1}{1+(x^2)^2} \frac{d}{dx}(x^2)$$

$$= \frac{1}{\tan^{-1} x^2} \cdot \frac{2x}{1+x^4}$$

Q#3

Find y', if $y = f(x)$ is implicit function.

$$y \ln x + x \sin^{-1} y = y e^x$$

$$\frac{d}{dx}\left[y \ln x + x \sin^{-1} y\right] = \frac{d}{dx}(y e^x)$$

$$\left(\frac{dy}{dx} \cdot \ln x + y \cdot \frac{1}{x}\right) + \left(1 \cdot \sin^{-1} y + x \cdot \frac{1}{\sqrt{1-y^2}} \cdot \frac{dy}{dx}\right)$$

$$= \frac{dy}{dx} \cdot e^x + y e^x$$

$$\frac{dy}{dx}\left[\ln x + \frac{x}{\sqrt{1-y^2}} - e^x\right] = -\frac{y}{x} - \sin^{-1} y + y e^x$$

$$\frac{dy}{dx} = \frac{-y/x - \sin^{-1} y + y e^x}{\ln x + \frac{x}{\sqrt{1-y^2}} - e^x}$$

Derivative of Hyperbolic function's

Q #1

Find y', $y = \tanh(8x^2+5)$

$$y' = \operatorname{sech}^2(8x^2+5) \cdot \frac{d}{dx}(8x^2+5)$$

$$= \operatorname{sech}^2(8x^2+5) \cdot 16x$$

$$= 16x \cdot \operatorname{sech}^2(8x^2+5)$$

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$

Q #2

Find y', if $y = \ln \cosh e^{x^4}$

$$y' = \frac{1}{\cosh e^{x^4}} \cdot d/dx \left(\cosh e^{x^4}\right)$$

$$= \frac{1}{\cosh e^{x^4}} \cdot \left(\sinh e^{x^4}\right) \cdot \frac{d}{dx}\left(e^{x^4}\right)$$

$$= \frac{1}{\cosh e^{x^4}} \cdot \left(\sinh e^{x^4}\right) \cdot e^{x^4}(4x^3)$$

$$= \frac{4x^3 \cdot e^{x^4} \cdot \sinh e^{x^4}}{\cosh e^{x^4}}$$

$$= 4x^3 e^{x^4} \cdot \tanh e^{x^4}.$$

Derivative of Inverse Hyperbolic fn:

$$\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$$

$$\frac{d}{dx}(\operatorname{sech}^{-1}x) = \frac{-1}{x\sqrt{1-x^2}}$$

Q#1:

Find y', if $y = \tanh^{-1}(5x^2)$

$$y' = \frac{1}{1-(5x^2)^2}\,\frac{d}{dx}(5x^2)$$

$$= \frac{10x}{1-25x^4}$$

Q#2:

Find y', if $y = \sinh^{-1}(\tan x)$

$$y' = \frac{1}{\sqrt{(\tan x)^2+1}}\,\frac{d}{dx}(\tan x)$$

$$= \frac{\sec^2 x}{\sqrt{\tan^2 x+1}}$$

$$= \frac{\sec^2 x}{\sqrt{\sec^2 x}}$$

$$= \frac{\sec^2 x}{\sec x}$$

$$= \sec x$$

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