lecture-02

Example: Prove that $i[T_1, T_2]$ is a Hermitian Operator, where T_1 & T_2 are hermitian operator.

proof:-

Let ψ be a wave fn & T_1, T_2 be two operators. First, we check,

$$\int \psi^* i [T_1, T_2] \psi \, d^3r$$

$$= \int \psi^* i (T_1 T_2 - T_2 T_1) \psi \, d^3r$$

$$= i \int \psi^* T_1 T_2 \psi \, d^3r - i \int \psi^* T_2 T_1 \psi \, d^3r$$

$$= i \int (T_1 \psi)^* T_2 \psi \, d^3r - i \int (T_2 \psi)^* T_1 \psi \, d^3r$$

$$= i \int (T_2 T_1 \psi)^* \psi \, d^3r - i \int (T_1 T_2 \psi)^* \psi \, d^3r$$

$$= - \int (i T_2 T_1 \psi)^* \psi \, d^3r + \int (i T_1 T_2 \psi)^* \psi \, d^3r$$

$$= \int \{ (i T_1 T_2 \psi)^* \psi + (i T_2 T_1 \psi)^* \psi \} \, d^3r$$

$$= \int \{ (i T_1 T_2 - i T_2 T_1) \psi \}^* \psi \, d^3r$$

$$= \int \{ i [T_1, T_2] \psi \}^* \psi \, d^3r$$

$\Rightarrow i[T_1, T_2]$ is Hermitian Operator.

Example:02: let T_1 & T_2 be two hermition operators then prove if $T_1 T_2$ is hermition operator then $[T_1, T_2] = 0$

Sol:-

let ψ be a wave fn, first we check $\int \psi^* T_1 T_2 \psi \, d^3r$

$$= \int (T_1 \psi)^* T_2 \psi \, d^3r \quad \because (T_1 \text{ is hermition})$$

$$= \int (T_2 T_1 \psi)^* \psi \, d^3r \quad \text{——} (1) \quad (T_2 \text{ is hermition})$$

Then we Take

$$\int \psi^* T_1 T_2 \psi \, d^3r$$

$$= \int (T_1 T_2 \psi)^* \psi \, d^3r \quad \text{——} (2)$$

Comparing (1) & (2) ; as both hold hermiticity condition

$$\Rightarrow T_2 T_1 = T_1 T_2$$

$$\Rightarrow T_1 T_2 - T_2 T_1 = 0 \quad \text{or}$$

$$[T_1, T_2] = 0.$$

Example:03: prove that K.E & P.E cannot be simultaneously calculated.

Proof:-

$K.E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} = \frac{-\hbar^2\nabla^2}{2m}$ | $p = \frac{\hbar}{i}\nabla$

$P.E = V(r)$

let ψ be a wave function

$$\left[-\frac{\hbar^2\nabla^2}{2m}, V(\underline{r})\right]\psi(\underline{r})$$

$$= \left\{-\frac{\hbar^2}{2m}\nabla^2 V(r) - V(r)\left(-\frac{\hbar^2}{2m}\nabla^2\right)\right\}\psi(r)$$

$$= -\frac{\hbar^2}{2m}\nabla^2(V.\psi) + V(r)\frac{\hbar^2}{2m}\nabla^2\psi$$

$$= -\frac{\hbar^2}{2m}\left(V(r).\nabla^2\psi\right) + \psi(r)\nabla^2(V(r)) + V(r)\frac{\hbar^2}{2m}\nabla^2\psi$$

$$= -\frac{\hbar^2}{2m}\psi(r)\nabla^2 V(r) \neq 0$$

so both operators cannot be simultaneously calculated

when $V(r) = $ const, then K.E & P.E) can be calculated simultaneously.

Assignment:-

Prove Jacobi Identity

$$[[T_1, T_2], T_3] + [[T_2, T_3], T_1] + [[T_3, T_1], T_2] = 0$$