Lecture-06

## Devission & Germer Experiment to prove wave particle properties:-

To discuss critical results of wave particle duality, a new dynamical system is formed.

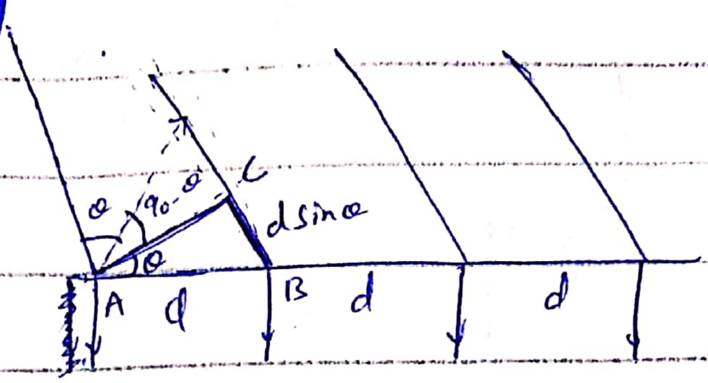
Wave particle duality was first proved by De-Broglie in 1924. In 1927, Devission & Germer proved wave-particle properties experimently. They passed a beam of electrons through a metal crystal grating.

Under certain condetions, beam of light travels in form of waves & under ~~centan~~ certain condetions, it travel like photons as particles.

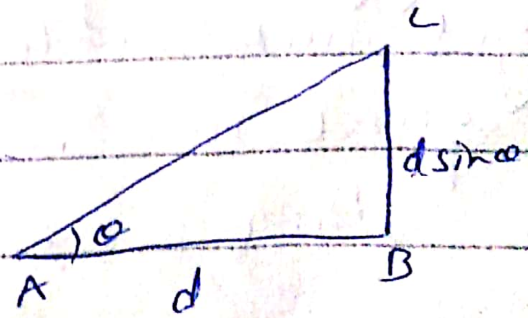
By considering light as wave, wave mechanics was established. If light consists of waves, it should satisfy two wave properties, diffraction, Interference

Beam waves have wave length λ. 2
which is also property of waves.

fig(a)



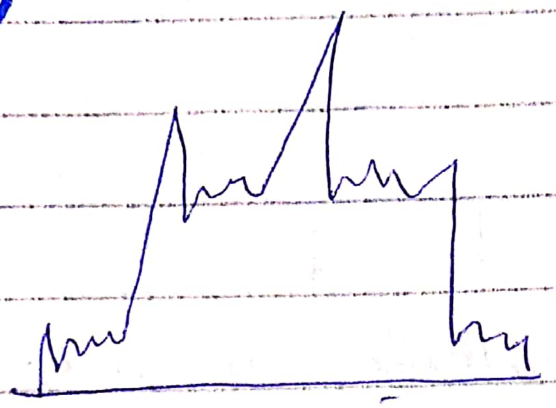

we have considered a nikal crystal grating. Beam of electrons is incident on plate at perpendicular.
Then waves of Beam are diffracted at an angle 'θ'. Distance b/w two nikal atoms is seperated by 'd'. In right angled Triangle ΔABC, we have




$BC = d\sin\theta$

so electrons beam consists of particles but obey wave property, which is diffraction. From where we can say that interference property also holds to prove wave particle duel nature.

fig(b)



a typical diffraction patteren, showing maxima & min arising from constructive or distructive interference.

## Bragg's Diffraction Peak:-

These peak were observed as Bragg's diffraction peaks. We can get max intensity if BC is equal to integral multiple of $\lambda$.

$d\sin\theta = n\lambda$, where

$\lambda = \frac{h}{p}$

(Its condition on $\theta$ for constructive interference to be at its strongest)

## G.P Thompson Experiment:-

Similar diffraction patterns were obtained by G.P Thompson when positively charged particles (proton, oxygen) were incident on thin metal.

## Summerfield-Whilson Principle:-

Due to this principle, we have to quantize a particle. This is generalization of third law of Bohr. This rule is also applied for non-circular (elliptic) orbits.

### Statement:-

Let $P_i$ denotes momenta of particles $i = 1, 2, \ldots, N$. Also consider $q_i$, their coordinates, this rule states;

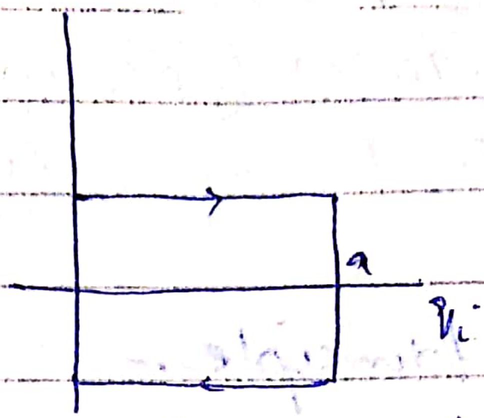
$\oint P_i \, dq_i = n_i h$, where $\oint$ denotes a closed cycle. We have to prove that:-

$$E = \frac{n^2 h^2}{8 m a^2}$$

i To prove iI, we consider a box of width a.

$2\pi m v r = nh$

$m v r = \frac{nh}{2\pi} = n\hbar$



In this box, a particle is moving freely in one dimension. In this box, particle moves back & forth motion. First we write formula of Total energy where P.E = 0 (particle is moving freely).

Total energy $= E = K.E + \cancel{P.E}^{0} = \frac{1}{2}mv^2$

$E = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$

Now, we apply Summerfield Whilson principle; as

$$2\int_0^a p\,dq = nh.$$

$$2pa = nh \Rightarrow p = \frac{nh}{2a}$$

Now formula of energy; as

$$E = \frac{1}{2m}\left(\frac{nh}{2a}\right)^2 = \frac{n^2h^2}{8ma^2}$$

# Sh Schrodinger equation :-

We will consider wave motion of a single particle. Let $\psi(x,t)$ be a wave fn. We know that wave equation is

$\psi_{xx} - c^2 \psi_{tt} = 0$. It is clear that its solution is

$$\psi(x,t) = e^{i(kx-\omega t)} \quad \text{(justify sol)}$$

In next theory, we will take frequency as $\nu$. Ay

$$\omega = 2\pi\nu \quad , \quad E = h\nu = \frac{h}{2\pi} \cdot 2\pi\nu = \hbar\omega$$

$$\text{Ay } k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c} = \frac{\omega}{c}$$

$$\Rightarrow \omega = kc \text{ or } k^2c^2 = \omega^2$$

We form wave equation of a packet when its localized (waves are present in specific space in specific time).

By inverse fourier we get

$$\psi(x,t) = \int e^{i(kx-\omega t)} f(k)\, d^3k \quad \text{—} \; (1)$$

Differentiating it w.r.t '$x$', twice time in vector form :-

$$\nabla^2\psi(x,t) = -k^2\int e^{i(kx-\omega t)} f(k)\, d^3k$$

multiplying by $-\frac{\hbar^2}{2m}$ on b.s; as

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x,t) = \frac{\hbar^2 k^2}{2m}\int e^{i(kx-\omega t)} f(k)\, d^3(k)$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x,t) = \hbar\omega\int e^{i(kx-\omega t)} f(k)\, d^3k \quad \text{--- (2)}$$

$$\because \left\{ \begin{array}{l} E = \frac{p^2}{2m} = \hbar^2 k^2/2m \\ p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar . k \quad \text{ \& } \quad E = \hbar\omega \quad \text{so} \\ \hbar\omega = \frac{\hbar^2 k^2}{2m} \end{array} \right\}$$

Differentiating (1) b.w.r.t 't'; as

$$\frac{\partial\psi(x,t)}{\partial t} = -i\omega\int e^{i(kx-\omega t)} f(k)\, d^3k$$

Multiplying by $i\hbar$ b.s; as

$$i\hbar\frac{\partial\psi}{\partial t} = \hbar\omega\int e^{i(kx-\omega t)} f(k)\, d^3k \quad \text{--- (3)}$$

Comparing (2) & (3); we get

$$-\frac{\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial\psi}{\partial t}$$

which is Schrodinger equation