Lecture 05

To find radius of an atom by Bohr's Law

$$r = \frac{4\pi\epsilon_0 n^2 \hbar^2}{me^2}$$

proof:-

Here 'e' denotes charge on an electron. Nucleus attracts electron having true charge. By Coulomb's Law

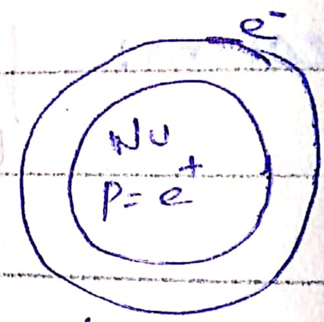
$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} \quad (1), \text{ where } k = \frac{1}{4\pi\epsilon_0}$$

By centripetal force

$$F = \frac{mv^2}{r} \quad (2)$$

Comparing both equations; as

$$\Rightarrow \frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$



$\because mvr = n\hbar$

$$v = \frac{n\hbar}{mr}$$

$$r = \frac{e^2}{4\pi\epsilon_0 mv^2}$$

(In Nu, charge of proton = charge of electron

put value of v according to 3rd law of bohr, we get; as

$$r = \frac{e^2}{4\pi\epsilon_0 m\left(\frac{n\hbar}{mr}\right)^2} = \frac{e^2 m r^2}{4\pi\epsilon_0 n^2 \hbar^2 r}$$

$$r = \frac{4\pi\epsilon_0 n^2 \hbar^2}{me^2}$$

when $n = 1$, then

$r = \frac{4\pi\epsilon_0 h^2}{me^2} = 0.53 \times 10^{-10}$ m

Mass of electron $= m = 9.1 \times 10^{-31}$ kg

Charge of electron $= e = 1.6 \times 10^{-19}$ C

Constant value of $K = \frac{1}{4\pi\epsilon_0}$ is 9×10^{9}

(n = Discrete No of orbits)

To find energy of an atom

$$E = \frac{-me^4}{2(4\pi\epsilon_0 n h)^2}$$

Proof:-

We write Total energy

$$E = K.E + P.E$$

$$= \frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

P.E is -ve as energy is given to overcome force of attraction

$$E = \frac{1}{2}\cdot\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} \quad \because mv^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

$$= \frac{e^2}{4\pi\epsilon_0 r}\left(\frac{1}{2} - 1\right) = \frac{-e^2}{2(4\pi\epsilon_0)r}$$

put value of $r = \frac{e^2}{4\pi\epsilon_0 m v^2}$

$$E = \frac{-e^2}{2(4\pi\epsilon_0)\left(\frac{e^2}{4\pi\epsilon_0 m v^2}\right)} = \frac{-me^4}{2(4\pi\epsilon_0 n\hbar)^2}$$

To find frequency

$$\omega = \frac{2\pi^2 me^4}{h^3(4\pi\epsilon_0)^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right), n_1 > n_2$$

proof:-

change in energy : $\Delta E = E_1 - E_2$

$$\Delta E = \frac{-me^4}{2(4\pi\epsilon_0 n_1\hbar)^2} - \frac{-me^4}{2(4\pi\epsilon_0 n_2\hbar)^2}$$

$$\hbar\omega = \frac{me^4}{2(4\pi\epsilon_0\hbar)^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

where $\Delta E = \hbar\omega$ (by Plank's Law)

$$\hbar\omega = \frac{me^4}{2(4\pi\epsilon_0)^2\left(\frac{h}{2\pi}\right)^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$\because \hbar = \frac{h}{2\pi}$$

$$\omega = \frac{2\pi^2 me^4}{h^3(4\pi\epsilon_0)^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

when electrons moves from 2^{nd} orbit to 1^{st} orbit, Then we get frequency $= w = 2.5 \times 10^{15} s^{-1}$

Now, we find its wave length

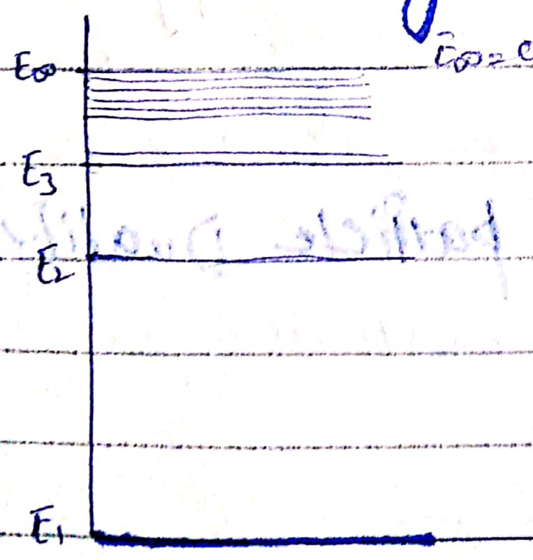
$$\lambda = \frac{c}{w} \Rightarrow \lambda = \frac{c}{2.5 \times 10^{15}} = \frac{3 \times 10^{8}}{2.5 \times 10^{15}}$$

$$\lambda = 1.2 \times 10^{-7} m$$

Range of visible light is $4 \times 10^{-7} m$ to $8 \times 10^{-7} m$

so radiation of electron giving radiation from 2^{nd} to 1^{st} orbit is ultra-violet radiation.

Schematic Diagram of Energy Levels:-



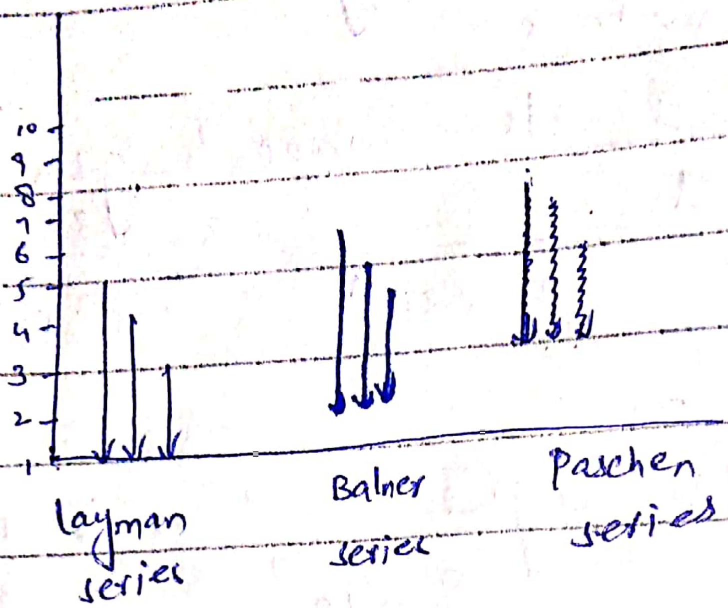
These energy levels are of hydrogen atom. E_1 is lowest energy level. E_∞ is Taken to be '0'

so That E_n is -ve $\forall$ n 's

$E_1 < E_2 < E_3 < \dots < 0$

$-13.6eV < -3.6eV < \dots < 0$

Spectral lines of series of H atom:-



Transition in H atom giving spectral lines in various series, each of these series was found to correspond to transition from various energy levels down to a particular value of n_f, with $n_f = 1$ for Balmer series & so on.

De Broglie wave particle Duality:-

If E is energy of radiation & ω is its frequency, then

$E = h\omega$ (plank's Law)

This energy is lost or absorb by an electron when it moves from one to another orbit. Energy also varies with momentum. It can also be written ; as

$E = PC$

Comparing both energies; we get

$PC = h\omega \Rightarrow P = \frac{h\omega}{c}$

$P = \frac{h}{\lambda} \Rightarrow P \propto \frac{1}{\lambda}$

$p = \frac{h}{\lambda}$

$\lambda = \frac{h \to \text{too small}}{p \to \text{too large}}$

So by De-Broglie principle, momentum varies inversly to wave-length. Phenomena of Q.M are given in terms of h. But classicaly forces & masses are large as compared to h. In such cases, $\lambda = 0$ as compared to other lengths. So λ is small in large objects. When objects are small, then particles moves in a wave with wave length λ in Q.M.

wavelength by De-Broglie form:-

Particle	K.E(MeV)	λ
Electron	10^{-6}	1.2×10^{-19}
	1	8.2×10^{-13}
protons	10^{-6}	2×10^{-11}
	1	2×10^{-14}
Oxygen Nuclei	10^{-6}	5×10^{-12}
	1	5×10^{-15}

Devission & Germer Experiment to prove wave particle properties:-

To discuss critical results of wave particle duality, a new dynamical system is formed.

Wave particle duality was first proved by De-Broglie in 1924.

In 1927, Davission & Germer proved wave-particle properties experimently.

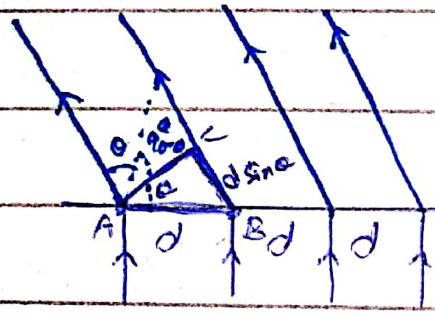
They passed a beam of electrons through a metal crystal grating.
bounded in definite nature lattice w.r.t

Under certain conditions, beam of light travels in form of waves & under certain conditions, it travel like photons as particles.

By considering light as wave, we have / wave mechanics was established of light consists of waves, it

should satisfy to wave properties as diffraction & interference.
Beam waves have wave length $\lambda \neq 0$ which is again property of waves

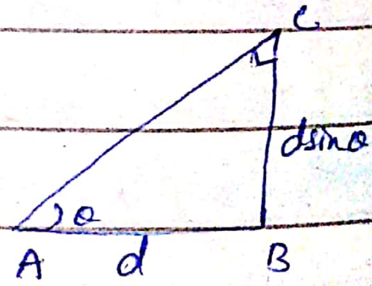
fig(a):-



We have considered a nikal crystal grating. Beam of electrons is incident on plate at perpendicular.

Then waves of beam are diffracted at an angle θ.
Distance between two nikal atoms is seperated by d. In right angled Triangle, ΔABC, we have

$$\boxed{BC = d\sin\theta}$$



So electron beam consists of particles but obey wave property, which is diffraction.

Scanned by CamScanner