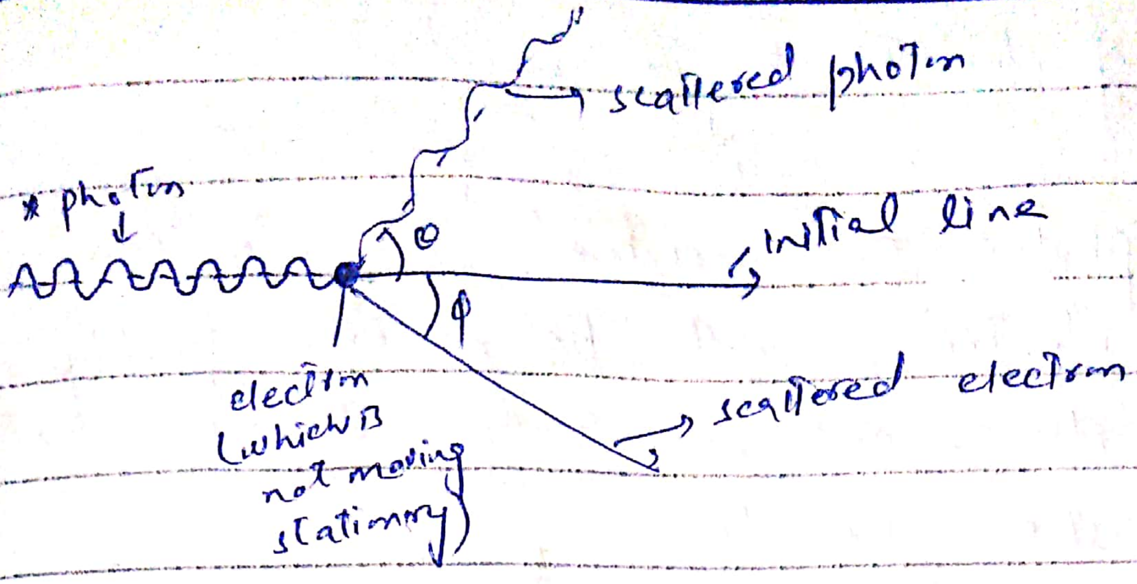
lecture-4

The Compton Effect:-

This effect was discovered by Compton in 1924. The base of this effect is in the fact that light consists in particles.



To prove formula of compton effect, we will use law of conservation of momentum with we will also use law of conservation of energy. let's consider m be the mass of electron. let ω & ω' be frequencies of photons before & after incident, respectively.

Now if we consider this photon a particle then we know that

Energy of photon before incident $= h\omega$

" " " after " $= h\omega'$

$E = h\omega$ of particle

Momentum of photon before incident $= \frac{h\omega}{c}$

" " after " " $= \frac{h\omega'}{c}$

Momentum of electron before " " $= 0$

" " after $= p$

$E = h\omega = mc^2$

$\Rightarrow mc = \frac{h\omega}{c}$

$\Rightarrow p = \frac{h\omega}{c}$

Energy of electron before incident = mc^2

" " " after = $\sqrt{p^2c^2+m^2c^4}$

when photon is incident on electron. Then electron will ~~be~~ move fastly. Then its mass 'm' is found by relativistic formula.

$$m' = \frac{m}{\sqrt{1-v^2/c^2}} \qquad (m')^2 = \frac{m^2}{\frac{c^2-v^2}{c^2}}$$

$$(m')^2 = \frac{m^2c^2}{c^2-v^2} \quad \text{— (A)}$$

Multiplying by c^2 on b.s; of (A)

$$c^2 m'^2 = \frac{m^2c^4}{c^2-v^2}$$

$$m^2c^4 = m'^2c^4 - m'^2c^2v^2$$

$$(mc^2)^2 = m^2c^4 + m'^2c^2v^2 \qquad \because p = mv$$

$$E^2 = p^2c^2 + m^2c^4$$

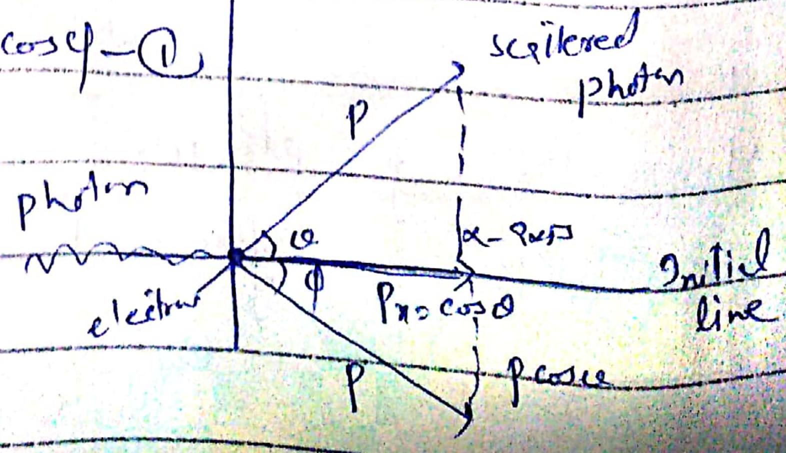
$$E = \sqrt{p^2c^2+m^2c^4}$$

Law of Conservation of Momentum:

of photon   of electron   total

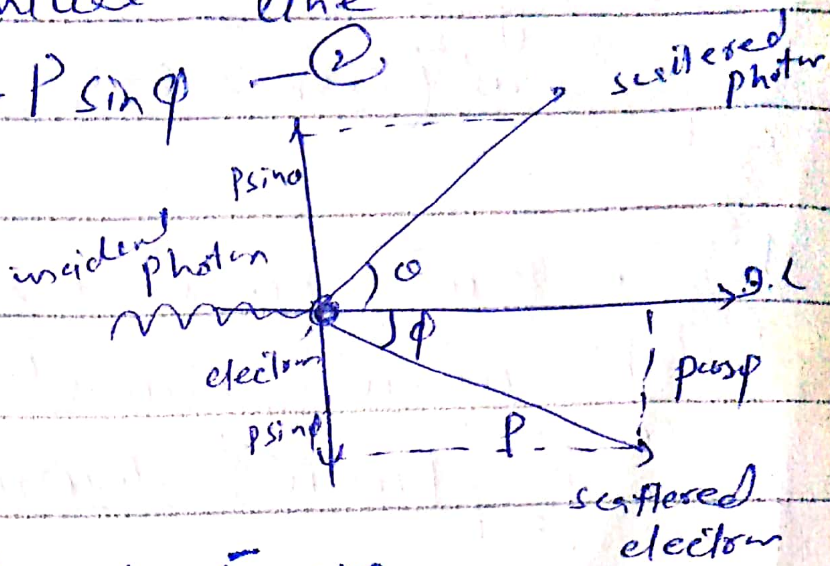
$$\frac{h\omega}{c} + 0 = \frac{h\omega'}{c}\cos\theta + P\cos\phi \quad \text{— (1)}$$

Now



perpendicular to initial line

$$0 + 0 = \frac{h\omega'}{c} \sin\theta + P\sin\phi \quad \text{—(2)}$$



Law of Conservation of Energy:-

$$h\omega + mc^2 = h\omega' + \sqrt{p^2c^2 + m^2c^4} \quad \text{—(3)}$$

Now using these three results we will proof Compton's formula which is

$$\Delta\lambda = \frac{2h}{mc} \sin^2\frac{\theta}{2} = \text{Change in wavelength}$$

Proof:-

As we know that

$$\frac{h\omega}{c} = \frac{h\omega'}{c}\cos\alpha + P\cos\phi \quad \text{--- (1)}$$

$$\frac{h\omega'}{c}\sin\alpha + P\sin\phi = 0 \quad \text{--- (2)}$$

$$mc^2 + h\omega = h\omega' + \sqrt{m^2c^4 + p^2c^2} \quad \text{--- (3)}$$

(1) $\Rightarrow$ $\frac{h\omega}{c} - \frac{h\omega'}{c}\cos\alpha = P\cos\phi$

$$\frac{h}{c}(\omega - \omega'\cos\alpha) = P\cos\phi \quad \text{--- (4)}$$

(2) $\Rightarrow$ $\frac{h\omega'}{c}\sin\alpha = P\sin\phi$ --- (5)

Squaring (4) & (5) & then adding, we get

$$\frac{h^2}{c^2}\{\omega^2 + \omega'^2(\cos^2\alpha + \sin^2\alpha) - 2\omega\omega'\cos\alpha\} = P^2(\cos^2\phi + \sin^2\phi)$$

$$\frac{h^2}{c^2}\{\omega^2 + \omega'^2 - 2\omega\omega'(1 - 2\sin^2\frac{\alpha}{2})\} = P^2$$

$$\frac{h^2}{c^2}\{\omega^2 + \omega'^2 - 2\omega\omega' + 4\omega\omega'\sin^2\frac{\alpha}{2}\} = P^2$$

$$\frac{h^2}{c^2}\{(\omega - \omega')^2 + 4\omega\omega'\sin^2\frac{\alpha}{2}\} = P^2 \quad \text{--- (6)}$$

$$\frac{h^2}{c^2}\{(\omega - \omega')^2$$

(3) $\Rightarrow$ $\{mc^2 + h(\omega - \omega')\}^2 = m^2c^4 + p^2c^4$

Dividing by c^2; we get

$$\{mc + \frac{h}{c}(\omega - \omega')\}^2 - m^2c^2 = p^2 \quad \text{--- (7)}$$

Comparing (6) & (7); as

$$\frac{h^2}{c^2}\{(\omega - \omega')^2 + 4\omega\omega'\sin^2\frac{\alpha}{2}\} = \cancel{m^2c^2} + \cancel{\frac{h^2}{c^2}(\omega - \omega')^2} + 2mc\cdot\frac{h}{c}(\omega - \omega') - \cancel{m^2c^2}$$

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$$\frac{h^2}{c^2} 4\omega\omega' \sin^2\frac{\alpha}{2} = 2m_e \frac{h}{c}(\omega-\omega')$$

$$\frac{2h}{mc}\sin^2\frac{\alpha}{2} = \frac{c(\omega-\omega')}{\omega\omega'}$$

or

$$\frac{c\omega}{\omega\omega_1} - \frac{c\omega'}{\omega\omega'} = \frac{2h}{mc}\sin^2\frac{\alpha}{2}$$

$$\frac{c}{\omega'} - \frac{c}{\omega} = \frac{2h}{mc}\sin^2\frac{\alpha}{2}$$

$c = \omega\lambda$

$\lambda = \frac{c}{\omega}$

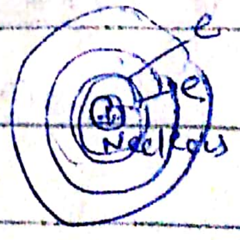
$\lambda' = \frac{c}{\omega'}$

$$\boxed{\Delta\lambda = \lambda' - \lambda = \frac{2h}{mc}\sin^2\frac{\alpha}{2}}$$ Required Result.

Rules of Neils Bohar:-

When Rutherford purposed his atomic model. He again faced two difficulties optical spectra & stability of matter Neils Bohar proposed his rules in 1913 to reduce problems of atom.

(i) when an electron revolved around nucleous in states, these states are called Discrete, stable or stationary states.

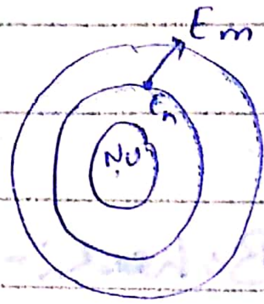


Rutherford

when an electron remains in its own orbit & is accelerating, then no radiation is emitted.

ii) Radiation is emitted when electrons moves from initial to final state.

Model of atom :-



$E = h\omega$

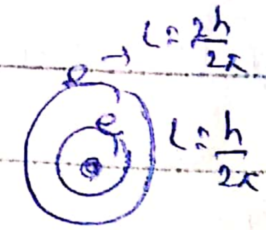
$\omega = E/h$

$\omega = \frac{E_n - E_m}{h}$

This is according to Ritz combination law.

iii) Electrons revolve only in those circular orbit for which angular momentum is integral multiple of $\frac{h}{2\pi}$

i.e



angular momentum of electron takes value as $0, \hbar, 2\hbar, 3\hbar, \ldots,$

where $\hbar = \frac{h}{2\pi}$

So we can say that its A.M is Quantized that is fixed.

A.M is basic article in Q.M.

$A.M = n\hbar, \; n = 0, 1, 2, 3, \ldots$

$r \times p = n\hbar \Rightarrow r_n m v_n = n\hbar$