Lecture-03

Plank's Law:-

So far we have proved Raleigh Jeans formula of energy. But this formula was correct for only low frequencies (ω). This paradox was corrected by plank in 1900. Plank suggested that radiation is exchanged with matter as energy in form of quanta, i.e

$E \propto \omega \Rightarrow E = h\omega$ (energy of one particle)

where 'h' is called plank's constant

$h = 6.625 \times 10^{-34}$ Js.

h appears in phenomena of quantum mechanics. Plank's law is start of Quantum Mechanics. so energy of each particle is in discrete packets, as $h\omega$. & Energy of range of particle will be

$0, h\omega, 2h\omega, 3h\omega, \ldots$

$E_n = nh\omega$

prob of energy: $P(E) = e^{-\frac{nh\omega}{kT}}$

$$\langle E \rangle = \frac{\sum_{n\geq 0} n\hbar\omega\, e^{-n\hbar\omega/kT}}{\sum_{n\geq 0} e^{-n\hbar\omega/kT}}$$

$$= \hbar\omega \frac{\sum_{n\geq 0} n e^{-n\hbar\omega/kT}}{\sum_{n\geq 0} e^{(-n\hbar\omega)/kT}}$$

$$= \frac{\hbar\omega\left(0 + e^{-\hbar\omega/kT} + 2e^{-2\hbar\omega/kT} + \cdots\right)}{1 + e^{-\hbar\omega/kT} + e^{-2\hbar\omega/kT} + \cdots}$$

Let's take substitute $e^{-\hbar\omega/kT} = x$, then

$$\langle E \rangle = \hbar\omega\left(\frac{x + 2x^2 + 3x^3 + \cdots}{1 + x + x^2 + \cdots}\right)$$

$$= \frac{\hbar\omega x\left(1 + 2x + 3x^2 + \cdots\right)}{1 + x + x^2 + \cdots}$$

$$= \frac{\hbar\omega x (1-x)^{-2}}{(1-x)^{-1}}$$

$$= \frac{\hbar\omega x}{(1-x)^{-1+2}} = \frac{\hbar\omega x}{(1-x)}$$

$$= \frac{\hbar\omega}{\frac{1}{x} - 1}$$

put value of $x = e^{-\hbar\omega/kT}$

$E_n = n\hbar\omega$

$\langle x \rangle = \dfrac{\sum_{n\geq 0} n x}{\sum_{n\geq 0} n}$

$\bar{\omega} = \dfrac{\sum_{i=1}^{n} P_i x_i}{\sum_{i=1}^{n} P_i}$

$\because (1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \cdots$

here $(1+(-x))^{-2} = \_\_\_\_$

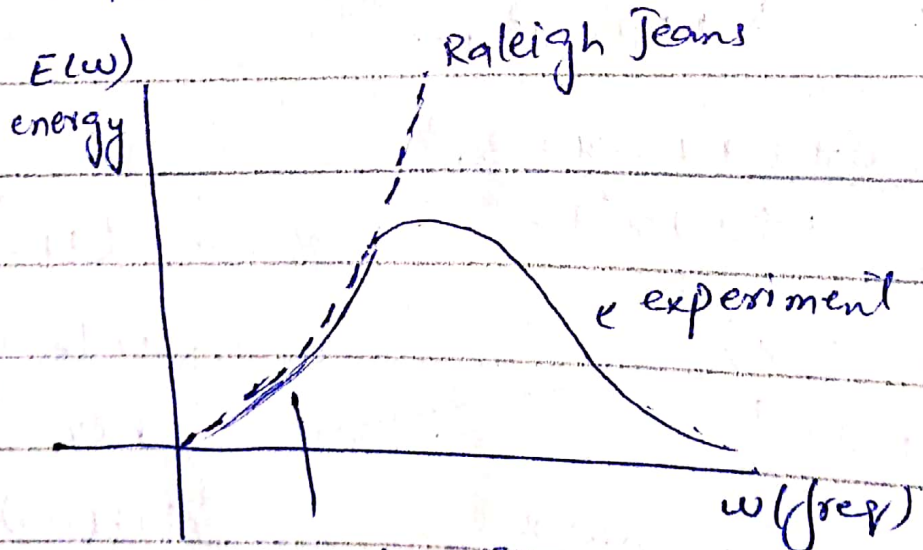
$$\langle E \rangle = \frac{h\omega}{\frac{1}{e^{-h\omega/kT} - 1}} = \frac{h\omega}{e^{h\omega/kT} - 1}$$

energy of all particles $= \frac{8\pi}{c^3}\omega^2 \langle E \rangle = \frac{8\pi\omega^2}{c^3}\left(\frac{h\omega}{e^{h\omega/\omega} - 1}\right)$

$$= \frac{8\pi h\omega^3}{c^3}\left(\frac{1}{e^{h\omega/kT} - 1}\right)$$

⇒ This is plank's law that is true for very small particles

Graphical Difference b/w Raleigh Jeans & Plank's Laws:-



experiment = Raleigh law

By equipartition of energy

$E \to kT$ if $\omega \to 0$ ——— (1)

Raleigh Jeans | $E \to 0$ as $w \to 0$ | ②
$E \to \infty$ as $w \to \infty$

according to Plank's law
① & ② hold (most)
⇒ energy must be fn of freq.
⇒ so that plank's took energy in packets / discrete form.
so plank's law gives finite total energy of radiation per unit volume.

Boltzman Stephan Law:

we know that

$E(w) = \frac{8\pi h w^3}{c^3}\left(\frac{1}{e^{hw/kT}-1}\right)$ from plank's law

$$\int_0^\infty E(w)\,dw = \int_0^\infty \frac{8\pi h w^3}{c^3}\left(\frac{1}{e^{hw/kT}-1}\right)dw$$

integration means to sum of energy of all particles or for all syste

$$= \frac{8\pi h}{c^3}\int_0^\infty \frac{w^3}{e^{hw/kT}-1}\,dw$$

let's we substitute $x = \frac{hw}{kT}$

$\Rightarrow w = \frac{kT}{h}x$, $dx = \frac{h}{kT}dw$

as $\omega: 0 \to \infty$ then $x = 0 \to \infty$

& $d\omega = \frac{kT}{h} dx$

substituting all values, we get

$$= \frac{8\pi h}{c^3} \int_0^\infty \frac{\left(\frac{kT}{h} x\right)^3}{e^x - 1} \left(\frac{kT}{h}\right) dx$$

$$= \frac{8\pi h}{c^3} \frac{k^4 T^4}{h^4} \int_0^\infty \frac{x^3}{e^x - 1} dx$$

as $\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{(\pi)^4}{15}$

so

$$= \frac{8\pi h}{c^3} \frac{k^4 T^4}{h^4} \frac{(\pi)^4}{15}$$

Total energy $= E = \frac{4}{c}\left(\frac{2(\pi)^5 (k)^4}{15 h^3 c^2}\right) T^4$

$$= \frac{4}{c} \delta T^4$$

where $\delta = \frac{2\pi^5 k^4}{15 h^3 c^2}$

δ is stephan's const.

⇒ Total radient heat power which is emitted from/by a black body is directly proportional to 4th power of temperature.

Photo-Electric Effect & concept of photon:-

When Ultra violet radiation is incident on metal
This radiation has wave length range 10nm to 400nm.
Then current flows due to emission of electrons. Even when there is retarding (resistant) potential present. This phenomene is called Photo-Electric Effect.

Stopping Potential:-

The largest retarding potential still allow electrons to be emitted is called stopping potential. This is denoted by V_s. V_s is directly proportional to energy by which electrons are emitted.

Concept of Photon:-

In 1905, Albert Einstein gave logical extension of

planck's law. Packets of light were called photon in discrete form. Photon are corpuscular (Latin word meaning particles). The Energy of each photon is hω.

let holding energy of an electron is W. Emission of electron will place if hω > W. Then emitted electrons have energy hω - W. so its clear that

$$V_s = h\omega - W$$

so by A. Mechanically, we came to know that V_s is dependent on ω.