

A sample of 900 plants is found to have a mean of 34 cm. Can it be reasonably regarded as a random sample from a large population with mean 32 cm and standard deviation 23 cm. Use 5 % level of significance.

Given:

Null hypothesis: $H_0 : \mu = 32$

Alternative hypothesis: $H_1 : \mu \neq 32$

Level of significance: $\alpha = 0.05$

Test - statistic: $Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

Critical region: $|Z| > 1.96$ ($Z < -1.96$ and $Z > 1.96$)

From the area table of normal distribution, we have $\frac{Z_{\alpha}}{2} = Z_{0.025} = 1.96$

Computations: Here, $n = 900$, $\bar{X} = 34$, $\sigma = 23$, and hence

$$Z = \frac{34 - 32}{\frac{23}{\sqrt{900}}} = \frac{2}{23} (30) = 2.61$$

Conclusion:

Since the calculated value of $Z = 2.61$ falls in the critical region, so we reject our null hypothesis $H_0 : \mu = 32$ at 5% level of significance. We may conclude that a random sample cannot be reasonably regarded from a population with mean 32 cm.

Suppose that the variance of the IQ's of the high school students in a

Q.5 Suppose that scores on an aptitude test used for determining admission to graduate study in statistics are known to be normally distributed with a mean of 500 and a population standard deviation of 100. If a random sample of 64 applicants from a college has a sample mean of 537, is there any evidence that their mean score is different from the mean expected of all applicants? Use $\alpha = 0.01$.

Solution:

1. Null hypothesis: $H_0: \mu = 500$
2. Alternative hypothesis: $H_1: \mu \neq 500$
3. Level of significance: $\alpha = 0.01$

Population σ
 $n = 64$

3. Test - statistic: $Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

4. Critical region: $|Z| > 2.575$ ($Z < -2.575$ and $Z > 2.575$)

(From the area table of normal distribution, we have $Z_{\frac{\alpha}{2}} = Z_{0.005} = 2.575$)

Computations: Here, $n = 64$, $\bar{X} = 537$, $\sigma = 100$, and hence

$$Z = \frac{537 - 500}{\frac{100}{\sqrt{64}}} = \frac{37}{100} (8) = 2.96$$

Conclusion:

Since the calculated value of $Z = 2.96$ falls in the critical region, so we reject our null hypothesis $H_0: \mu = 500$ at 1% level of significance. On the basis of the evidence we may conclude that their mean score is different from the mean expected of all applicants.

Q.7 A random sample of 25 values gives the average of 83. Can this be regarded as drawn from the normal population with mean 80 and standard deviation 7 at 5% level of significance?

Solution:

Null hypothesis: $H_0: \mu = 80$

Alternative hypothesis: $H_1: \mu \neq 80$

Level of significance: $\alpha = 0.05$

Test - statistic:
$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

Critical region: $|Z| > 1.96$ ($Z < -1.96$ and $Z > 1.96$)

(From the area table of normal distribution, we have $Z_{\frac{\alpha}{2}} = Z_{0.025} = 1.96$)

Computations: Here, $n = 25$, $\bar{X} = 83$, $\sigma = 7$ and hence

$$Z = \frac{83 - 80}{\frac{7}{\sqrt{25}}} = \frac{3}{7} \times 5 = 2.14$$

Conclusion:

Since the calculated value of $Z = 2.14$ falls in the critical region, so we reject our null hypothesis $H_0: \mu = 80$ at 5% level of significance. We conclude that this sample cannot be regarded as drawn from a normal population with $\mu = 80$ and $\sigma = 7$.

$H_0: \mu = 80$
 $H_1: \mu \neq 80$

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Q.8. A random sample of 64 drinks from a soft-drink machine has an average content of 21.9 deciliters, with a standard deviation of 1.42 deciliters. Test the hypothesis that $\mu = 22.2$ deciliters against the alternative hypothesis $\mu < 22.2$, at the 5 % level of significance.

Solution:

1. Null hypothesis: $H_0 : \mu = 22.2$
2. Alternative hypothesis: $H_1 : \mu < 22.2$
3. Level of significance: $\alpha = 0.05$

$H_0: \mu = 22.2$
 $\mu < 22.2$
 S
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3. Test - statistic: $Z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$

4. Critical region: $Z < -1.645$
 (From the area table of normal distribution, we have $-Z_\alpha = -Z_{0.05} = -1.645$)

5. Computations: Here, $n = 64$, $\bar{X} = 21.9$ and $S = 1.42$; we get

$$Z = \frac{21.9 - 22.2}{\frac{1.42}{\sqrt{64}}} = \frac{-0.3}{1.42} (8) = -1.69$$

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6. Conclusion: Since the calculated value of $Z = -1.69$ falls in critical region, so we reject our null hypothesis $H_0: \mu = 22.2$ at 5 % level of significance.

... were driven on the average 16300 miles a