

PROBABILITY

History:

The foundation of probability were laid by two French mathematicians of the 17th century - Blaise Pascal (1623-1662) and Pierre De Fermat (1601-1665)

Probability:

A numerical measure of the chance that an uncertain event will happen is called probability.

Example: toss a coin, draw a card, and throw a dice.

It is clear what we mean when we make a statements of the type that it is likely to rain today. Or I have a fair chances of passing annual examination.

Set:

A set is well defined collection or list of distinct objects and term distinct means that each object must appear only once.

For example: a group of students, the books in library.

Members/Elements:

The objects that are in a set called members of elements of that set.

Sets are usually denoted by capital letters such as A, B, C, Y . while 3 elements are represented by small letters such as a, b, c, d, y .

$A = \{a, b, x, y\}$ or $B = \{1, 2, 3, 4, 7\}$ where A and B are two sets and a, b, x, y are elements/member of set A and $1, 2, 3, 4, 7$ are elements of set B .

The no. of a set A , written as $n(A)$.

If $A = \{a, b, x, y\}$ then the total elements of set A are 4 then $n(A) = 4$.

ho gi ^{like} ^{the} ^{chance} ^{to} ^{probability} $0-1$ ^{have} ^{distinct}

Sample space \rightarrow All possible outcomes of an experiment

MOTOWOTOFOS

said. as H/WO-C/WO

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$n(A)$ = probability
 $n(S)$ = Sample space

n = no of elements
So here n of A is (6)

$S = 7$
 $A = 6$
 $B = 4$

Random Experiment:

Experiment:

The term experiment means a planned activity or process whose results yield a set of data.

A single performance of an experiment is called a trial. The result obtained from experiment or trial called outcome.

Random Experiment:

An experiment which produces diff results even though it is repeated a large no: of times under essentially similar conditions, called a random experiment.

Example: tossing of fair coin, Throwing a balanced dice.

Sample Space:

A set consisting of all possible outcomes that can result from random experiment called sample space

1- If we throw a dice

$A = \{1, 2, 3, 4, 5, 6\}$ sample space

All possible outcomes of our experiment.

2- If we toss a single coin there are

2 possible outcomes of experiment head or

$n(A)$ = Interest

METOWOTOPUS

n = Head?

H/WO-C/WO

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2 up Probability of $A = \frac{n(A)}{n(S)}$ (Event like safe age)

Now we were see head (like to coin)

$$\text{Probability of } A = \frac{n(A)}{n(S)}$$

So its probability is $1/2$

Assignment:

i) $S = \{\text{ball, pen, table, coin, die, card, book}\}$
 $n(S) = \{7\}$

ii) $A = \{1, 3, 5, 7, 8, 9, 0, 4, 2, 12\}$
 $n(A) = \{10\}$

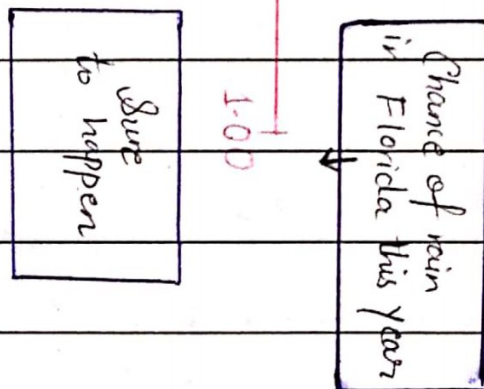
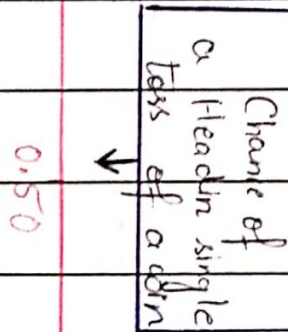
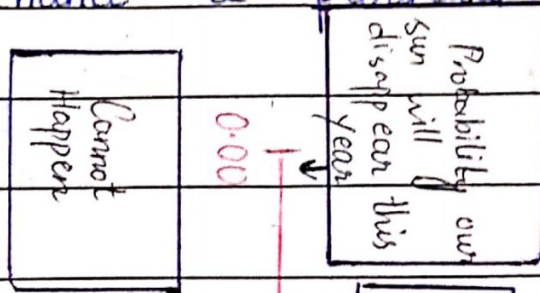
iii) $B = \{\text{book, city, clock, teacher}\}$
 $n(B) = \{4\}$

Probability

- Empirical probability
- Classical probability
- Rules of Addition
- Rules of Multiplication
- Complement Rule
- Principles of Counting
 - ✓ Multiplication Formula
 - ✓ Permutation
 - ✓ Combination.

⇒ Probability:

Probability is a value b/w '0' and '1' inclusive that represents the possibility / chance a particular event will happen.



Three key words are used in study of probability

- Experiment
- Outcome
- Event

Experiment:


A process that generates possible observations

Outcome:

A particular result of an experiment.

Event: (If we seeing of our own interest)

A collection of one or more outcomes of an experiment

	
Experiment	Roll a die
All possible outcomes	Observe a 1
	" " 2
	" " 3
	" " 4
	" " 5
	" " 6
Some possible events	Observe an even number
	" a No: greater than 4
	" " " 3 or less

Approaches to assigning Probabilities

→ **Classical Prob:** (We talk about present)

Classical probability is based on the assumption that the outcomes of an experiment are equally likely.

Probability of an event = $\frac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}}$

→ **Example:**

Consider an experiment of rolling a six-sided die. What is probability of the event

"an even No. of spots appear face up"?

The possible outcomes are:



possible outcomes = sample space

$$\begin{aligned} \text{Probability of an even number} &= \frac{3}{6} \\ &= 0.5 \end{aligned}$$

Example:

A fair coin is tossed. Find the probability

1, Head appears

2, tail "

3, No head appears

All possible outcomes = H, T

A = Head appears = H

B = Tail " = T

C = No head appears = T

$$P(A) = \frac{1}{2} = \frac{n(A)}{n(S)} \quad P(B) = \frac{1}{2} = \frac{n(B)}{n(S)} \quad P(C) = \frac{1}{2} = \frac{n(C)}{n(S)}$$

⇒ Two coins are tossed:

Find the probability:

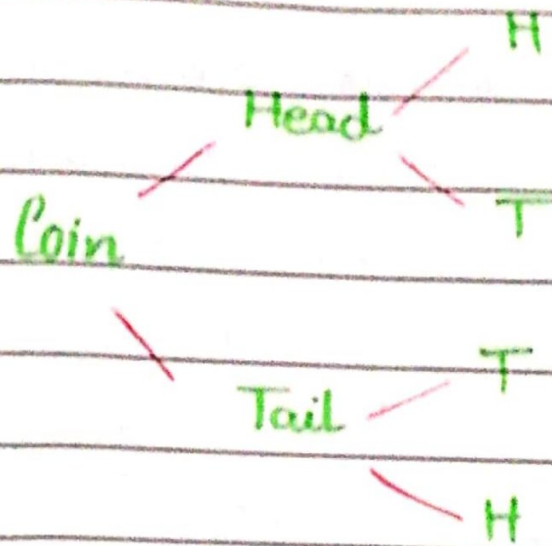
1, No head appears

2, 1 head appear

3, 2 head appears

4, 1 tail and 1 head appears

5, 1 or more than 1 head appears



T
R
E
E
D
I
A
G
R
A
M

All possible outcomes = {HH, HT, TH, TT}

A = No Head appears = TT

B = 1 head appears = HT, TH

C = 2 head appears = HH

D = 1 tail and 1 head appear.

= TH, HT

E = 1 or more than 1 head appear

= HT, TH, HH

$$P(A) = 1/4$$

$$P(B) = 2/4 = 1/2$$

$$P(C) = 1/4$$

$$P(D) = 2/4 = 1/2$$

$$P(E) = 3/4$$

	H	T
H	HH	HT
T	TH	TT


⇒ Example:

If a die is rolled. What is the probability.

1- Greater than 4 appear

2- Multiple of 3 appear

3- More than 7 appear

all possible outcomes = 

A = Greater than 4 appear = 5, 6

B = Multiple of 3 appear = 3, 6

C = More than 7 appear = 0

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Probability have range
0 → 1 and probability

$$P(B) = \frac{2}{6} = \frac{1}{3}$$

not less than 0.

$$P(C) = 0$$

Class Probability → *that tell about future.*

⇒ Empirical Probability:

The probability of an event happening is the fraction of the time similar events happened in the past

Empirical = No of times event occurs

Probability Total no of observations

Important

(Example on Next Page)

Best Quality Notes

→ Find the probability that one throw of a die will

a) Have 3 on top

b) Have 5 on top

c) Greater than 4

d) An odd number

Sample space is $S = \{1, 2, 3, 4, 5, 6\}$

Assume that it is a well balanced die, that is, each of six sides will have an equal chance to turn on the top when die stops in a throw. Then sample points are the six dots and sample space is

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$P(\text{Having a '3'}) = \frac{\text{No. of favourable cases} = 1}{\text{No. of possible cases} = 6} = \frac{1}{6}$$

$$P(\text{Having a '5'}) = \frac{1}{6}$$

$$P(\text{More than '4'}) = \frac{2}{6} = \frac{1}{3}$$

of these 6 outcomes 2 outcomes result event more than 4 that is face 5 or 6

$$P(\text{An odd no.}) = \frac{3}{6} = \frac{1}{2}$$

→ An _____ contain 8 balls of which 5 are white & 3 are black. A ball is drawn at random. What probability that the ball drawn is white?

Sol:

$$P(A) = \frac{\text{Favourable to event A}}{\text{No of possible cases}} \Rightarrow \frac{5}{8}$$

Let the ball numbered as 1 to 8 and selection of a ball at random can produce 8 different outcomes as any of the balls may turn up.

Let an event A be defined as the ball is white of these 8 diff mutually exclusive and equally likely outcomes. 5 outcomes result in favour of event A.

Example of Empirical Probability:

On February 1, 2003, the space shuttle columbia exploded. This was second disaster in 113 space missions for NASA. On basis of this information, what is the probability that a future mission is successfully

completed?

$$\text{Probability of } \alpha \text{ successful flight} = \frac{\text{No of successful flights}}{\text{Total no. of flights}}$$
$$P(A) = \frac{111}{113} \Rightarrow 0.98$$

Based on past experience, the probability is 0.98 that a future space shuttle mission will be safely completed.

Bez probability v. near to 1 so successful. 98% completed successfully

⇒ **Mutually exclusive events:**

Jin koi occur honai koi chances bhi same hon or common point bhi na ho so

Two

or more events are said to be mutually exclusive if they cannot occur together, i.e. they have no common point

e.g. A & B are mutually exclusive. A die is rolled and A = Even no B = odd no

⇒ **Collectively exhaustive:**

Two or more

events are said to be collectively exhaustive events, if they are mutually exclusive & their union is complete set of outcomes

e.g. $A = \{2, 4, 6\}$, $B = \{1, 3, 5\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

Best Quality Notes

→ Jis mei common a jagai ^{mutually} exclusive events
 → Collectively exhaustive ^{non equal to}
 Mean jo union lain to is ka sample space jagai
 $A \cup B \Rightarrow$ dono sets hai different nahi to hatao
 $A \cap B \Rightarrow$ Jo common hai

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From main (slides)

(1,1) Mean 7 dies hain roll kiya, one die
 pai 1 or 2nd die pai bhi 1 a jagai

(3,4) One die pai 3 one die pai 4

Total possible outcomes of 2 die when
 roll $6 \times 6 = 36$ 36 outcomes

$A \cap B$
 ↓
 Not mutually
 exclusive
 events

Jitni
 outcomes
 a single
 hai

Second die

When 2 dice rolled

First die	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

i) A = Both dice have same no:

ii) B = less than 5

iii) C = In one 1 must come

iv) $A \cup B$

v) $A \cap B$

i - A = {1,1}, {2,2}, {3,3}, {4,4}, {5,5}, {6,6}

ii - B = {1,1}, {1,2}, {1,3}, {2,1}, {3,1}, {2,2}

iii - C = {1,1}, {2,1}, {3,1}, {4,1}, {5,1}, {6,1},
 {1,2}, {1,3}, {1,4}, {1,5}, {1,6}

portant

$$\text{iv- } A \cup B = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (1,2), (1,3), (2,1), (3,1)\}$$

$$\text{v) } A \cap B = \{(1,1), (2,2)\}$$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

$$P(C) = \frac{11}{36}$$

$$P(A \cup B) = \frac{10}{36}$$

$$P(A \cap B) = \frac{2}{36}$$

⇒ General rule of Addition:

The outcomes of an experiment may not be mutually exclusive. The rule for two non-mutually exclusive events designated A and B is written:

$$P(A \text{ or } B) = p(A) + p(B) - p(A \cap B)$$

or
and $A \cap B \Rightarrow$ Not mutually exclusive.

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\rightarrow or = Mean two event are to be added
us ka Union lain gai
and = Mean we take intersection mean
Multiplication rule apply.

\Rightarrow Special Rule of Addition:

To apply special rule of addition, the events must mutually exclusive.

$$w \quad P(A \text{ or } B) = P(A) + P(B)$$

For three mutually exclusive events designated A, B & C rule is written:

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$$

\Rightarrow Complement Rule:

$$P(A) + P(\bar{A}) = 1$$

$$P(A) = 1 - P(\bar{A})$$

$$P(\bar{A}) = 1 - P(A)$$

It is used to determine the problem of an event occurring by subtracting the prob of the event not occurring from 1

\Rightarrow Mutually exclusive

Two terms/events \hookrightarrow exclusive

Both are unique \rightarrow mean no common point

Formula will be

\hookrightarrow No intersection exists

$$P(A \text{ or } B) = P(A) + P(B)$$

Mutually exclusive $\Rightarrow P(A) \text{ or } P(B)$
Not Mutually exclusive $\Rightarrow P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

\Rightarrow General rule of Addition: \times

Non-mutually exclusive:
not Both events Unique

Both events are not unique \rightarrow some common point is existing somewhere.
 \downarrow
mean intersection exists

then above formula becomes

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

$P(A \cup B)$

\hookrightarrow we subtracted the common point from formula

\Rightarrow Addition law for not mutually exclusive

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Question # 1

A die is tossed. Find probability that the face is a prime or is even number.

Solution

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{2, 3, 5\} \quad B = \{2, 4, 6\}$$

$$A \cap B = \{2\}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{6} + \frac{3}{6} - \frac{1}{6}$$

$$= \frac{5}{6}$$

A die is rolled. Find following probabilities

i) Face is multiple of 3 or multiple of 5

ii) The face is complete square or it is maximum face

Sol:

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let $A =$ multiple of 3 so, $A = \{3, 6\}$

$B =$ multiple of 5 $B = \{5\}$

or mean = addition so

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

$$P(A) = \frac{2}{6} \quad P(B) = \frac{1}{6}$$

$$P(A \cup B) = \frac{2}{6} + \frac{1}{6}$$
$$= \frac{1}{2}$$

ii) $A =$ complete square = $\{1, 4\}$

$B =$ Max face = $\{6\}$

$$P(A \cup B) = P(A) + P(B)$$

$$\Rightarrow P(A) = \frac{2}{6} \quad P(B) = \frac{1}{6}$$

$$\frac{2}{6} + \frac{1}{6} \Rightarrow \frac{1}{2}$$

Home assignment:

Two coins are tossed. Find probability that both faces are heads or at least one is head.

Sol:

	H	T
H	HH	HT
T	HT	TT

$2^2 = 4$ possible outcomes.

Let A = represent event "both are heads"

B = represent event "at least one is head"

$$A = \{(H, H)\}$$

$$B = \{(HH), (HT), (TH)\}$$

$$A \cap B = \{(HH)\}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) = \frac{1}{4} \quad P(B) = \frac{3}{4} \quad P(A \cap B) = \frac{1}{4}$$

$$= \frac{1}{4} + \frac{3}{4} - \frac{1}{4} \Rightarrow \frac{3}{4} \text{ required probability.}$$

A coin is tossed twice. Let A be event that at least one tail appears, B be the event that at most one tail appears and C be the event that exactly 2 tails appear.

Kum sai kum, ziada jithai mrzi
piada sai ziada)
→ Jitha sai utna to hai us sai km bhi no to haka ga jo
(H, H)
also

Total possibilities = 4

$P(A)$

$A = (TH), (HT), (TT)^*$

$B = (TH), (HT), (HH)$ AU

$C = (T, T)$

	H	T
H	HH	HT
T	TH	TT

- i) Are B & C mutually exclusive events? Yes
- ii) Are A & B equally likely events? Yes
- iii) Are B & C collectively exhaustive events? Yes

A die is thrown. Consider following events:

$A =$ even no: occurs $B =$ prime no: occurs

$C =$ odd no: occurs $D =$ no: is divisible by 5

- i - Are A & B equally likely events?
- ii - Are A & B mutually exclusive events
- iii) " " & C mutually exclusive events?
- iv) Are A & C collectively " " ?
- v) Which of above events simple &

which are compound? $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$ $D = \{5\}$

$B = \{2, 3, 5\}$

$C = \{1, 3, 5\}$

- i _____ v. A, B, C = compound
- ii No $D =$ simple
- iii Yes
- iv Yes

Two die thrown once following events are defined

A = sum of spots is 6

B = " " " is 8

C = same no appear

D = diff no appear

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$A = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$$

$$B = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$

$$C = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

$$D = \{(1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,3), (2,4),$$

$$(2,5), (2,6), (3,1), (3,2), (3,4), (3,5), (3,6), (4,1), (4,2),$$

$$(4,3), (4,5), (4,6), (5,1), (5,2),$$

$$(5,3), (5,4), (5,6)\}$$

iii) Yes

$$iv) \{(1,1), (1,2), (1,3)\} \text{ Yes}$$

$P(A \& B) = P(A) \cdot P(B)$
If equal = events independent

Special rule of Multiplication:

A and B
⇒ Independent events:

Two or more events are said to be independent if they happen together and do not effect each other. The selection of events will be with replacement.
n(S) remain same

⇒ Dependent events:

Two or more events are said to be dependent if they happen one after another and probability of 2nd event is effected due to first. The selection of events will be without replacement.
n(S) → change ho jata hai

The special rule of multiplication requires that 2 events A & B are independent.

It is written symbolically as

$$P(A \& B) = P(A) \cdot P(B)$$

For 3 independent events, A, B, C the special rule of multiplication used to determine probability that all 3 events

$P(\bar{A} \cap \bar{B}) = P(\bar{A}) P(\bar{B})$ said to be independent

will occur is

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$

Example:

Two coins are rolled. Show that the event "head on first coin" and "both faces are same" are independent.

Sol:

Sample space = $\{HH, HT, TH, TT\}$

$$A = \{HH, HT\}, P(A) = 2/4 = 1/2$$

$$B = \{HH, TT\}, P(B) = 2/4 = 1/2$$

$$P(\bar{A} \cap \bar{B}) = \frac{1}{4} \Rightarrow \frac{1}{4} \xrightarrow[\text{Common}]{\text{1 out of 4}}, P(A) \times P(B) = \frac{1}{4}$$

So this is independent $\because P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$

Example:

A six faced die is rolled. Show that the event "face is even" and the "event is more than 4" are independent

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{2, 4, 6\}$$

$$B = \{5, 6\}$$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

$$P(B) = \frac{2}{6} = \frac{1}{3}$$

$$P(\bar{A} \cap \bar{B}) = \frac{1}{6}$$

$$6$$

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

So Independent.

if outcomes large then no. of possible outcomes
 don't use formula
 Multiplication formula
 if for a total 3 items: 8x8x8
 if for 4 items: 8x8x8x8

⇒ Principle of Counting:

If no. of possible outcomes in an experiment is small, it is relatively easy to count them. If however, there are a larger no. of possible outcomes, it would be difficult to count all the possibilities.

To facilitate counting, we discuss 3 formulas

- Multiplication formula } to use
- Permutation formula } when order matters, to use
- Combination formula } when order doesn't matter, to use

→ Multiplication formula:

If there are 'm' ways of doing one thing and 'n' ways of doing another thing, there are $m \times n$ ways of doing both.

In terms of formula:

Total no. of arrangements = $m \times n$

$n!$ = Total
 $r!$ = Interest (Jitnai select kr rha)
 $P_r = \frac{n!}{(n-r)!}$

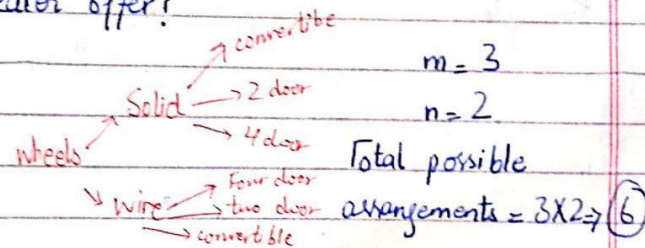
This can be extended to more than two events, for 3 events m, n and k:

Total no. of arrangements = $m \times n \times k$

Example:

An automobile dealer wants you to advertise that for \$29,999 you can buy a convertible, a two door (system) sedan or a four-door model with your choice of either wire wheel covers or solid wheel covers.

How many different arrangements of models and wheel covers can the dealer offer?



Example:

A developer of a new subdivision offers home buyers a choice of Tudor, rustic, colonial & traditional exterior styling in ranch, two story and split level floor plans. In how many

different ways can a buyer order one of these homes?

$$m=4, n=3$$

$$\text{Total possible arrangement} = 4 \times 3 = 12$$

→ Permutation formula:

An arrangement of 'r' objects selected from a single group of 'n' possible objects. In it order of the objects selected from a group is important.

Arrangement matter karo to permutation if arrangement matter na karo to then combination me use.
 ${}^n P_r = \frac{n!}{(n-r)!}$
 $n(A) = \text{mean favourable me pe}$
 $n(S) = 6 \text{ by } 6$

Example:

Referring to group of three electronic parts that are to be assembled in any order in how many diff ways can be assembled?

$${}^n P_r = \frac{n!}{(n-r)!}$$

$$(n-r)!$$

$${}^3 P_3 = \frac{3!}{(3-3)!}$$

$$(3-3)!$$

$$= \frac{3!}{1!} = 6$$

$$1!$$

Example:

Betta Machine shop increase has eight screw machines but only three spaces available in production are for the machines. In how many different ways can 8 machines be arranged in 3 spaces available?

$${}^n P_r = \frac{n!}{(n-r)!}$$

$$(n-r)!$$

$${}^8 P_3 = \frac{8!}{(8-3)!} \Rightarrow \underline{\underline{336}}$$

$$(8-3)!$$

Events: An event is an individual outcome or any no. of outcomes (sample points) of a random experiment or a trial. In set terminology any subset of a sample space S of experiment, is called an event.

Simple event → An event that contains exactly one sample point, defined as simple event.

Compound event: Contain more than one sample point & produced by union of simple events.

Example: occurrence of a 6 when die is thrown, is simple event, while occurrence of sum of 10 with pair of dice, is compound event, it decomposed into 3 simple events (4,6), (5,5) & (6,4)

Regression → We check dependency of independent variable on dependent variable.

variable → person to person vary
 e.g. CGPA → Study hrs interesting sub, concept clearance so this is dependent variable.

dose given to patient we have to see B.P.
 B.P is dependent & dose is independent.

J-P
 MCR → Regression introduced by-? Francis Galton.
 Rest example:

dependent variable $Y = a + \beta x + \epsilon$ (Regression equation)
 intercept slope error term

ZZZ These all words are diff so error said as natural errors & chances must

intercept → (starting point)
 If value $\beta = 0$ then ^{dependent} ~~ant~~ value equal to intercept comes
 $Y = a + \beta x$
 If slope 0 then $Y = a$

β -slope (If one time x change we see what effect come on dependent variable)

x mai 1 unit change karo mai y mai kya change ayai ga slope tells

Types → Simple (1 dependent & 1 independent)

Multiple → If one dependent & more than one independent.

Types of models

Exact models → like physics model

Probabilistic model → statistical term mai jab error include kr dain so

said as statistical model (Bcz variability exists naturally) in stats so said as elastic models

$Y = a + \beta x + \epsilon$ (this is statistical model)

(Lecture) → Principle of least squares: (We find by it)

minimize error It

(Regression) We have to find a & b

$Y = a + bx + e$
 $Y = a + \beta x + e$ } Same

To find a $a = \bar{Y} - \bar{b}\bar{x}$

To find b y on x

$b = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$

Fig y ki jaga x ko rkhai gai to y independent

→ y on x (Y dependent)

→ x on y (x dependent)

derivative ki jagah

→ Combination formula:

If the order of the selected objects is not, any selection is called a combination. The formula to count the no. of 'r' object combinations from a set of objects is:

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

Example: A pollster randomly selected 4 of 10 available people. How many diff groups of 4 are possible?

$${}^{10} C_4 = 210$$

$$\therefore n = 10$$

$$\therefore r = 4$$

Example: Find ${}^7 C_3$

$${}^7 C_3 = \frac{7!}{3!(7-3)!} \Rightarrow 7! \Rightarrow 35$$

Example: In a lottery game, three numbers are randomly selected from a tumbler of balls numbered 1 through 50.

a. How many permutations are possible.

b. How many combinations are possible.

$$n = 50$$

$$r = 3$$

$${}^n P_r = {}^{50} P_3$$

$$= 17600$$
$${}^n C_r = {}^n C_3$$
$$= 19600$$

Lecture

Regression and Correlation

Regression:

The term regression was introduced by the English biometrician, **Sir Francis Galton**.

Definition: An equation that expresses the linear relationship b/w two variables is called regression equation.

$$Y = a + bx$$

↓ dependent variable ↓ slope (Regression coefficient) → independent variable
↑
y-intercept

- It is relationship b/w independent & dependent variable
- It investigates the dependence of one variable called dependent variable, one or more variables, called independent variable.
- The dependent variable is assumed to be a random variable whereas the independent variables are assumed to have fixed values