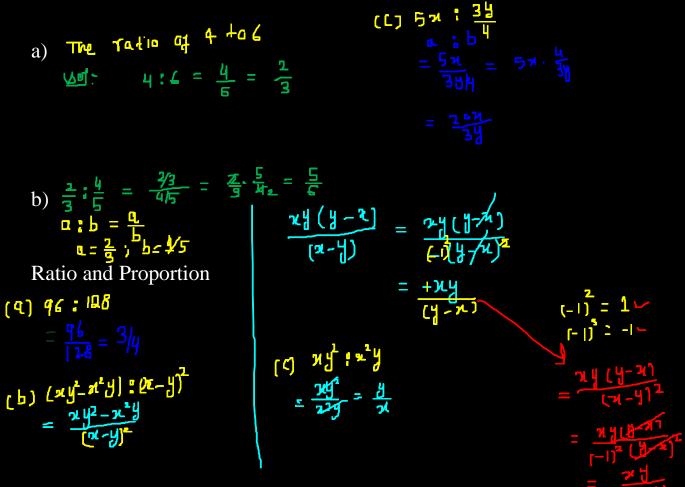# Ratio, Proportion, and Variation

Ratio:

The ratio of two number a and b, written as a : b is the fraction a/b provided $b \neq 0$ thus

$$a:b = \frac{a}{b} \; ; \; b \neq 0$$

Example: 01

a) The ratio of 4 to 6

Sol: $4 : 6 = \frac{4}{6} = \frac{2}{3}$

[c] $5x : \frac{3y}{4}$

$a : b = \frac{5x}{\frac{3y}{4}} = 5x \cdot \frac{4}{3y}$

$= \frac{20x}{3y}$

b) $\frac{2}{3} : \frac{4}{5} = \frac{2/3}{4/5} = \frac{2}{3} \cdot \frac{5}{4} = \frac{5}{6}$

$a : b = \frac{a}{b}$

$a = \frac{2}{3} \; ; \; b = \frac{4}{5}$

Ratio and Proportion

[a] $96 : 128$

$= \frac{96}{128} = 3/4$

[b] $(xy^2 - x^2 y) : (x - y)^2$

$= \frac{xy^2 - x^2 y}{(x-y)^2}$

$\frac{xy(y-x)}{(x-y)} = \frac{xy(y-x)}{(-1)(y-x)^2}$

$= \frac{+xy}{(y-x)}$

[c] $xy^2 : x^2 y$

$= \frac{xy^2}{x^2 y} = \frac{y}{x}$

$(-1)^2 = 1 \checkmark$

$(-1)^3 = -1 \checkmark$

$= \frac{xy(y-x)}{(x-y)^2}$

$= \frac{xy(y-x)}{(-1)^2 (y-x)^2}$

$= \frac{xy}{(y-x)}$

In each of the following proportions determine the value of x.

PROPORTION:

A proportion is an equality of two ratios.

$$a:b = c:d$$

Means

4th proportion

extremes

3rd proportion

Example:02

a) $(3-x):(x+1) = 2:1$

$$\frac{3-x}{x+1} \times \frac{2}{1}$$

$$2(x+1) = 1(3-x)$$

$$2x+2 = 3-x$$

$$2x+x = 3-2$$

$$3x = 1$$

$$\boxed{x = 1/3}$$

b) $(x+3):10 = (3x-2):8$

$$\frac{x+3}{10} \times \frac{3x-2}{8}$$

$$8(x+3) = 10(3x-2)$$

$$8x+24 = 30x-20$$

$$30x - 8x = 24+20$$

$$22x = 44$$

$$\boxed{x = 2}$$

c) $(x-1):(x+1) = (2x-4):(x+4)$

$$\frac{(x-1)}{x+1} \times \frac{2x-4}{x+4}$$

$$(x-1)(x+4) = (2x-4)(x+1)$$

$$5x = x^2$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$x = 0 \qquad x-5 = 0$$

$$\boxed{x = 5}$$

$$a:b = c:d \qquad d = x = ?$$

Find the fourth proportional to each of the following sets of numbers. In each case let x be the fourth proportional

a) $2, 3, 6,$

$$a:b = c:d$$
$$2:3 = 6:x$$

$$\frac{2}{3} \bowtie \frac{c}{x}$$

$$2x = 18$$
$$x = \frac{18}{2} = 9$$

$$\boxed{x = 9}$$

$a = 9$
$b = 3$
$c = 6$
$\boxed{d = x}$

$\boxed{d = 9}$

Hence $\boxed{2:3 = 6:9}$

(b) $4, -5, 10$

$$a:b = c:d$$
$$4:-5 = 10:x$$

$$-\frac{4}{5} \bowtie \frac{10}{x}$$

$$-10 \times 5 = 4x$$
$$-\frac{50}{4} = x$$

$$\boxed{x = \frac{-25}{2}}$$

$a = 4$
$b = -5$
$c = 10$
$d = x$

Find the mean proportional between 2 and 8.

$$\boxed{a:b = c:d}$$

$$a:x = x:d$$
$$2:x = x:8$$

$$\frac{2}{x} \bowtie \frac{x}{8}$$

$$\sqrt{16} = \sqrt{x^2}$$
$$4 = |x|$$
$$\boxed{x = \pm 4}$$

$b = ?$ $a = 2$
$c = ?$ $d = 8$

$b = 4$ | $b = -4$
$c = -4$ | $c = 4$

$\boxed{T \propto V}$
$$T = KV$$

VARIATION:

They represent a specific type of function called variation functions.

The three general types of variation functions are direct, inverse, and joint

1) If x varies directly as y^2 $\Rightarrow$ $x \propto y^2$ $\Rightarrow$ $\boxed{x = Ky^2}$

2) If x varies inversely as y^2 $\Rightarrow$ $\boxed{x = K/y^2}$

3) If x varies jointly as y and z $\Rightarrow$ $\boxed{x = Kyz}$

4) If x varie directly as y^2 and inversely as z

$$x = \frac{Ky^2}{z}$$

if jointly
$$x = Ky^2z$$

5)

Find 3rd proportial

a) 2, 3

$$2 : 3 = 3 : x$$

$$\frac{2}{3} = \frac{3}{x}$$

$$2x = 9$$

$$\boxed{x = 9/2}$$

$$c = x = 9/2$$

(b) $-2, 8/3$

$$-2 : \frac{8}{3} = \frac{8}{3} : x$$

$$-2 \cdot \frac{3}{8} = \frac{8}{3x}$$

$$-\frac{6}{8} = \frac{8}{3x}$$

$$-18x = 64$$

$$x = -\frac{64}{18} \quad \frac{32}{9}$$

$$\boxed{x = -32/9}$$