Topic: Polynomial Function

ZEROS OF POLYNOMIAL EQUATIONS

There are three types

1) Remainder Theorem
2) Factor theorem
3) Synthetic division

1) Remainder Theorem:

$x - r = 0 \quad \boxed{x = r}$

If r is any constant and if a polynomial P(x) is divided by (x - r), the remainder is P(r).

i.e $\boxed{P(r) = R}$

Example: 01 Find the remainder when $x^3 + 4x^2 - 2x + 5$ divided by $(x-1)$.

Sol:-

$P(x) = x^3 + 4x^2 - 2x + 5$

$P(1) = 1^3 + 4 \cdot 1^2 - 2(1) + 5$

$P(1) = 1 + 4 - 2 + 5$

$\boxed{P(1) = 8}$

$\boxed{R = 8}$

$x - 1 = 0$

$\boxed{x = 1}$

$x - 1 = x - r$

$-1 = -r$

$\boxed{1 = r}$

$x = r$

$\boxed{x = 1}$

Example:02 Find the value of k, if the polynomial $x^3 + kx^2 - 7x + 6$ has a $\boxed{R = -4}$ when divided by $(x+2)$

Sol:-

$\boxed{P(r) = R}$

$P(-2) = -4 \longrightarrow (1)$

$4k + 12 = -4$

$4k = -4 - 12$

$4k = -16$

$k = -16/4$

$\Rightarrow \boxed{k = -4}$

$x + 2 = 0$

$\boxed{x = -2}$

$P(x) = x^3 + kx^2 - 7x + 6$

$P(-2) = (-2)^3 + k(-2)^2 - 7(-2) + 6$

$= -8 + 4k + 14 + 6$

$\boxed{P(-2) = 4k + 12}$

Example 03 :- when $2x^3-3x^2-x+8$ divided by $(x+1)$

Sol:-

$x+1=0$

$\boxed{x=-1}$

$P(x) = R$

$R = 2x^3-3x^2-x+8$

$= 2(-1)^3-3(-1)^2-(-1)+8 \quad = -2-3+1+8$

$\boxed{R = 4}$

2) Factor Theorem:

If r is a root of the equation P(x) ~~¼~~ 0, i.e. if $\boxed{P(r) = 0}$, then (x - r) is a factor of P(x).

Conversely, if (x - r) is a factor of P(x), then r is a root of $\boxed{P(x) = 0}$, or P(r) =0

$P(a) = 0$

$x = a$

$(x-a)$ is $P(x)$

$(x-a)$

$P(a) = 0$ root

Example:04

Show That $(x-2)$ is a factor of x^4-13x^2+36

Sol:-

$P(x) = x^4-13x^2+36$

$x-2=0$

$\boxed{x=2}$

$P(2) = 2^4-13\cdot 2^2+36$

$= 16-52+36$

$= 52-52$

$\boxed{P(2) = 0}$ $\quad x=2$ is the root of the $P(x)$

Example: 05 Find factor of the polynomial, when $-3, -2, -1, 0, 1, 2, 3$

$x^3 + 4x^2 + x - 6$

Sol:-

$P(x) = x^3 + 4x^2 + x - 6$

at $x = 0$

$P(0) = 0 + 0 + 0 - 6 = -6 \neq 0$

at $x = 1$

$P(1) = 1 + 4 + 1 - 6 = 6 - 6 = 0$

$\boxed{P(1) = 0}$ ✓

at $x = -1$

$P(-1) = -1 + 4 - 1 - 6 = -4 \neq 0$

at $x = 2$

$P(2) = 8 + 16 + 2 - 6 = 20 \neq 0$

at $x = -2$

$P(-2) = -8 + 16 - 2 - 6 = -16 + 16 = 0$

$\boxed{P(-2) = 0}$ ✓

at $x = 3$

$P(3) = 27 + 36 + 3 - 6 = 60 \neq 0$

at $x = -3$

$P(-3) = -27 + 36 - 3 - 6 = -36 + 36 = 0$

$\boxed{P(-3) = 0}$ ✓

$P(1) = 0 \Rightarrow x = 1$, $(x-1) = 0$

$P(-2) = 0$, $x = -2$, $(x+2) = 0$

$P(-3) = 0$, $x = -3$, $(x+3) = 0$

Hence $(x-1)(x+2)$ $(x+3)$ is factor of $P(x)$

3) Synthetic Division:

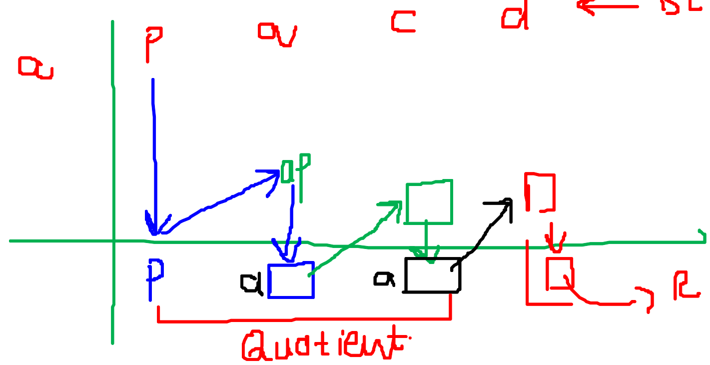
Synthetic division is a simplified method of dividing a polynomial P(x) by $\boxed{(x - r)}$, where r is any assigned number. By this method the values of the coefficients of the quotient and the value of the remainder can readily be determined.

$px^3 + qx^2 + cx + d$ by $(x - a)$

$x^3 + 1 = 0$

$x^3 + 0x^2 + 0 \cdot x + 1 = 0$

p q c d ← 1st line



Exp:- $x^4 - 10x^2 - 2x + 4 \div$ by $(x+3)$ ✓

$(x - r) = (x - (-3))$

$\boxed{x = -3}$

$x + 3 = 0$

$x = -3$

$\boxed{a = -3}$

$P(-3) = (-3)^4 - 10(-3)^2 + 6 + 4$

$= 0$

$(-3)(-3) = 9$

$(-1)(-3) = 3$

$x^4 + 0x^3 - 10x^2 - 2x + 4$

-3	1	0	-10	-2	4
		-3	9	3	-3
	1	-3	-1	1	$\boxed{1}$

Q R

Remainder = 1

Quotient = $x^3 - 3x^2 - x + 1$

Exp:- If $(x-2)$ & $(x+2)$ are factor of $x^4 - 13x^2 + 36$;

Sol:-

$p(x) = x^4 + 0x^3 - 13x^2 + 0x + 36$

$x - 2 = 0$ $\quad$ $x + 2 = 0$

$\boxed{x = 2}$ $\quad$ $\boxed{x = -2}$

S.D :-

$$
\begin{array}{r|rrrrr}
2 & 1 & 0 & -13 & 0 & 36 \\
  &   & 2 & 4 & -18 & -36 \\
\hline
  & 1 & 2 & -9 & -18 & \vert\ 0
\end{array}
$$

$$
\begin{array}{r|rrrr}
-2 & 1 & 2 & -9 & -18 \\
   &   & -2 & 0 & 18 \\
\hline
   & \underbrace{1 \quad 0 \quad -9}_{Q} & & & \vert\ 0 \rightarrow R
\end{array}
$$

$\therefore$ Quotient = $x^2 + 0x - 9$

$= x^2 - 9$

$= (x+3)(x-3)$

$R = 0$