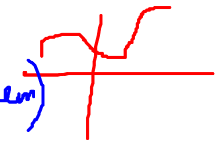
Continuity of a function:

(i) **Continuity at a point:**

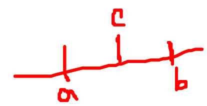
A function is said to be continuous at a point x =a

If $\boxed{\lim_{x \to a} f(x) = f(a)}$



(ii) Continuity in an interval: / intermediate (theorem)

A function is said to be continuous in [a,b] if satisfies the following three conditions:-



$[1, 3]$, $a = 1$, $b = 3$

$x \in (a, b)$

$c ; f(c) = y_0$

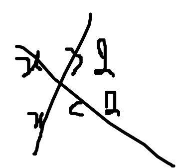
1) $f(x)$ is continuous $\forall\, x \in (a, b)$
2) $\lim_{x \to a^+} f(x) = f(a)$
3) $\lim_{x \to b^-} f(x) = f(b)$

pts

Example: 01 check continuity at $\boxed{x = 2}$

$\boxed{\lim_{x \to c} f(x) = f(c)}$

$$\checkmark f(x) = \begin{cases} \checkmark \dfrac{x^2 - 4}{x - 2} & if \;\; x \neq 2 \\ 2 & if \;\; x = 2 \end{cases}$$



Sol :-

$a^2 - b^2 = (a+b)(a-b)$

$x^2 - 4 = (x+2)(x-2)$

for $x = 2$

$\boxed{f(2) = 2}$

Now at $x \neq 2$

$= \lim_{x \to 2} \left(\dfrac{x^2 - 4}{x - 2} \right) = \left(\dfrac{0}{0} \right)$

$= \lim_{x \to 2} \left(\dfrac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} \right) ; x \neq 0 \;\; = \lim_{x \to 2} (x+2) = 2 + 2 = 4$

then $\lim_{x \to 2} f(x) = f(2)$

$4 \neq 2$ $\;\therefore f(x)$ is not continuous at $x = 2$

Example: 02

continuity at a $x=0$

$$f(x) = \begin{cases} (1+3x)^{\frac{1}{x}} & ; \quad x \neq 0 \\ e^{3} & ; \quad x = 0 \end{cases}$$

Sol:- $\boxed{\lim_{x \to a} f(x) = f(a)}$

$1/0$

$(1+0)^{\infty} = 1^{\infty}$

$\lim_{x \to a} [f(x)]^{g(x)} = e^{\lim_{x \to a} g(x)[f(x)-1]}$

(1) $f(0) = e^{3}$

(2) $\lim_{x \to 0} f(x) = \lim_{x \to 0} (1+3x)^{1/x} = e^{\lim_{x \to 0} \frac{1}{x}(1+3x-1)} = e^{\lim_{x \to 0} \frac{1}{x}(3x)} = e^{3}$

(3) $\lim_{x \to 0} f(x) = f(0)$

$e^{3} = e^{3}$ $\therefore$ $f(x)$ is continuous at $x=0$

Continuity of Polynomials and Rational Functions:

Every polynomial is continuous at every point of the real lines and every rational number is continuous at every point where its denominator is different from zero.

∞

Example: 03 Rational Function

$$f(x) = x^{4} + 20; \qquad g(x) = 5x(x-2)$$

$\because$ $f(x)$ & $g(x)$ is continuous at every value of x

But $\frac{f(x)}{g(x)} = \frac{x^{4}+20}{5x(x-2)}$ $\therefore$ is continuous at every value of x except $x=0$, $x=2$ (where denominator is zero)

$x=0$, $x=2$

$x(x-2) \neq 0$

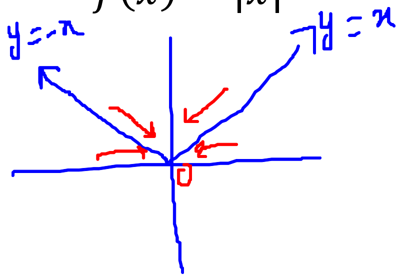
$x \neq 0$; $(x-2) \neq 0$

$\boxed{x \neq 0}$; $\boxed{x \neq 2}$

Example: 04

$$f(x) = |x|$$

$$|x| = \begin{cases} x & ; x > 0 \\ -x & ; x < 0 \end{cases}$$



$|x|$ is continuous at every value of x. $(0, 0)$

But origin :- $\lim_{x \to 0} |x| = 0$

Continuity of Composition function:

$\lim_{x \to a} f(x) = L$ (exist) = continuity

if $= \infty$ Discontinuity

Exp:-

(a) $\lim_{x \to \pi} \sin(\pi - \sin x) = ?$

Sol:-

$$\lim_{x \to \pi} \sin(\pi - \sin x) = \sin(\pi - \sin \pi)$$
$$= \sin(\pi - 0)$$
$$= \sin(\pi)$$
$$= 0 = L \quad \therefore \text{ is continuous.}$$

(b) $\lim_{x \to 0} \sin\left(\frac{\pi}{2}\left(\cos(\tan x)\right)\right)$

Sol :-

$$\lim_{x \to 0} \sin\left(\frac{\pi}{2}\left(\cos(\tan x)\right)\right) = \sin\left[\frac{\pi}{2}\left(\cos(\tan(0))\right)\right]$$
$$= \sin\left(\frac{\pi}{2}\left(\cos(0)\right)\right)$$
$$= \sin\left(\frac{\pi}{2}(1)\right)$$
$$= \sin\left(\frac{\pi}{2}\right)$$
$$= 1 = L \quad \text{is continuous.}$$

(c) $\lim_{y \to 1} \sec\left(y\sec^2 y - \tan^2 y - 1\right)$

Sol :-

$$\lim_{y \to 1} \sec\left(y\sec^2 y - \tan^2 y - 1\right)$$

$\sec^2 y = \tan^2 y + 1$

$$= \lim_{y \to 1} \sec\left(y\sec^2 y - (\tan^2 y + 1)\right)$$
$$= \lim_{y \to 1} \sec\left(y\sec^2 y - \sec^2 y\right)$$
$$= \sec\left(\sec^2(1) - \sec^2(1)\right)$$
$$= \sec(0)$$
$$= 1 = L$$

Continuity from Graph:

Sol :-

$f(-1) = 0.5$ ✓

$f(3) = 2$ ✓

$f(a) = 0.5$

$f(b) = 2$

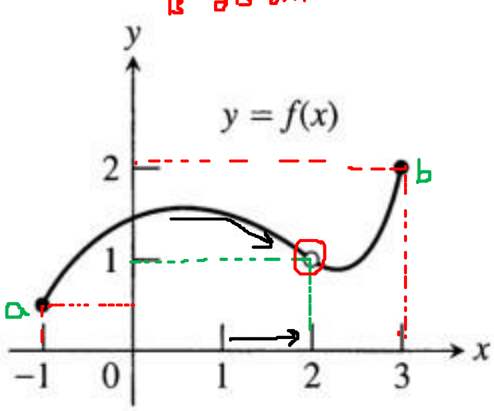
$\lim_{x \to 2^-} f(x)$

① $f(b) = \lim_{x \to b^-} f(x)$

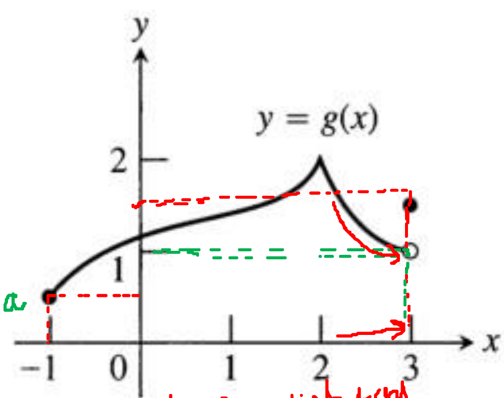
② $f(a) = \lim_{x \to a^+} f(x)$

① $2 \neq 1$

1. this $f(x)$ is discontinuous at $x = 2$



2. is discontinuous at $x = 3$



$f(b) = \lim_{x \to b^-} f(x)$

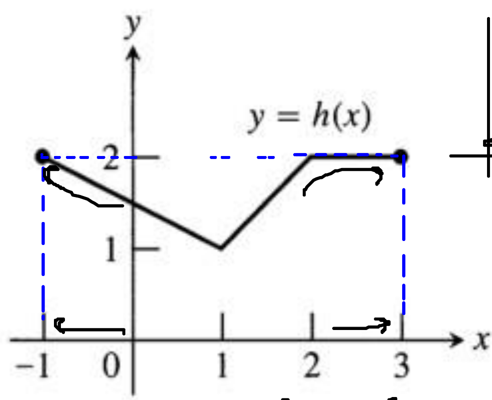
$1.5 \neq 1$

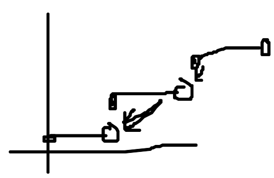
① $f(b) = \lim_{x \to b^-} f(x)$

$f(3) = 1.5$

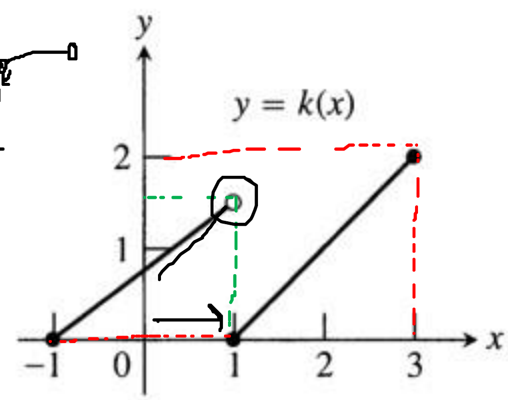
$\lim_{x \to 3^-} f(x) = 1$

3.





4.



$[-1, 3]$

$f(a) = f(-1) = 2 = \lim_{x \to -1^+} f(x) = 2$

$f(b) = f(3) = 2 = \lim_{x \to 3^-} f(x) = 2$

$\therefore$ is continuous

$[-1, 1]$ or $[1, 3]$

1) $f(-1) = 0$

2) $f(1) = 0 = \lim_{x \to 1^-} f(x) = 1.5$

$0 \neq 1.5$

-: Points of Discontinuous :- $\frac{1}{0} = \infty$

15. $y = \frac{1}{x-2} - 3x$

$x - 2 = 0$

$\boxed{x = 2}$

is discontinuous

16. $\frac{1}{(x+2)^2} + 4$

$(x+2)^2 = 0$

$x + 2 = 0$

$\boxed{x = -2}$ Dis.cont.

17. $y = \frac{x+1}{x^2-4x+3}$

$x^2 - 4x + 3 = 0$

$3x^2$; $-3x$; $-x$

$x^2 - 3x - x + 3 = 0$

$x(x-3) - 1(x-3) = 0$

$(x-1)(x-3) = 0$

$x - 1 = 0$; $x - 3 = 0$

$\boxed{x = 1}$; $\boxed{x = 3}$

18. $y = \frac{1}{|x|+1} - \frac{x^2}{2}$

$|x| + 1 \neq 0$

Defined $\forall x$

continuous