Some standard limits: -

(i) $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$

(ii) $\lim_{x\to 0} \frac{e^{mx}-1}{x} = e$

(iii) $\lim_{x\to 0} \frac{a^x - 1}{x} = \log a$

(iv) $\lim_{x\to 0} (1+ax)^{\frac{1}{x}} = e^a$

(v) $\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^x = e^a$

(vi) $\lim_{x\to 0} \frac{\sin mx}{x} = m$

(vii) $\lim_{x\to\infty} \frac{Sinx}{x} = 0$

(viii) $\lim_{x\to o} \frac{\tan x}{x} = 1$

(ix) $\lim_{x\to a} \left(\frac{a^x+b^x}{2}\right)^{1/x} = \sqrt{ab}$

(x) $\lim_{x\to o} (\cos x + a\sin bx)^{1/x} = e^{ab}$

(xi) $\lim_{x\to o} \frac{1-\cos ax}{x^2} = \frac{a^2}{2}$

(xii) $\lim_{x\to a} [f(x)]^{g(x)} = \lim_{x\to a} g(x)[f(x)-1]$

1. $\lim_{x\to 0} \frac{1-\cos 3x}{x \sin 2x} = \frac{\lim_{x\to 0} (1-\cos 3x)}{\lim_{x\to 0} (x \sin 2x)} = \frac{\lim_{x\to 0} \frac{(1-\cos 3x)}{x^2}}{\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)} = \frac{3^2/2}{2} = \frac{9}{4}$

$= \frac{\lim_{x\to 0} (1-\cos 3x)\, x}{\lim_{x\to 0} x^2 \left(\frac{\sin 2x}{x}\right) x}$; $x \neq 0$

$P(x) = \frac{1-\cos 3x}{x \cdot x}\, x$

$Q(x) = \frac{\sin 2x}{x}\, x$

$P(x) = \left(\frac{1-\cos 3x}{x^2}\right) x$

$Q(x) = \left(\frac{\sin 2x}{x}\right) x$

$\sin^2 x = \sin$ $(\sin 2x)^2$

2. $\lim_{x\to \pi/2} \frac{\sin^2(2x)}{(x-\frac{\pi}{2})^2}$ is

a) 1 b) 2 c) 4 ✓ (d) −4

Sol :- $\lim_{x\to \pi/2} \frac{\left[\sin\left(2\left(\frac{\pi}{2}-x\right)\right)\right]^2}{\left(\frac{\pi}{2}-x\right)^2}$ $\to 0$

$\lim_{x\to 0} \frac{\sin mx}{x} = ?$

$\sin\left(\frac{\pi}{2}-x\right) = \cos x$

$\sin(\pi - x) = \sin x$

$(-1)\left(\frac{\pi}{2}-x\right)^2$ $(-1)(-1) = 1$

$\sin^2 x = (\sin x)^2$

$\frac{\pi}{2} - x = t$

$x \to \pi/2$

$\pi/2 - \pi/2 = t$

$t \to 0$

$= \lim_{t\to 0} \frac{[\sin 2t]^2}{t^2}$

$= \lim_{t\to 0} \left(\frac{\sin 2t}{t}\right)^2 = (2)^2 = 4$

$$\lim_{x\to c} (f(x))^{r/s} = \left(\lim_{x\to c} f(x)\right)^{r/s}$$

10. $\lim_{x\to 0} x^x = ?$ $\qquad 0^0$

$\because \log x^x = x\log x$

Sol:- $y = x^x$

$\log y = x\log x$

$\lim_{x\to 0}\log y = \lim_{x\to 0}(x\log x)$

$= \lim_{x\to 0}(-x)$

$\frac{0}{0}$

$\lim_{x\to 0}\frac{\log x}{1/x} = \frac{1/x}{-1/x^2}$

$= -x$

$(x^{-1})' = \frac{1}{x}$

$-1\,x^{-2}$

$\lim_{x\to 0}\log y = 0$

$\lim_{x\to 0} e^{\log y} = e^0$

$\lim_{x\to 0} y = e^0$

$$\boxed{\lim_{x\to 0} x^x = 1}$$

Q $\lim_{x\to 0}[1-\sin x]^{1/\sin x}$ $\qquad$ ($f(x) = 1-\sin x$, $g(x) = \frac{1}{\sin x}$)

$(1-0)^{1/0} = 1^\infty$

$\lim_{x\to a}[f(x)]^{g(x)} = e^{\lim_{x\to a} g(x)[f(x)-1]}$

Sol:- $\lim_{x\to 0}[1-\sin x]^{1/\sin x} = e^{\lim_{x\to 0}\frac{1}{\sin x}[1-\sin x-1]}$

$\frac{1}{\sin x}(-\sin x)$

$= e^{\lim_{x\to 0}(-1)}$

$= e^{-1}$

$= 1/e$

$\lim_{x\to a} K = K$

$\Rightarrow \lim_{x\to 0}[1-\sin x]^{1/\sin x} = 1/e$

Q: $\lim_{x\to 0}[\cos x]^{1/x^2} = (1)^{1/0} = (1)^\infty$ $\qquad$ ($f(x) = \cos x$, $g(x) = \frac{1}{x^2}$)

$\lim_{x\to a}[f(x)]^{g(x)} = e^{\lim_{x\to a} g(x)[f(x)-1]}$

Sol:- $\lim_{x\to 0}[\cos x]^{1/x^2} = e^{\lim_{x\to 0}\frac{1}{x^2}[\cos x-1]}$

$= e^{-\lim_{x\to 0}\frac{1}{x^2}[1-\cos x]}$

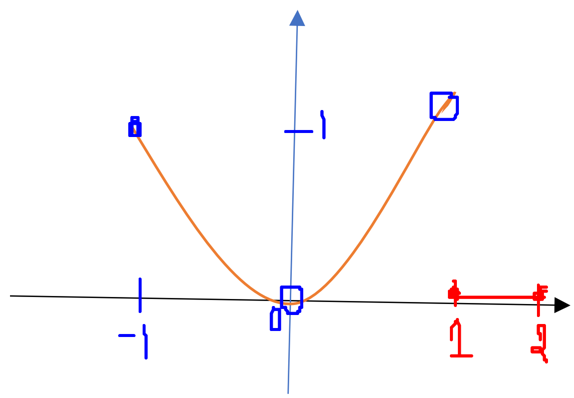
$= e^{-1/2}$

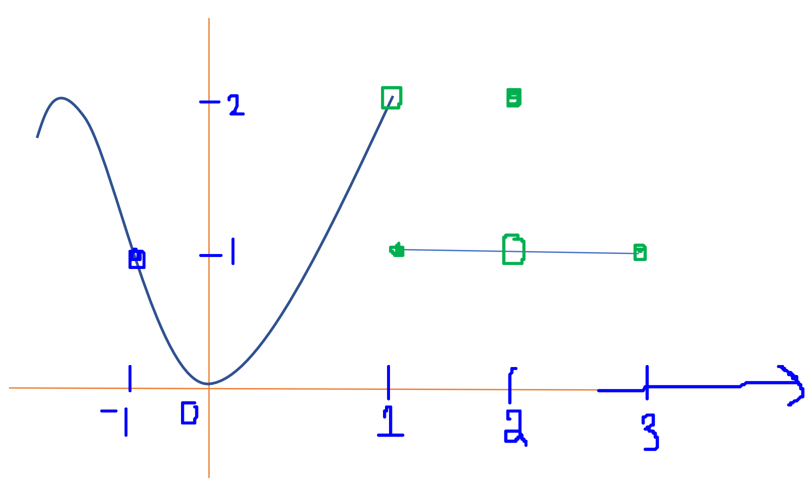
$= e$

$\lim_{x\to 0}\frac{1-\cos x}{x^2}$

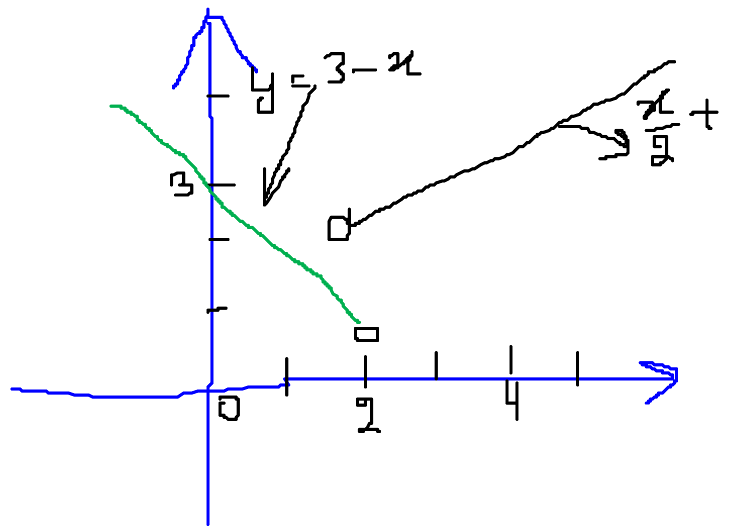
$= \frac{x^2}{2}$

Find limits Graphically:





$$f(x) = \begin{cases} 3 - x, & x < 2 \\ \dfrac{x}{2} + 1, & x > 2 \end{cases}$$



INFINITE LIMITS AND VERTICAL ASYMPTOTES