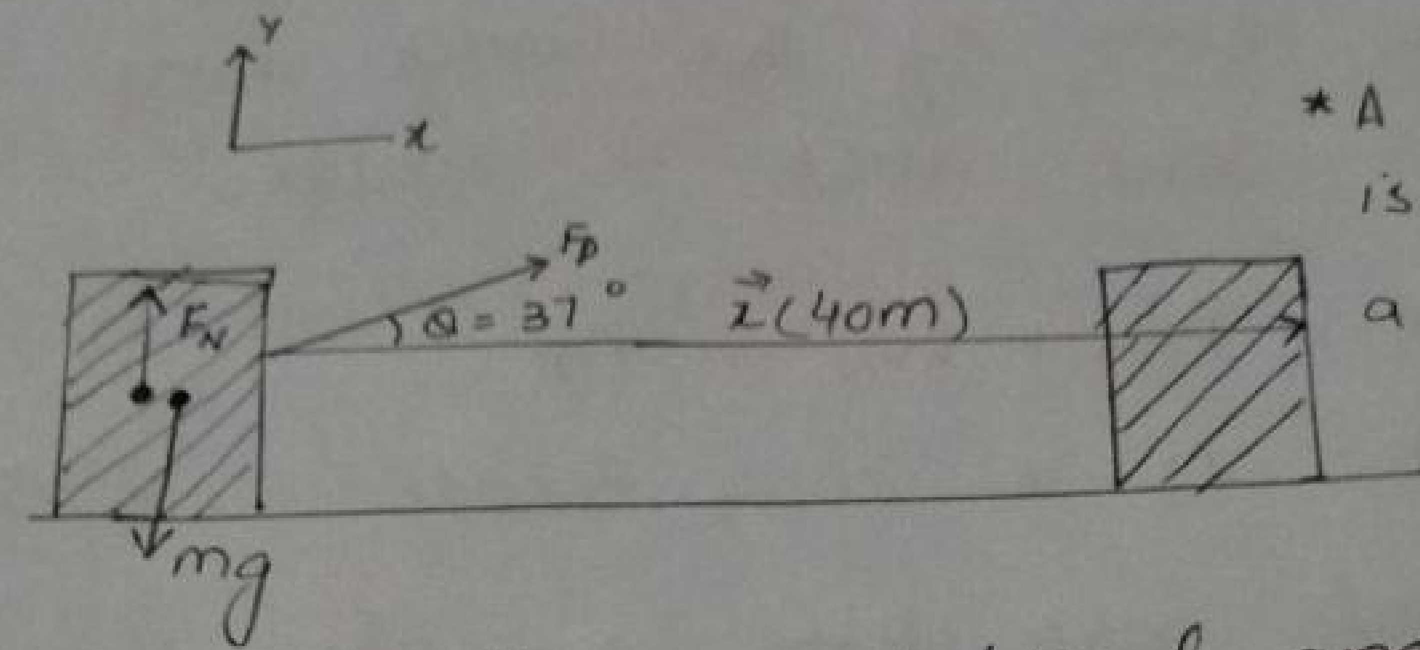
Example 7-1:-



★ A 50kg crate is pulled along a smooth floor

(a)

- The work done by the gravitational and normal force is zero. So they are perpendicular to the displacement $\vec{x}$ ($\vec{x}$, $\theta = 90°$)

$W = \vec{F} \cdot \vec{d}$ $\vec{d} = \vec{x}$, $F = mg$

$W_G = Fd\cos\theta$

$W_G = mgx\cos\theta$

$\theta = 90°$

$W_G = mgx\cos 90° \Rightarrow \boxed{W_G = 0}$

$W_N = F_N x \cos 90°$

$\boxed{W_N = 0}$

- The work done by ~~normal~~ force Person

$W_P = F_P x \cos\theta$

$W_P = (100)(40)\cos 37°$

$\boxed{W_P = 3200\ J}$

(b) The net work can be calculated in two equivalent ways

① The net work done on an object is the algebric sum of the walk done by each force, Since work is scalar. (2)

$$W_{net} = W_G + W_N + W_P$$

$$W_{net} = 0 + 0 + 3200 = 3200 J$$

② The net work done can also be calculated by first determining the net force on the object and then taking its components along the displacement

$$(F_{net})_x = F_P \cos\theta$$

So net workdone

$$W_{net} = (F_{net})x = (F_P \cos\theta)x$$

$$= (100N)(\cos 37^\circ)(40m)$$

$$\boxed{W_{net} = 3200 J}$$

In vertical component there is no displacement and no work done.

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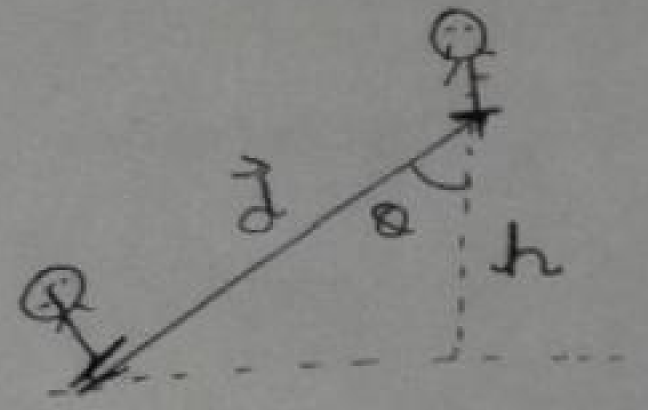
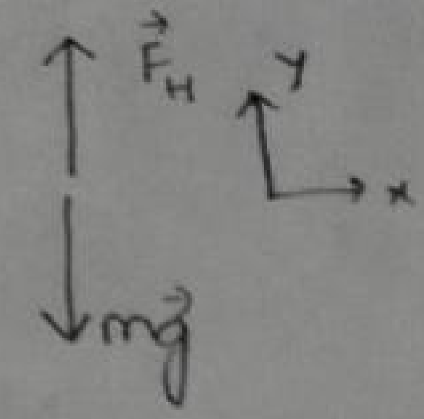
Example 7.2:-

(a) Applaying 2nd law of motion in vertical direction $a_y = 0$



$$\Sigma F_y = m a_y$$

$$F_H - mg = 0$$

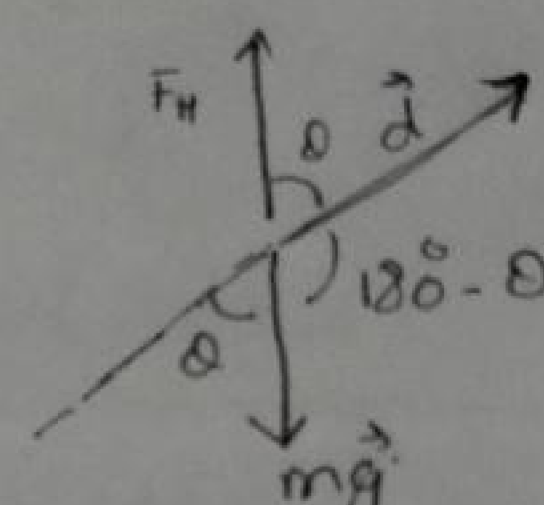


$$F_H = mg$$

$$= (15.0\,kg)(9.8\,m/s^2) = 147$$

$$\boxed{F_H = 147N}$$

$$W_H = F_H(d\cos\theta)$$



$$d\cos\theta = h$$

$$W_H = F_H(d\cos\theta)$$

$$W_H = F_H h = (147\,N)(10.0\,m) = 1470\,J$$

- Work done depends only on the change in elevation and not on the angle of the hill, 'θ'. The hiker would do the same work to lift the pack vertical the same hight 'h'.

(b) Work done by gravity on backpack is

$$W_G = F_G d\cos(180° - \theta)$$

$$\cos(180° - \theta) = -\cos\theta$$

$$W_G = F_G d(-\cos\theta)$$

$$= F_G(-h) \Rightarrow -mgh = -(15.0)(9.8)(10) = -14$$

$$W_{net} = W_G + W_H = -1470 + 1470 = 0$$

Example 7.4

$$\vec{F_p} = F_x \hat{i} + F_y \hat{j}$$

$$= (F_p \cos\theta)\hat{i} + (F_p \sin\theta)\hat{j}$$

$$\vec{F_x} = \vec{F_p} \cos\theta \, \hat{i}$$

$$\vec{F_y} = F_p \sin\theta \, \hat{j}$$

$$\vec{d} = (100 m)\hat{i}$$

$$\vec{F_p} = (17 N)\hat{i} + (10 N)\hat{j}$$

$$W = \vec{F_p} \cdot \vec{d}$$

$$= (17 N)(100) + (10 N)(0) + (0)(0)$$

$$\boxed{W = 1700 J}$$

Example 7.6 :-

$$F(x) = F_0\left(1 + \frac{1}{6}\,\frac{x^2}{x_0}\right)$$

- $F_0 = 2.0\,N$, $x_0 = 0.0070m$, and 'x' is the position of the end of the arm. If the arm moves from $x_1 = 0.010m$ to $x_2 = 0.050m$, how much work did the motor do?

- We integrate to find work done by the motor:

$$W_m = F_0 \int_{x_1}^{x_2} \left(1 + \frac{x^2}{6x_0^2}\right) dx$$

$$W_m = F_0 \int_{x_1}^{x_2} 1\,dx + \int_{x_1}^{x_2} \frac{1}{6x_0} \cdot x^2\,dx$$

$$W_m = F_0 \left(x\Big|_{x_1}^{x_2} + \frac{1}{6x_0}\,\frac{x^3}{3}\Big|_{x_1}^{x_2}\right.$$

$$W_m = F_0 \left(x + \frac{1}{6x_0}\,\frac{x^3}{3}\right)\Big|_{x_1}^{x_2}$$

$$W_m = 2.0\left((x_2 - x_1) + \frac{1}{6(x_0)} \cdot \frac{1}{3}\left(x_2^3 - x_1^3\right)\right)$$

$$W_m = 2.0\left[(0.050 - 0.010) + \frac{1}{18(0.070)}\,(0.050)^3 - (0.010)^3\right]$$

$$\boxed{W_m = 0.36\,J}$$

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$$\boxed{W_m = 0.36\,J}$$

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Example 7.5

- The force $F = Kx$ holds at each point, including x_{max}. Hence F_{max} occurs at $x = x_{max}$

(a) First we need to calculate the spring constant 'K':

$$F = Kx$$

$$K = \frac{F_{max}}{x_{max}} = \frac{75N}{0.030m} = 2.5 \times 10^3 \, N/m$$

- The work done by the person on the spring is

$$W = \frac{1}{2} K x^2_{max}$$

$$W = \frac{1}{2}(2.5 \times 10^3)(0.030)^2$$

$$\boxed{W = 1.1.J}$$

(b) The force that the person exerts is still $F_p = Kx$, though now both 'x' and F_p are negative (x is positive to the right). The work done is

$$W_p = \int_{x=0}^{x=0.030} F_p(x)\,dx = \int_{x=0}^{x=-0.030} Kx\,dx$$

$$= \frac{1}{2}Kx^2 \Big|_0^{-0.030} = \frac{1}{2}(2.5 \times 10^3 N/m)(-0.030+0)$$

$$\boxed{W_p = 1.1 J}$$

Which is same as for stretching it.

Example: 7.7 :-

- We use $K = \frac{1}{2}mv^2$ and work energy principle

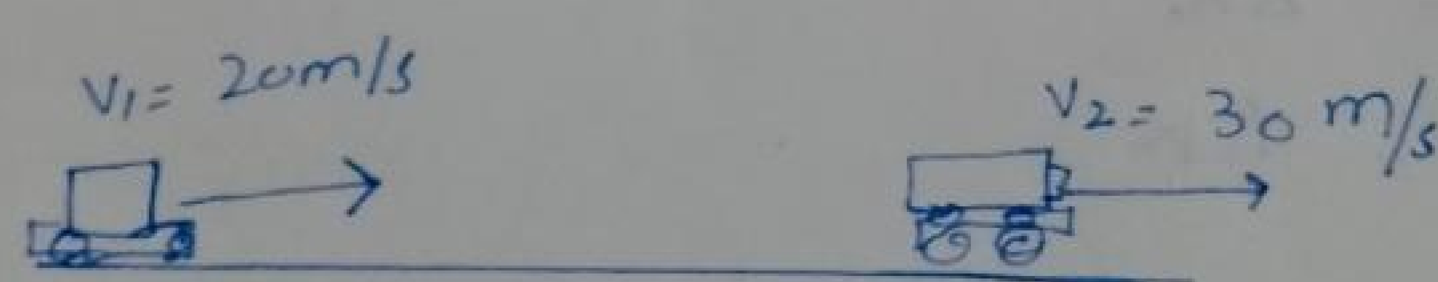
$$W_{net} = \Delta K$$
$$W_{net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

(a) The Kinetic energy of ball after the thrown is

$$K = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(0.145\,kg)(25\,m/s^2)$$
$$\boxed{K = 45\,J}$$

(b) Since the initial Kinetic energy was 'zero', the net work done is just equal to the final Kinetic energy 45 J

Example 7.8



- The net work needed is equal to increase in Kinetic energy.

$$W = K_2 - K_1$$
$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$
$$= \frac{1}{2}(100)(30)^2 - \frac{1}{2}(1000\,kg)(20)$$
$$\boxed{W = 2.5\times10^5\,J}$$

Example 7.9 :-

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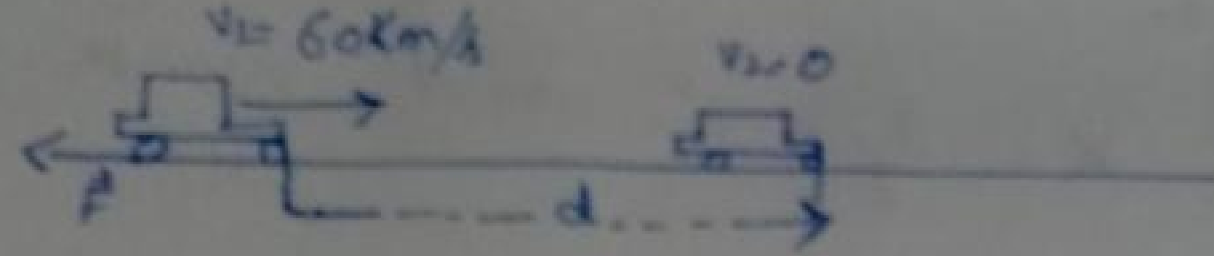

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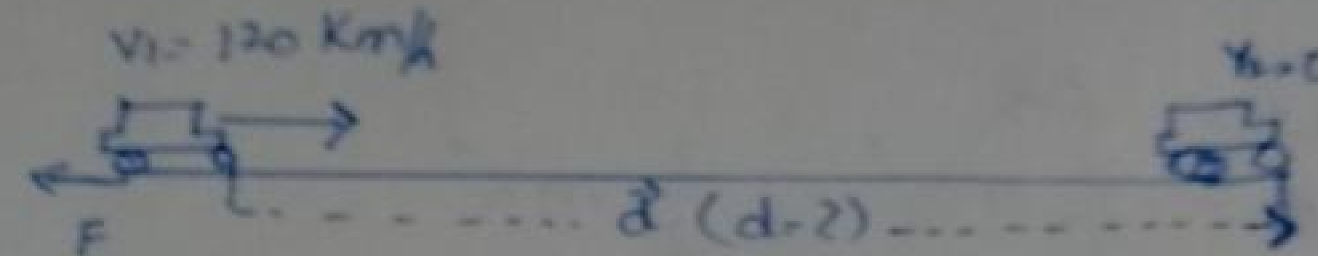


• The net stopping force F is approximately constant, the net work needed to stop the car Fd, is proportional to the distance traveled and, we apply the work energy principle noting $\vec{F}$ and $\vec{d}$ are in opposite directions and that the final speed of the car is 'zero'.

$$W_{net} = Fd\cos\theta$$

$$= Fd\cos 180^\circ$$

$$W_{net} = -Fd$$

$$\because \Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$W = \Delta K$$

$$-Fd = \Delta K$$

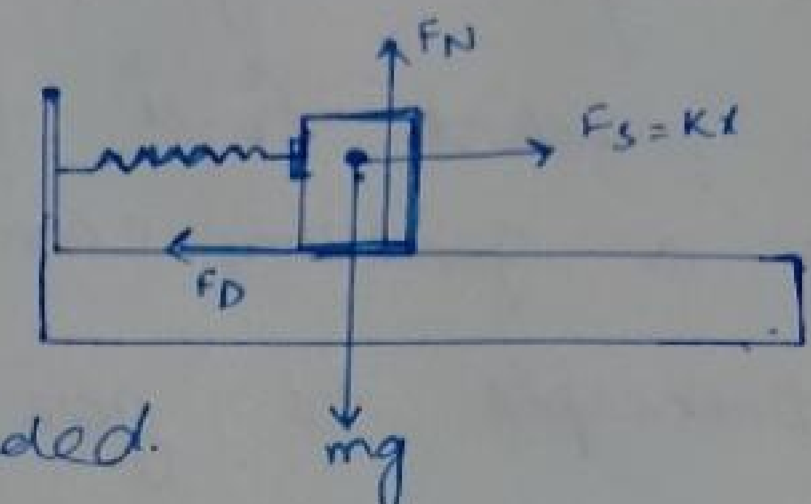
$$-Fd = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$-Fd = 0 - \frac{1}{2}mv_1^2$$

$$\cancel{-}Fd = \cancel{-}\frac{1}{2}mv_1^2 \Rightarrow Fd = \frac{1}{2}mv_1^2$$

Thus since the force and mass are constant we see that the stopping distance 'd' increases with the square of speed $d \propto v^2$. If car initial speed doubled, the stopping distance $(2)^2 = 4$ times as great or 20m

Example 7.10:-



(a) The word needed to compress the spring a distance $x = 0.110m$

$$W = \frac{1}{2} Kx^2$$

$$W = \frac{1}{2}(360 N/m)(0.110m)$$

$$\boxed{W = 2.18 J}$$

(b) In returning to its uncompressed length, the spring does 2.18 J of work on the block (same calcultion as (a) only in reverse). According to work-energy principle, the block acquires kinetic energy of 2.18J.

$$K = \frac{1}{2} mv^2$$

$$2K = mv^2 \Rightarrow v^2 = \frac{2K}{m}$$

$$v = \sqrt{\frac{2K}{m}}$$

$$v = \sqrt{\frac{2 \times (2.18)}{1.85}} \Rightarrow \boxed{v = 1.54 m/s}$$

) There are two forces on the block. that exerted by a force such as friction is complicated. For one thing, heat (or, rather, thermal energ) is produced - try rubbing your hands together.

- Nonetheless, The product $\vec{F}_D \cdot \vec{d}$ for The drag force even when it is friction, can be used in the work-energy principle to give correct results for particle like objects. The spring does '2.18 J' of work on The block.
- The work done by friction or drag force on block in The negative 'x' direction is

$$W_D = -F_D x$$
$$= -(7.0N)(0.110) = \boxed{-0.77\ J = W_D}$$

- The work is negative because the drag force acts in the opposite to The displacement 'x'.
- The net work done on block is

$$W_{net} = 2.18\ J - 0.77\ J$$
$$\boxed{W_{net} = 1.41\ J}$$

From work energy principle with $v_2 = v$, $v_1 = 0$

$$W = \Delta K$$
$$W_{net} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$
$$W_{net} = \frac{1}{2} m v^2 - 0$$
$$W_{net} = \frac{1}{2} m v^2 \Rightarrow \frac{2W_{net}}{m} = v^2$$
$$v = \sqrt{2W_{net}/1.85} = 1.23\ m/s$$

- For the block's speed at the moment it separtes from The spring $(x=0)$