$$x^2 + 22x + 112 = 0$$
$$x^2 + 22x = -112$$
$$x^2 + 22x + 121 = -112 + 121$$
$$(x + 11)^2 = 9$$
$$x + 11 = \pm\sqrt{9}$$
$$x + 11 = \pm 3$$
$$x + 11 = 3 \quad \text{or} \quad x + 11 = -3$$
$$x = -8 \quad \text{or} \quad x = -14$$

The solution set is $\{-14, -8\}$.

Classroom Example
Solve $x^4 + 2x^2 - 360 = 0$.

EXAMPLE 4 Solve $x^4 - 4x^2 - 96 = 0$.

Solution

An equation such as $x^4 - 4x^2 - 96 = 0$ is not a quadratic equation, but we can solve it using the techniques that we use on quadratic equations. That is, we can factor the polynomial and apply the property "$ab = 0$ if and only if $a = 0$ or $b = 0$" as follows:

$$x^4 - 4x^2 - 96 = 0$$
$$(x^2 - 12)(x^2 + 8) = 0$$
$$x^2 - 12 = 0 \quad \text{or} \quad x^2 + 8 = 0$$
$$x^2 = 12 \quad \text{or} \quad x^2 = -8$$
$$x = \pm\sqrt{12} \quad \text{or} \quad x = \pm\sqrt{-8}$$
$$x = \pm 2\sqrt{3} \quad \text{or} \quad x = \pm 2i\sqrt{2}$$

The solution set is $\{\pm 2\sqrt{3}, \pm 2i\sqrt{2}\}$.

Remark: Another approach to Example 4 would be to substitute y for x^2 and y^2 for x^4. The equation $x^4 - 4x^2 - 96 = 0$ becomes the quadratic equation $y^2 - 4y - 96 = 0$. Thus we say that $x^4 - 4x^2 - 96 = 0$ is of *quadratic form*. Then we could solve the quadratic equation $y^2 - 4y - 96 = 0$ and use the equation $y = x^2$ to determine the solutions for x.

Solving Word Problems Involving Quadratic Equations

Before we conclude this section with some word problems that can be solved using quadratic equations, let's restate the suggestions we made in an earlier chapter for solving word problems.

Suggestions for Solving Word Problems

1. Read the problem carefully, and make certain that you understand the meanings of all the words. Be especially alert for any technical terms used in the statement of the problem.
2. Read the problem a second time (perhaps even a third time) to get an overview of the situation being described and to determine the known facts, as well as what is to be found.
3. Sketch any figure, diagram, or chart that might be helpful in analyzing the problem.
4. Choose a meaningful variable to represent an unknown quantity in the problem (perhaps l, if the length of a rectangle is an unknown quantity), and represent any other unknowns in terms of that variable.
5. Look for a guideline that you can use to set up an equation. A guideline might be a formula such as $A = lw$ or a relationship such as "the fractional part of a job done by Bill plus the fractional part of the job done by Mary equals the total job."

6. Form an equation that contains the variable and that translates the conditions of the guideline from English to algebra.
7. Solve the equation and use the solutions to determine all facts requested in the problem.
8. **Check all answers back into the original statement of the problem.**

Keep these suggestions in mind as we now consider some word problems.

Classroom Example
A margin of 1 inch surrounds the front of a card, which leaves 39 square inches for graphics. If the height of the card is three times the width, what are the dimensions of the card?

EXAMPLE 5

A page for a magazine contains 70 square inches of type. The height of a page is twice the width. If the margin around the type is to be 2 inches uniformly, what are the dimensions of a page?

Solution

Let x represent the width of a page. Then $2x$ represents the height of a page. Now let's draw and label a model of a page (Figure 6.8).

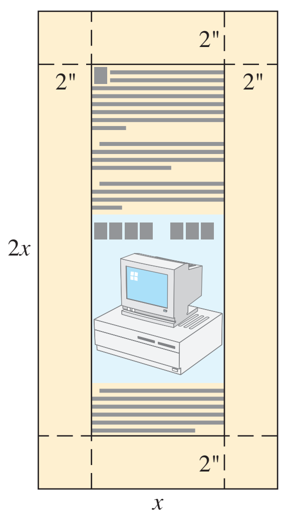

Figure 6.8

Width of typed material → $(x - 4)$; Height of typed material → $(2x - 4)$; Area of typed material → 70

$$(x-4)(2x-4) = 70$$
$$2x^2 - 12x + 16 = 70$$
$$2x^2 - 12x - 54 = 0$$
$$x^2 - 6x - 27 = 0$$
$$(x-9)(x+3) = 0$$
$$x - 9 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 9 \quad \text{or} \quad x = -3$$

Disregard the negative solution; the page must be 9 inches wide, and its height is $2(9) = 18$ inches.

Let's use our knowledge of quadratic equations to analyze some applications of the business world. For example, if P dollars is invested at r rate of interest compounded annually for t years, then the amount of money, A, accumulated at the end of t years is given by the formula

$$A = P(1 + r)^t$$

This compound interest formula serves as a guideline for the next problem.

Classroom Example
Suppose that \$2500 is invested at a certain rate of interest compounded annually for 2 years. If the accumulated value at the end of 2 years is \$2704, find the rate of interest.

EXAMPLE 6

Suppose that \$2000 is invested at a certain rate of interest compounded annually for 2 years. If the accumulated value at the end of 2 years is \$2205, find the rate of interest.

Solution

Let r represent the rate of interest. Substitute the known values into the compound interest formula to yield

$$A = P(1 + r)^t$$
$$2205 = 2000(1 + r)^2$$

Solving this equation, we obtain

$$\frac{2205}{2000} = (1 + r)^2$$
$$1.1025 = (1 + r)^2$$
$$\pm\sqrt{1.1025} = 1 + r$$
$$\pm 1.05 = 1 + r$$
$$1 + r = 1.05 \quad \text{or} \quad 1 + r = -1.05$$
$$r = -1 + 1.05 \quad \text{or} \quad r = -1 - 1.05$$
$$r = 0.05 \quad \text{or} \quad r = -2.05$$

We must disregard the negative solution, so that $r = 0.05$ is the only solution. Change 0.05 to a percent, and the rate of interest is 5%.

Classroom Example
After hiking 9 miles of a 10-mile hike, Sam hurt his foot. For the remainder of the hike, his rate was two miles per hour slower than before he hurt his foot. The entire hike took $2\frac{3}{4}$ hours. How fast did he hike before hurting his foot?

EXAMPLE 7

On a 130-mile trip from Orlando to Sarasota, Roberto encountered a heavy thunderstorm for the last 40 miles of the trip. During the thunderstorm he drove an average of 20 miles per hour slower than before the storm. The entire trip took $2\frac{1}{2}$ hours. How fast did he drive before the storm?

Solution

Let x represent Roberto's rate before the thunderstorm. Then $x - 20$ represents his speed during the thunderstorm. Because $t = \frac{d}{r}$, then $\frac{90}{x}$ represents the time traveling before the storm, and $\frac{40}{x - 20}$ represents the time traveling during the storm. The following guideline sums up the situation.

Time traveling before the storm	+	Time traveling after the storm	=	Total time
↓		↓		↓
$\frac{90}{x}$	+	$\frac{40}{x - 20}$	=	$\frac{5}{2}$

Solving this equation, we obtain

$$2x(x - 20)\left(\frac{90}{x} + \frac{40}{x - 20}\right) = 2x(x - 20)\left(\frac{5}{2}\right)$$
$$2x(x - 20)\left(\frac{90}{x}\right) + 2x(x - 20)\left(\frac{40}{x - 20}\right) = 2x(x - 20)\left(\frac{5}{2}\right)$$
$$180(x - 20) + 2x(40) = 5x(x - 20)$$
$$180x - 3600 + 80x = 5x^2 - 100x$$
$$0 = 5x^2 - 360x + 3600$$
$$0 = 5(x^2 - 72x + 720)$$
$$0 = 5(x - 60)(x - 12)$$
$$x - 60 = 0 \quad \text{or} \quad x - 12 = 0$$
$$x = 60 \quad \text{or} \quad x = 12$$

We discard the solution of 12 because it would be impossible to drive 20 miles per hour slower than 12 miles per hour; thus Roberto's rate before the thunderstorm was 60 miles per hour.

Classroom Example
James bought a shipment of monitors for $6000. When he had sold all but 10 monitors at a profit of $100 per monitor, he had regained the entire cost of the shipment. How many monitors were sold and at what price per monitor?

EXAMPLE 8

A computer installer agreed to do an installation for $150. It took him 2 hours longer than he expected, and therefore he earned $2.50 per hour less than he anticipated. How long did he expect the installation would take?

Solution

Let x represent the number of hours he expected the installation to take. Then $x + 2$ represents the number of hours the installation actually took. The rate of pay is represented by the pay divided by the number of hours. The following guideline is used to write the equation.

Anticipated rate of pay	−	$2.50	=	Actual rate of pay
↓	↓	↓	↓	↓
$\frac{150}{x}$	−	$\frac{5}{2}$	=	$\frac{150}{x+2}$

Solving this equation, we obtain

$$2x(x+2)\left(\frac{150}{x} - \frac{5}{2}\right) = 2x(x+2)\left(\frac{150}{x+2}\right)$$

$$2(x+2)(150) - x(x+2)(5) = 2x(150)$$

$$300(x+2) - 5x(x+2) = 300x$$

$$300x + 600 - 5x^2 - 10x = 300x$$

$$-5x^2 - 10x + 600 = 0$$

$$-5(x^2 + 2x - 120) = 0$$

$$-5(x+12)(x-10) = 0$$

$$x = -12 \quad \text{or} \quad x = 10$$

Disregard the negative answer. Therefore he anticipated that the installation would take 10 hours.

This next problem set contains a large variety of word problems. Not only are there some business applications similar to those we discussed in this section, but there are also more problems of the types we discussed in Chapters 3 and 4. Try to give them your best shot without referring to the examples in earlier chapters.

Concept Quiz 6.5

For Problems 1–5, choose the method that you think is most appropriate for solving the given equation.

1. $2x^2 + 6x - 3 = 0$
2. $(x+1)^2 = 36$
3. $x^2 - 3x + 2 = 0$
4. $x^2 + 6x = 19$
5. $4x^2 + 2x - 5 = 0$

A. Factoring
B. Square-root property (Property 6.1)
C. Completing the square
D. Quadratic formula

Problem Set 6.5

For Problems 1–20, solve each quadratic equation using the method that seems most appropriate to you. **(Objective 1)**

1. $x^2 - 4x - 6 = 0$
2. $x^2 - 8x - 4 = 0$
3. $3x^2 + 23x - 36 = 0$
4. $n^2 + 22n + 105 = 0$
5. $x^2 - 18x = 9$
6. $x^2 + 20x = 25$
7. $2x^2 - 3x + 4 = 0$
8. $3y^2 - 2y + 1 = 0$
9. $135 + 24n + n^2 = 0$
10. $28 - x - 2x^2 = 0$

11. $(x - 2)(x + 9) = -10$ **12.** $(x + 3)(2x + 1) = -3$

13. $2x^2 - 4x + 7 = 0$ **14.** $3x^2 - 2x + 8 = 0$

15. $x^2 - 18x + 15 = 0$ **16.** $x^2 - 16x + 14 = 0$

17. $20y^2 + 17y - 10 = 0$ **18.** $12x^2 + 23x - 9 = 0$

19. $4t^2 + 4t - 1 = 0$ **20.** $5t^2 + 5t - 1 = 0$

For Problems 21–40, solve each equation. **(Objective 1)**

21. $n + \frac{3}{n} = \frac{19}{4}$ **22.** $n - \frac{2}{n} = -\frac{7}{3}$

23. $\frac{3}{x} + \frac{7}{x - 1} = 1$ **24.** $\frac{2}{x} + \frac{5}{x + 2} = 1$

25. $\frac{12}{x - 3} + \frac{8}{x} = 14$ **26.** $\frac{16}{x + 5} - \frac{12}{x} = -2$

27. $\frac{3}{x - 1} - \frac{2}{x} = \frac{5}{2}$ **28.** $\frac{4}{x + 1} + \frac{2}{x} = \frac{5}{3}$

29. $\frac{6}{x} + \frac{40}{x + 5} = 7$ **30.** $\frac{12}{t} + \frac{18}{t + 8} = \frac{9}{2}$

31. $\frac{5}{n - 3} - \frac{3}{n + 3} = 1$ **32.** $\frac{3}{t + 2} + \frac{4}{t - 2} = 2$

33. $x^4 - 18x^2 + 72 = 0$ **34.** $x^4 - 21x^2 + 54 = 0$

35. $3x^4 - 35x^2 + 72 = 0$ **36.** $5x^4 - 32x^2 + 48 = 0$

37. $3x^4 + 17x^2 + 20 = 0$ **38.** $4x^4 + 11x^2 - 45 = 0$

39. $6x^4 - 29x^2 + 28 = 0$ **40.** $6x^4 - 31x^2 + 18 = 0$

For Problems 41–68, set up an equation and solve each problem. **(Objective 2)**

41. Find two consecutive whole numbers such that the sum of their squares is 145.

42. Find two consecutive odd whole numbers such that the sum of their squares is 74.

43. Two positive integers differ by 3, and their product is 108. Find the numbers.

44. Suppose that the sum of two numbers is 20, and the sum of their squares is 232. Find the numbers.

45. Find two numbers such that their sum is 10 and their product is 22.

46. Find two numbers such that their sum is 6 and their product is 7.

47. Suppose that the sum of two whole numbers is 9, and the sum of their reciprocals is $\frac{1}{2}$. Find the numbers.

48. The difference between two whole numbers is 8, and the difference between their reciprocals is $\frac{1}{6}$. Find the two numbers.

49. The sum of the lengths of the two legs of a right triangle is 21 inches. If the length of the hypotenuse is 15 inches, find the length of each leg.

50. The length of a rectangular floor is 1 meter less than twice its width. If a diagonal of the rectangle is 17 meters, find the length and width of the floor.

51. A rectangular plot of ground measuring 12 meters by 20 meters is surrounded by a sidewalk of a uniform width (see Figure 6.9). The area of the sidewalk is 68 square meters. Find the width of the walk.

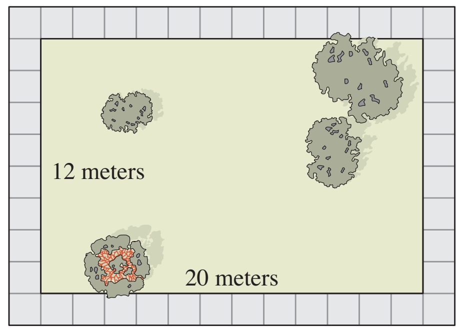

Figure 6.9

52. A 5-inch by 7-inch picture is surrounded by a frame of uniform width. The area of the picture and frame together is 80 square inches. Find the width of the frame.

53. The perimeter of a rectangle is 44 inches, and its area is 112 square inches. Find the length and width of the rectangle.

54. A rectangular piece of cardboard is 2 units longer than it is wide. From each of its corners a square piece 2 units on a side is cut out. The flaps are then turned up to form an open box that has a volume of 70 cubic units. Find the length and width of the original piece of cardboard.

55. Charlotte's time to travel 250 miles is 1 hour more than Lorraine's time to travel 180 miles. Charlotte drove 5 miles per hour faster than Lorraine. How fast did each one travel?

56. Larry's time to travel 156 miles is 1 hour more than Terrell's time to travel 108 miles. Terrell drove 2 miles per hour faster than Larry. How fast did each one travel?

57. On a 570-mile trip, Andy averaged 5 miles per hour faster for the last 240 miles than he did for the first 330 miles. The entire trip took 10 hours. How fast did he travel for the first 330 miles?