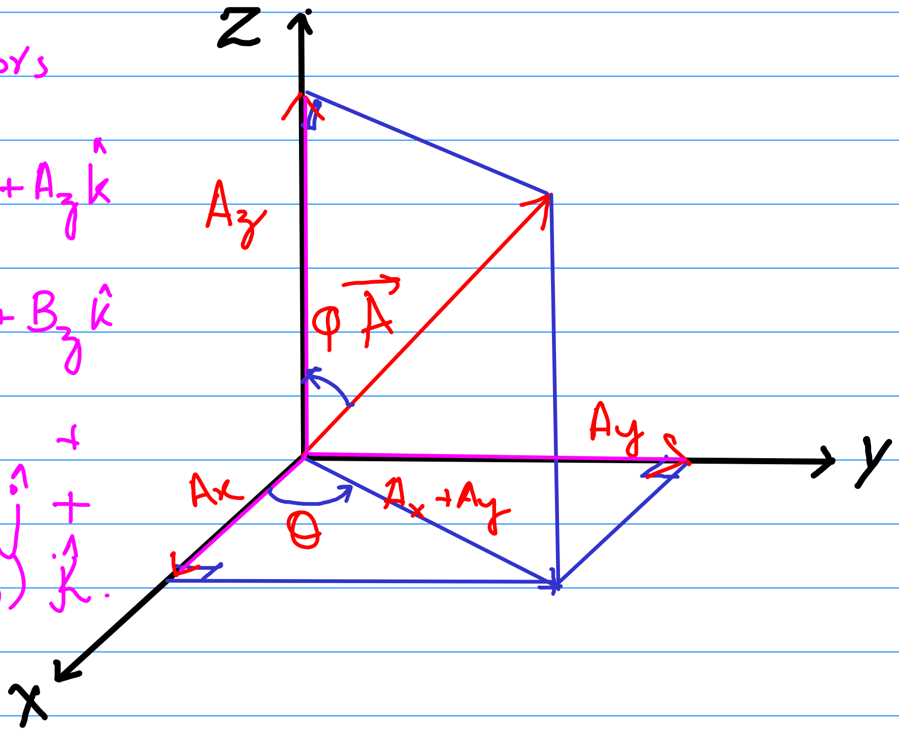
Vectors in Three Dimensions:

Addition of vectors



$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} +$
$(A_y + B_y)\hat{j} +$
$(A_z + B_z)\hat{k}.$

Multiplication of Vectors:

→ Dot Product / Scalar Product

$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$ $\Rightarrow$ $\hat{i}\cdot\hat{i} = 1$, $\hat{j}\cdot\hat{j} = 1$, $\hat{k}\cdot\hat{k} = 1$

→ Cross Product / Vector Product

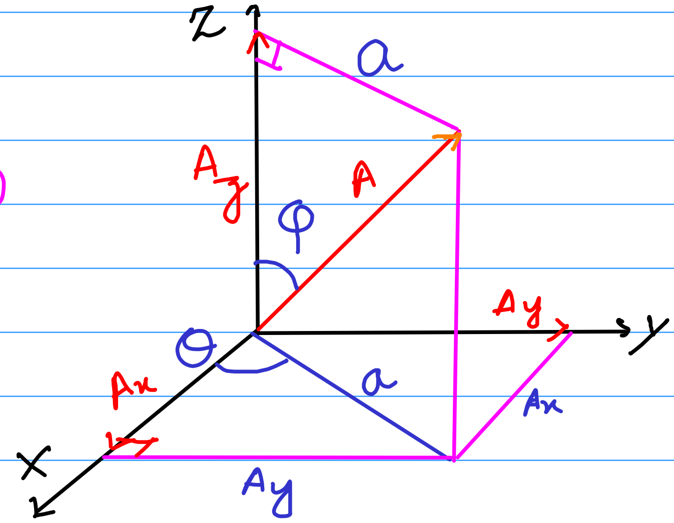
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_yB_z - B_yA_z)\hat{i} - \hat{j}(A_xB_z - B_xA_z) + \hat{k}(A_xB_y - B_xA_y).$$

Components of vector in three dimensions:

$\rightarrow$ $\cos\theta = \frac{A_x}{a}$ —— (1)

$\sin\theta = \frac{A_y}{a}$ —— (2)



$\cos\varphi = \frac{A_z}{A}$ —— (3)

$\sin\varphi = \frac{a}{A}$ —— (4)

$\Rightarrow$ $A_z = A\cos\varphi$

from eq (4) $a = A\sin\varphi$

$\Rightarrow$ $A_x = a\cos\theta = A\cos\theta\sin\varphi.$

$\Rightarrow$ $A_y = a\sin\theta = A\sin\theta\sin\varphi.$

$\vec{A}\cdot\vec{B}$ ✓

$\vec{A}\times\vec{B}$ ✓

Scalar Triple Product:

$\vec{A}$, $\vec{B}$ and $\vec{C}$

$\vec{A}\cdot\cancel{(\vec{B}\cdot\vec{C})}$ $\Longrightarrow$

$$\left.\begin{aligned}
&\vec{A}\cdot(\vec{B}\times\vec{C})\\
&\vec{A}\cdot(\vec{C}\times\vec{B})\\
&\vec{B}\cdot(\vec{A}\times\vec{C})\\
&\vec{B}\cdot(\vec{C}\times\vec{A})\\
&\vec{C}\cdot(\vec{A}\times\vec{B})\\
&\vec{C}\cdot(\vec{B}\times\vec{A})
\end{aligned}\right\}$$ Scalar quantity

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = ?$$

$$= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot\big((B_yC_z - B_zC_y)\hat{i} - \hat{j}(B_xC_z - C_xB_z) + \hat{k}(B_xC_y - C_xB_y)\big)$$

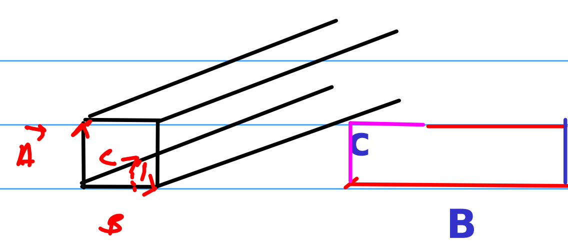
$$= A_x(B_yC_z - B_zC_y) - A_y(B_xC_z - C_xB_z) + A_z(B_xC_y - C_xB_y).$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \Longrightarrow ?$$

$$\Rightarrow \vec{A}\cdot(\vec{B}\times\vec{C})$$

$$\vec{A}\cdot(BC\sin\theta)$$

$$A\,(BC\sin\theta)$$



Volume of parallelpiped.

area of parall.

Vector triple Product:

$\vec{A} \times (\vec{B} \times \vec{C})$ or $\vec{A} \times (\vec{C} \times \vec{B})$

$\vec{B} \times (\vec{A} \times \vec{C})$ or $\vec{B} \times (\vec{C} \times \vec{A})$

$\vec{C} \times (\vec{A} \times \vec{B})$ or $\vec{C} \times (\vec{B} \times \vec{A})$.