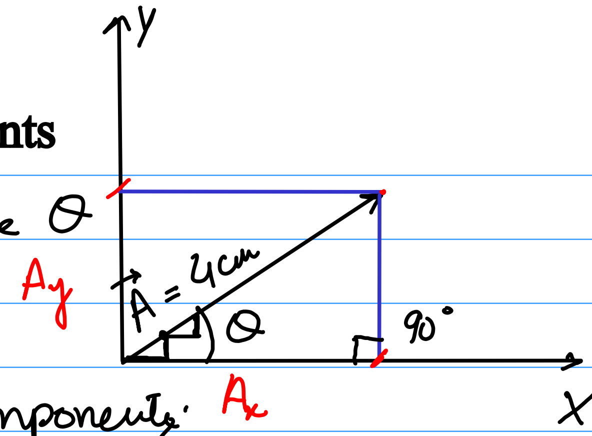
Vectors and their components

Vector $\vec{A}$ is making an angle θ with x-axis



$\theta \rightarrow$ orientation

To find its cartesian components: components of $\vec{A}$ along x and y-axis

$$\sin\theta = \frac{Perp}{Hyp}, \quad \cos\theta = \frac{Base}{Hyp}.$$

$$Hyp = |\vec{A}| = A, \quad Perp = A_y, \quad Base = A_x$$

$$\sin\theta = \frac{A_y}{A}, \quad \cos\theta = \frac{A_x}{A}$$

$$A_y = A\sin\theta \quad \text{and} \quad A_x = A\cos\theta.$$

$$\tan\theta = \frac{Perp}{Base} = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right).$$

Adding Two Vectors by its Components.

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j}$$

$\hat{i}, \hat{j}$ are unit vectors $\Rightarrow |\hat{i}| = |\hat{j}| = 1$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$

Multiplication of Two Vectors

- → Scalar Multiplication
- → Vector Multiplication

if two vector are multiplying

$\vec{A}$ multiplying $\vec{B}$ $\longrightarrow$ $\vec{A}\cdot\vec{B}$

$\longrightarrow$ $\vec{A}\times\vec{B}$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$ $\quad\quad$ $\vec{B} = B_x\hat{i} + B_y\hat{j}$

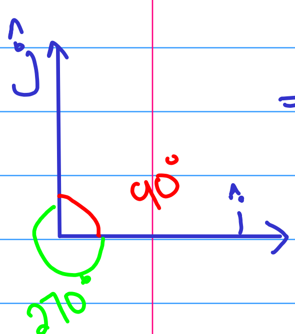
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y$$
$$= (A_x\hat{i} + A_y\hat{j})\cdot(B_x\hat{i} + B_y\hat{j})$$
$$= A_x\hat{i}\cdot B_x\hat{i} + A_x\hat{i}\cdot B_y\hat{j} + A_y\hat{j}\cdot B_x\hat{i} + A_y\hat{j}\cdot B_y\hat{j}$$
$$= A_xB_x\,\hat{i}\cdot\hat{i} + \underline{A_xB_y\,\hat{i}\cdot\hat{j}} + \underline{A_yB_x\,\hat{j}\cdot\hat{i}} + A_yB_y\,\hat{j}\cdot\hat{j}$$
$$= A_xB_x + A_yB_y.$$

$\hat{j}\cdot\hat{j} = \hat{i}\cdot\hat{i} = 1\cdot 1\cdot\cos 0^\circ = 1$

$\hat{j}\cdot\hat{i} = 0$

$\hat{i}\cdot\hat{j} = 0$

$$\vec{A}\times\vec{B} = (A_x\hat{i} + A_y\hat{j})\times(B_x\hat{i} + B_y\hat{j})$$



$$= A_xB_x\,\hat{i}\times\hat{i} + A_xB_y\,\hat{i}\times\hat{j} + A_yB_x\,\hat{j}\times\hat{i}$$
$$+ A_yB_y\,\hat{j}\times\hat{j}$$

$\hat{j}\times\hat{j} = |\hat{j}|\cdot|\hat{j}|\sin\theta = 0 = \hat{i}\times\hat{i}$

$\hat{i}\times\hat{j} = 1$

$\hat{j}\times\hat{i} = -1$

$$= A_xB_y - A_yB_x.$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} \\ A_x & B_x \\ A_y & B_y \end{vmatrix}$$
$$= A_xB_y - A_yB_x.$$

$\longrightarrow$ $\vec{A}\cdot\vec{B} = AB\cos\theta$

$\longrightarrow$ $\vec{A}\times\vec{B} = AB\sin\theta$