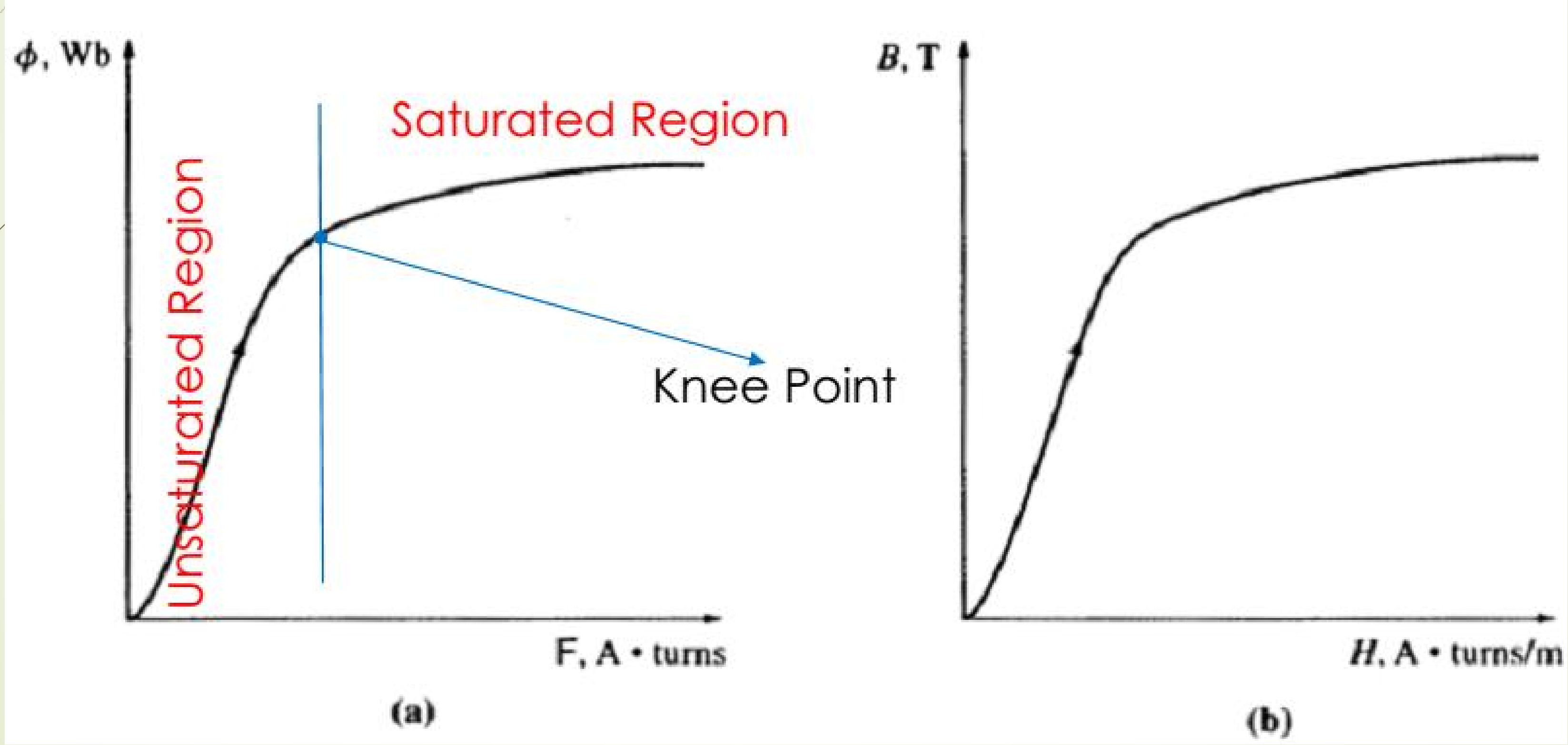
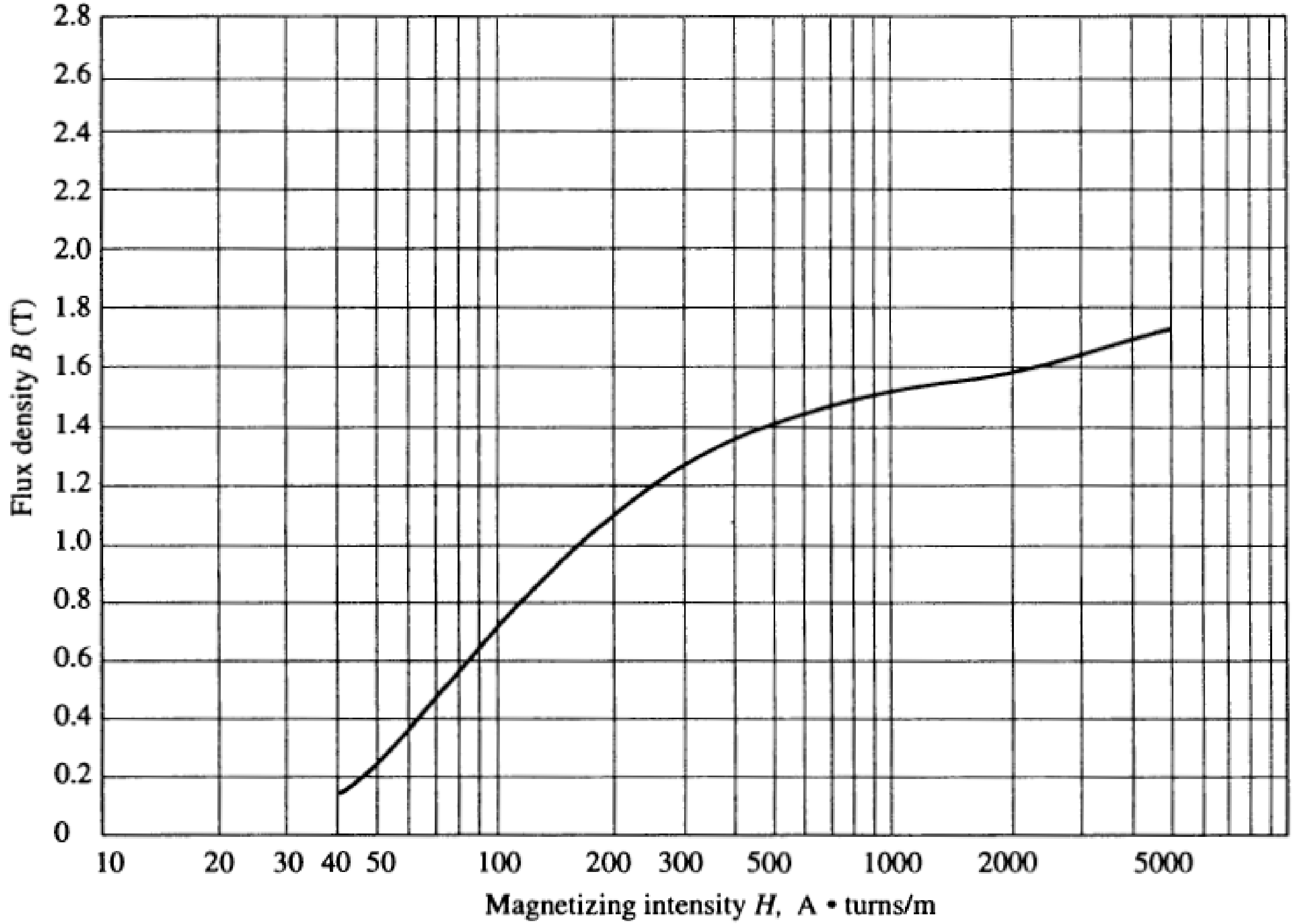
Magnetic behavior of ferromagnetic material – Saturation Curve/Magnetization Curve

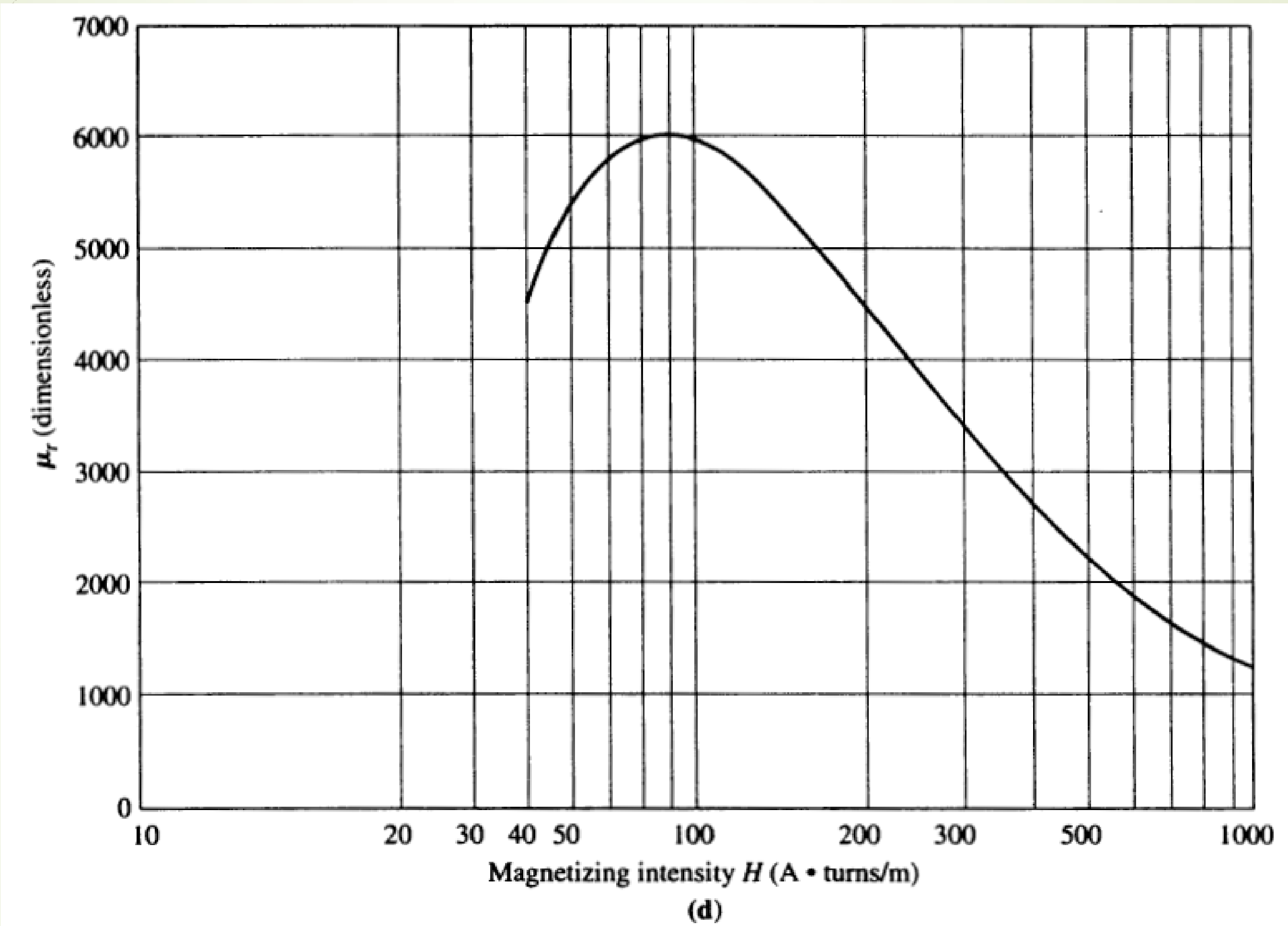


Magnetic curve for a typical steel



(c)

A plot of relative permeability mr



Example 1-4

Example 1–4. Find the relative permeability of the typical ferromagnetic material whose magnetization curve is shown in Figure 1–10c at (a) $H = 50$, (b) $H = 100$, (c) $H = 500$, and (d) $H = 1000$ A • turns/m.

Example 1-5

Example 1–5. A square magnetic core has a mean path length of 55 cm and a cross-sectional area of 150 cm^2. A 200-turn coil of wire is wrapped around one leg of the core. The core is made of a material having the magnetization curve shown in Figure 1–10c.

(a) How much current is required to produce 0.012 Wb of flux in the core?
(b) What is the core's relative permeability at that current level?
(c) What is its reluctance?

Solution

(a) The required flux density in the core is

$$B = \frac{\phi}{A} = \frac{1.012 \text{ Wb}}{0.015 \text{ m}^2} = 0.8 \text{ T}$$

From Figure 1–10c, the required magnetizing intensity is

$$H = 115 \text{ A} \cdot \text{turns/m}$$

From Equation (1–20), the magnetomotive force needed to produce this magnetizing intensity is

$$\mathcal{F} = Ni = Hl_c$$
$$= (115 \text{ A} \cdot \text{turns/m})(0.55 \text{ m}) = 63.25 \text{ A} \cdot \text{turns}$$

so the required current is

$$i = \frac{\mathcal{F}}{N} = \frac{63.25 \text{ A} \cdot \text{turns}}{200 \text{ turns}} = 0.316 \text{ A}$$

(b) The core's permeability at this current is

$$\mu = \frac{B}{H} = \frac{0.8 \text{ T}}{115 \text{ A} \cdot \text{turns/m}} = 0.00696 \text{ H/m}$$

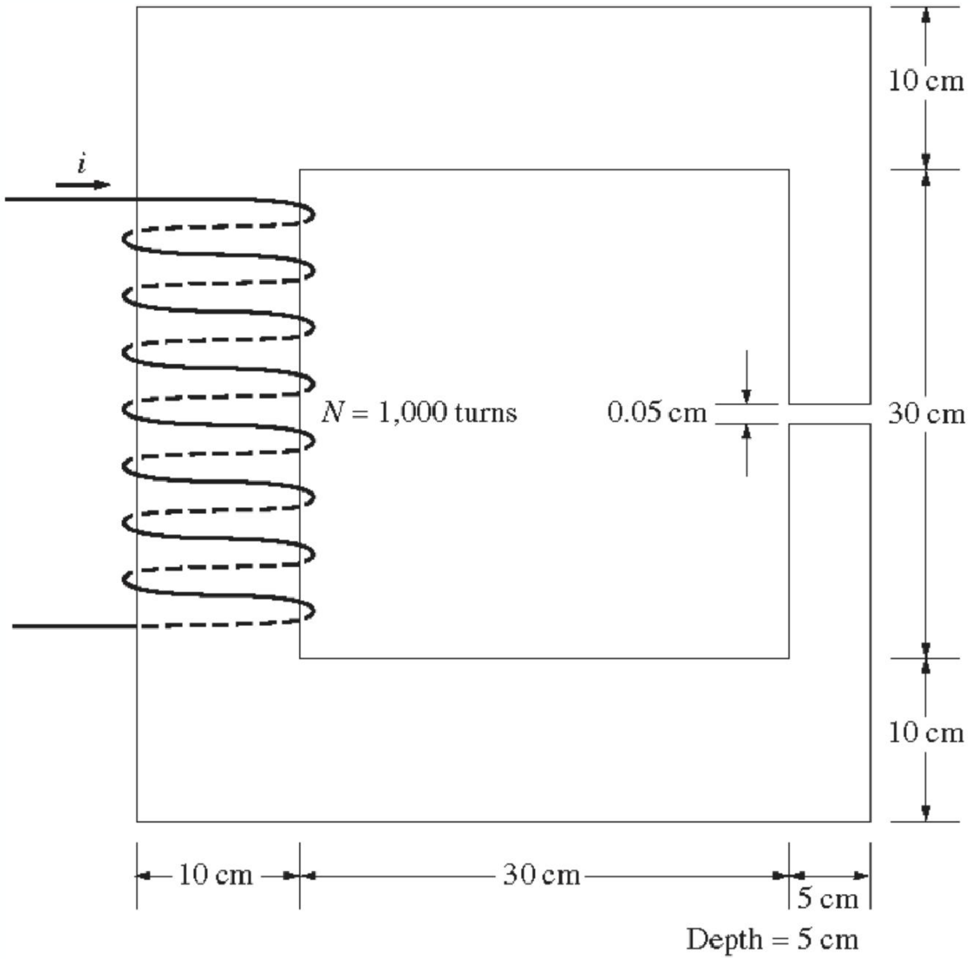
Therefore, the relative permeability is

$$\mu_r = \frac{\mu}{\mu_0} = \frac{0.00696 \text{ H/m}}{4\pi \times 10^{-7} \text{ H/m}} = 5540$$

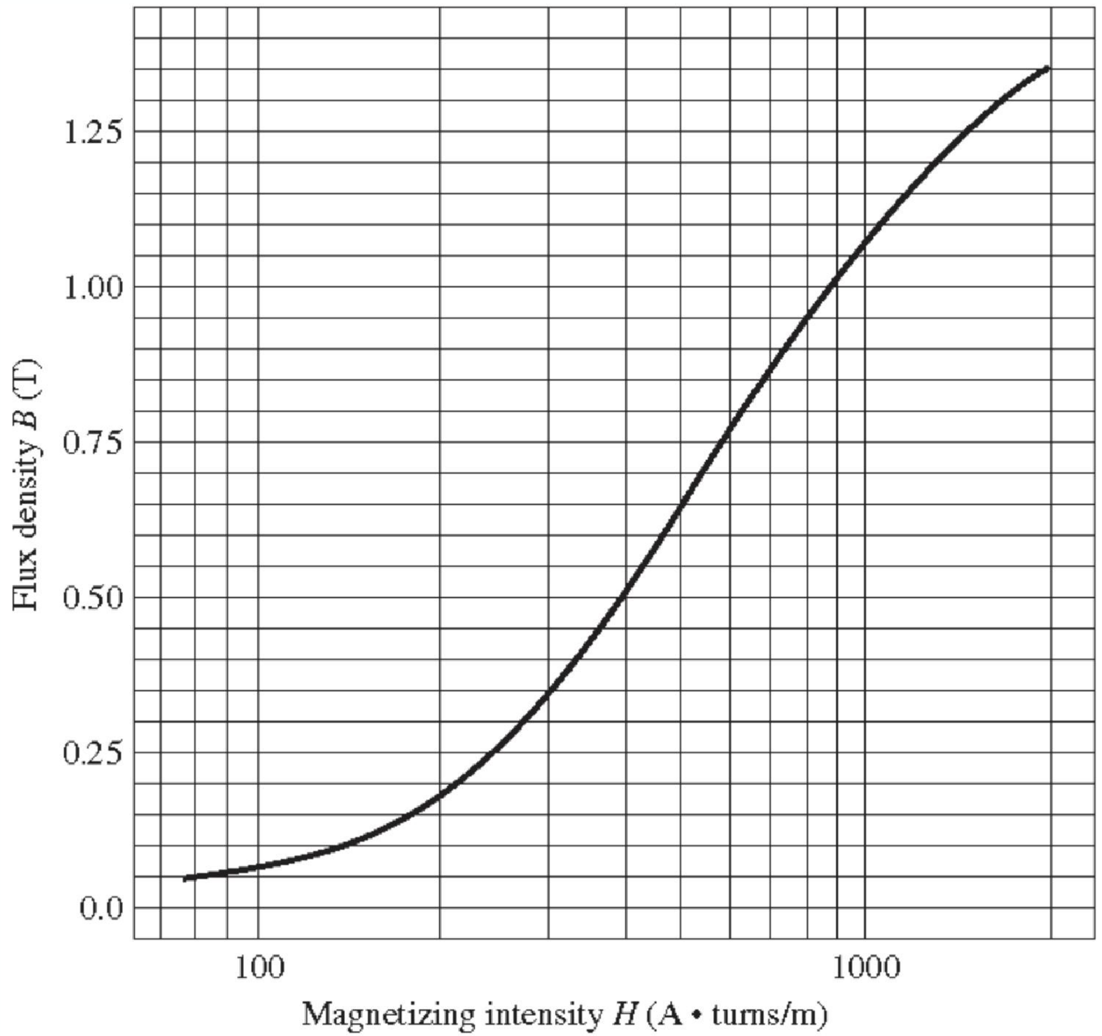
(c) The reluctance of the core is

$$\mathcal{R} = \frac{\mathcal{F}}{\phi} = \frac{63.25 \text{ A} \cdot \text{turns}}{0.012 \text{ Wb}} = 5270 \text{ A} \cdot \text{turns/Wb}$$

1–14. A two-legged magnetic core with an air gap is shown in Figure P1–11. The depth of the core is 5 cm, the length of the air gap in the core is 0.05 cm, and the number of turns on the coil is 1000. The magnetization curve of the core material is shown in



Magnetization Curve



An air-gap flux density of 0.5 T requires a total flux of

$$\phi = BA_{\text{eff}} = (0.5\text{ T})(0.05\text{ m})(0.05\text{ m})(1.05) = 0.00131\text{ Wb}$$

This flux requires a flux density in the right-hand leg of

$$B_{\text{right}} = \frac{\phi}{A} = \frac{0.00131\text{ Wb}}{(0.05\text{ m})(0.05\text{ m})} = 0.524\text{ T}$$

The flux density in the other three legs of the core is

$$B_{\text{top}} = B_{\text{left}} = B_{\text{bottom}} = \frac{\phi}{A} = \frac{0.00131\text{ Wb}}{(0.10\text{ m})(0.05\text{ m})} = 0.262\text{ T}$$

The magnetizing intensity required to produce a flux density of 0.5 T in the air gap can be found from the equation $B_{\text{ag}} = \mu_o H_{\text{ag}}$:

$$H_{\text{ag}} = \frac{B_{\text{ag}}}{\mu_0} = \frac{0.5\text{ T}}{4\pi \times 10^{-7}\text{ H/m}} = 398\text{ kA}\cdot\text{t/m}$$

The magnetizing intensity required to produce a flux density of 0.524 T in the right-hand leg of the core can be found from Figure P1-9 to be

$$H_{\text{right}} = 410\text{ A}\cdot\text{t/m}$$

The magnetizing intensity required to produce a flux density of 0.262 T in the top, left, and bottom legs of the core can be found from Figure P1-9 to be

$$H_{\text{top}} = H_{\text{left}} = H_{\text{bottom}} = 240\ \text{A}\cdot\text{t/m}$$

The total MMF required to produce the flux is

$$\mathcal{F}_{\text{TOT}} = H_{\text{ag}}\, l_{\text{ag}} + H_{\text{right}}\, l_{\text{right}} + H_{\text{top}}\, l_{\text{top}} + H_{\text{left}}\, l_{\text{left}} + H_{\text{bottom}}\, l_{\text{bottom}}$$

$$\mathcal{F}_{\text{TOT}} = (398\ \text{kA}\cdot\text{t/m})(0.0005\ \text{m}) + (410\ \text{A}\cdot\text{t/m})(0.40\ \text{m}) + 3(240\ \text{A}\cdot\text{t/m})(0.40\ \text{m})$$

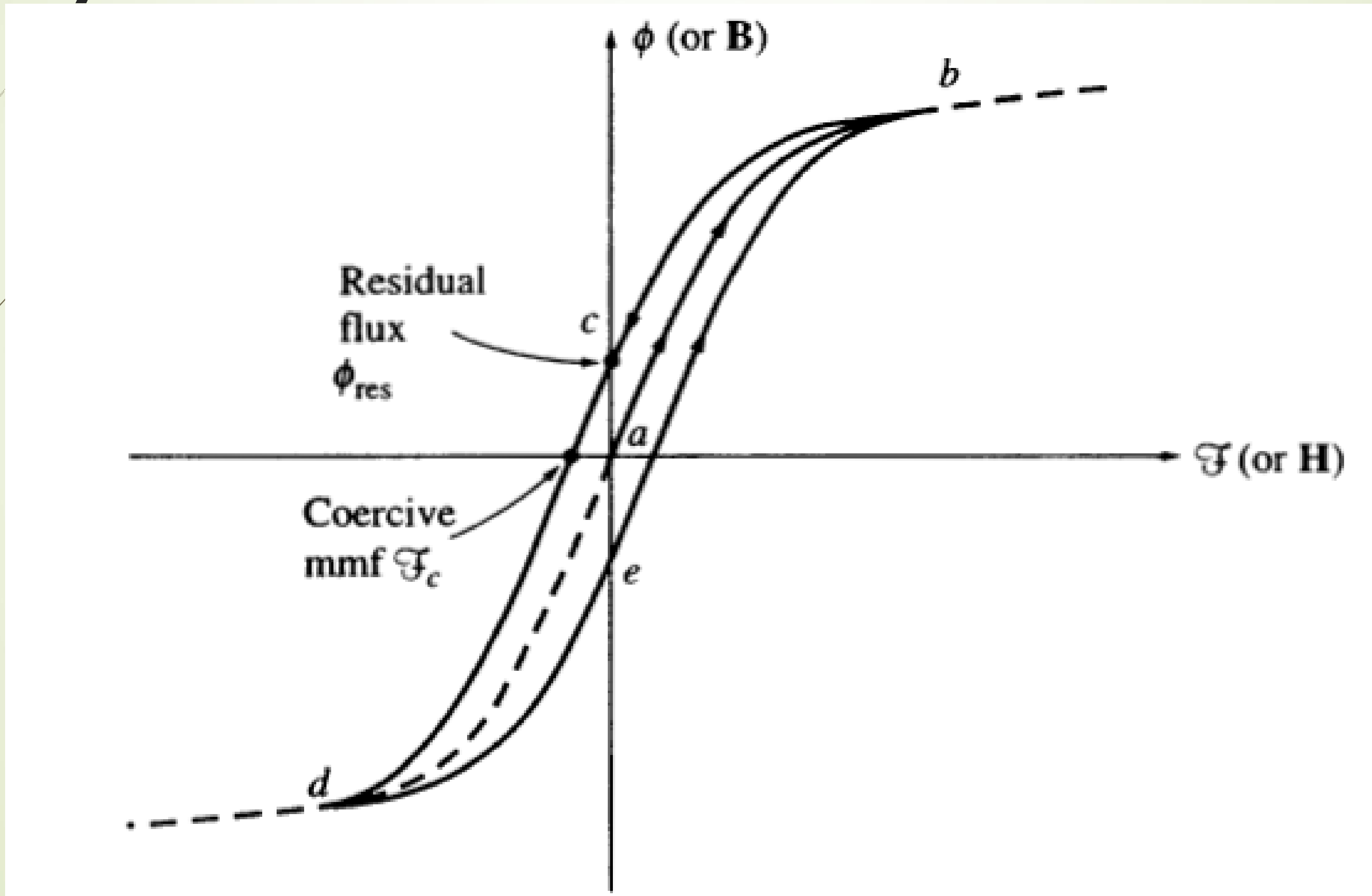
$$\mathcal{F}_{\text{TOT}} = 278.6 + 164 + 288 = 651\ \text{A}\cdot\text{t}$$

and the required current is

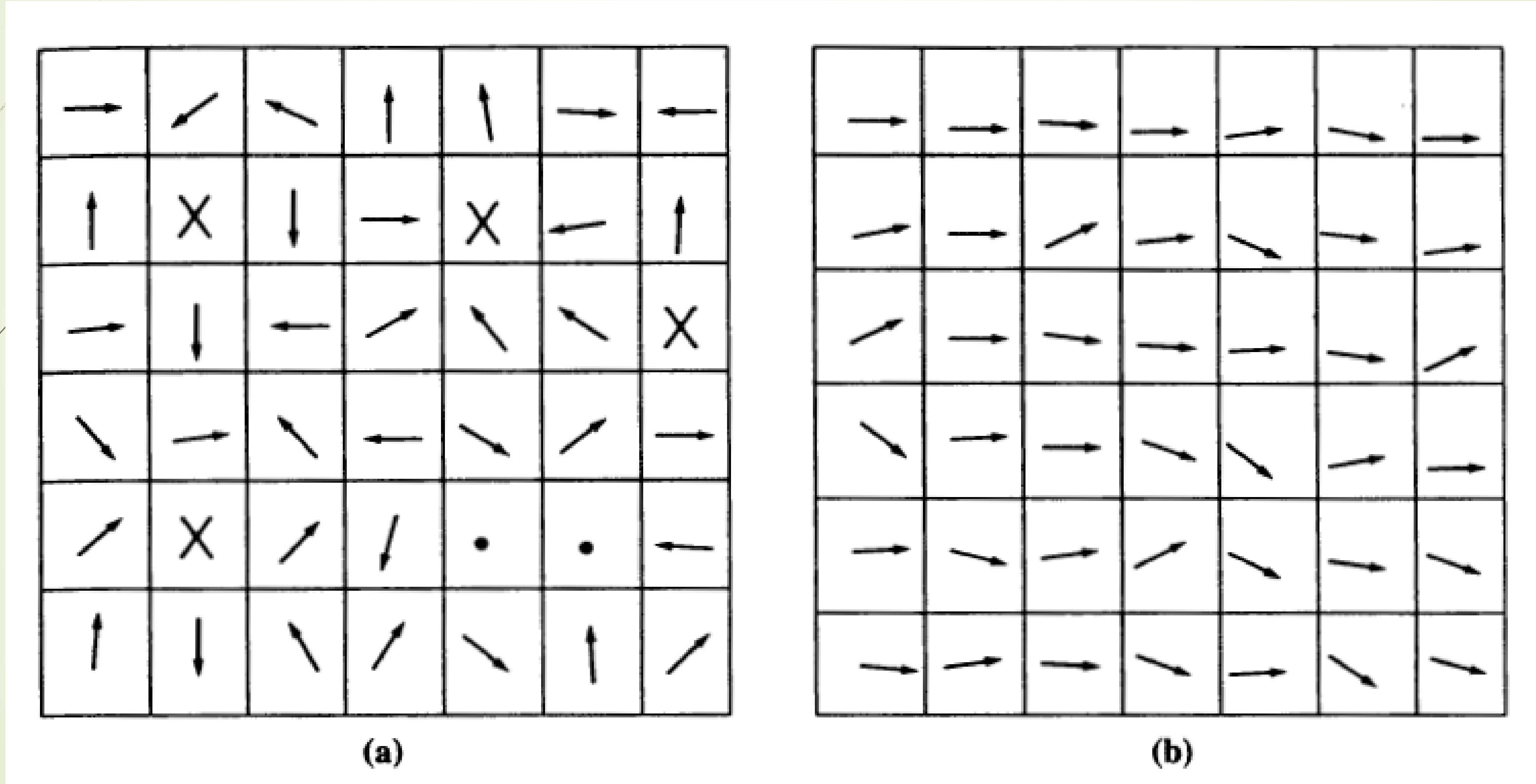
$$i = \frac{\mathcal{F}_{\text{TOT}}}{N} = \frac{651\ \text{A}\cdot\text{t}}{1000\ \text{t}} = 0.651\ \text{A}$$

The flux densities in the four sides of the core and the total flux present in the air gap were calculated above.

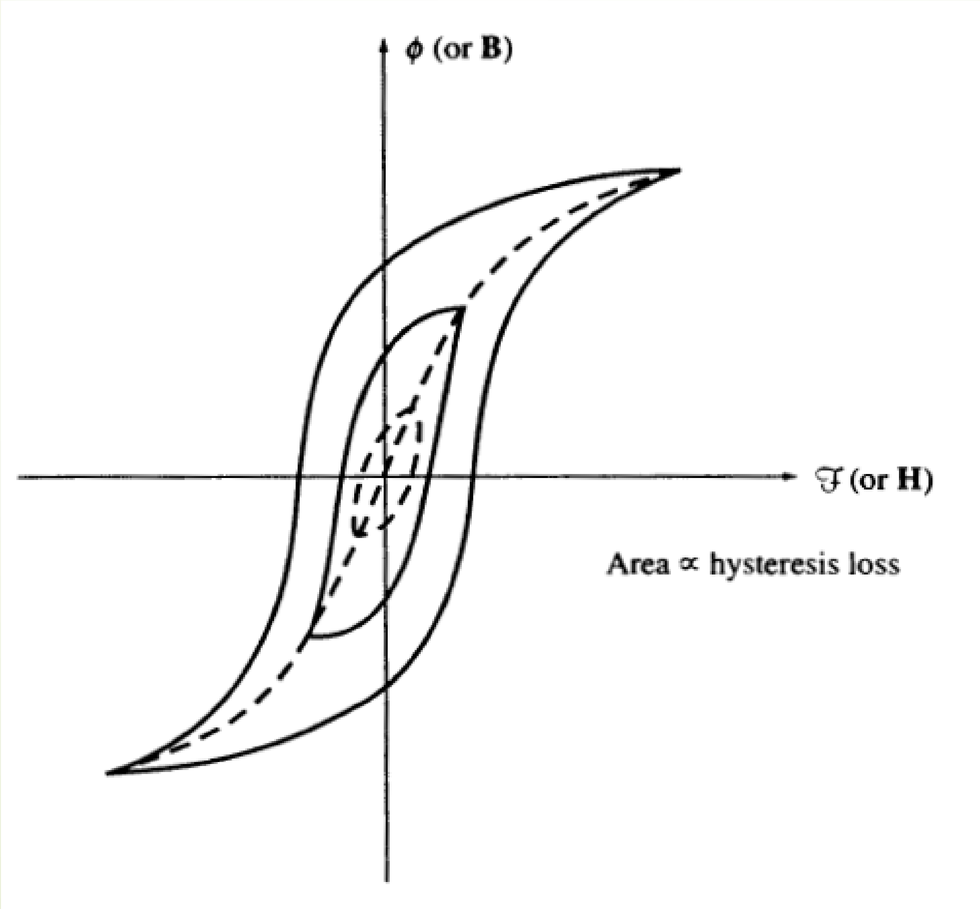
Energy loss in ferromagnetic core – hysteresis loss



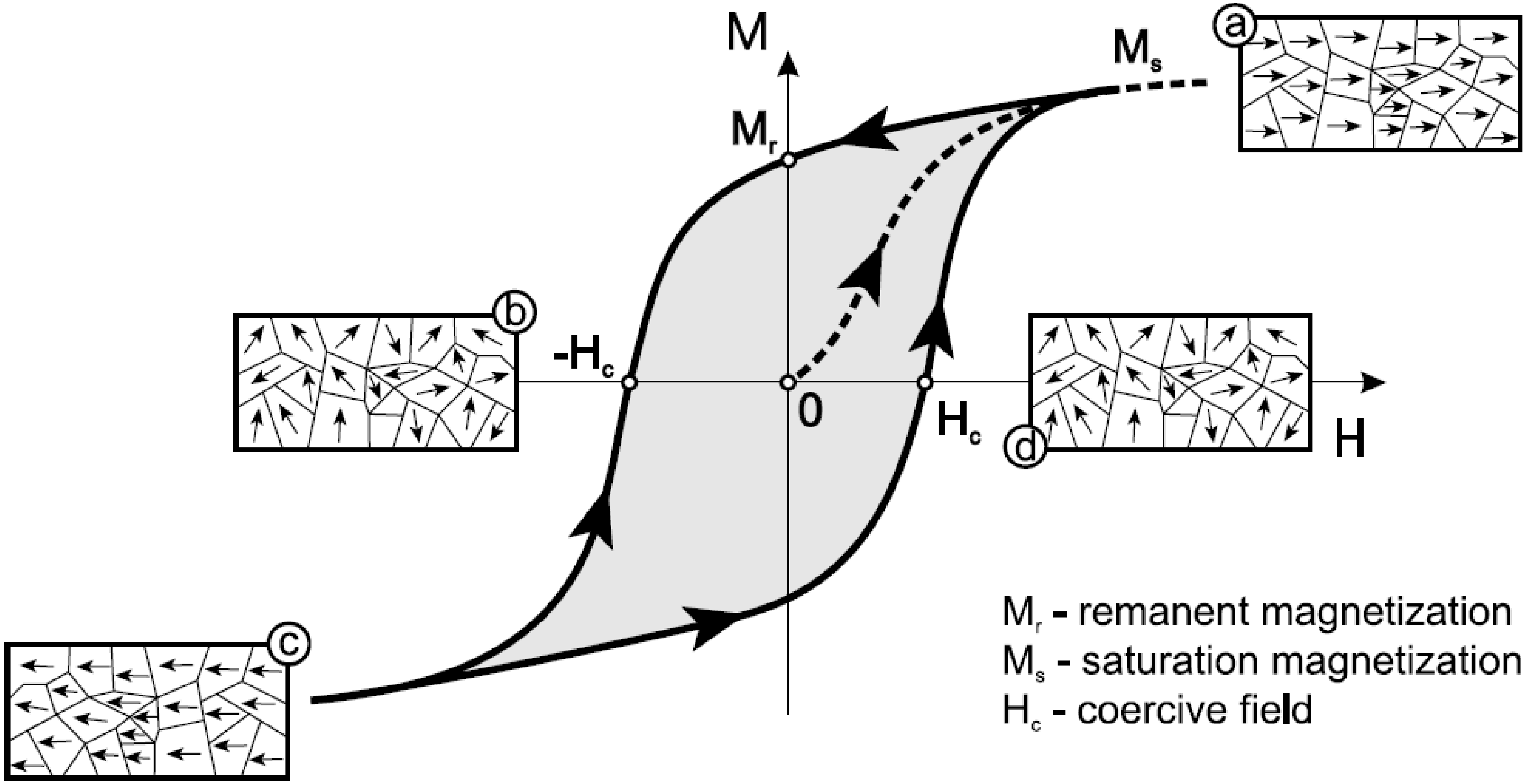
Hysteresis loop – residual flux



The effect of magnetomotive force on the hysteresis loop



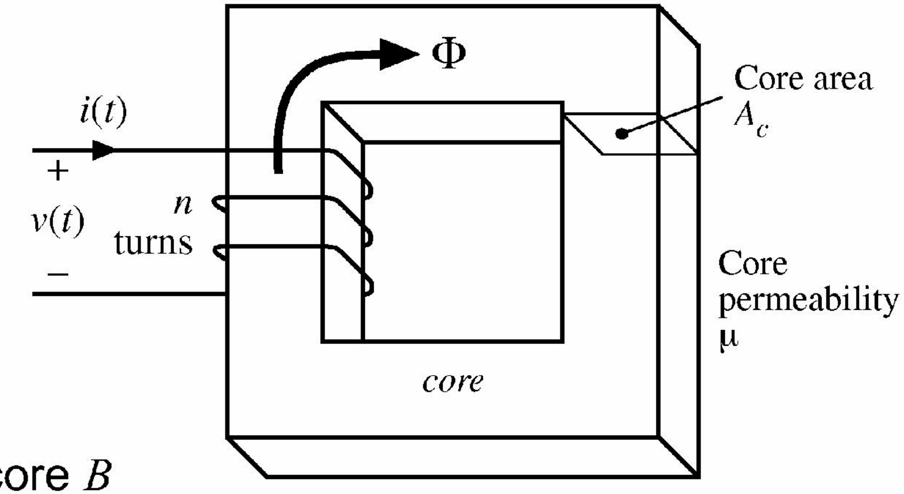
Magnetization curve



Hysteresis loss

Energy per cycle W flowing into n-turn winding of an inductor, excited by periodic waveforms of frequency f:

$$W = \int_{one\ cycle} v(t)i(t)\,dt$$



Relate winding voltage and current to core B and H via Faraday's law and Ampere's law:

$$v(t) = nA_c \frac{dB(t)}{dt} \qquad H(t)\ell_m = ni(t)$$

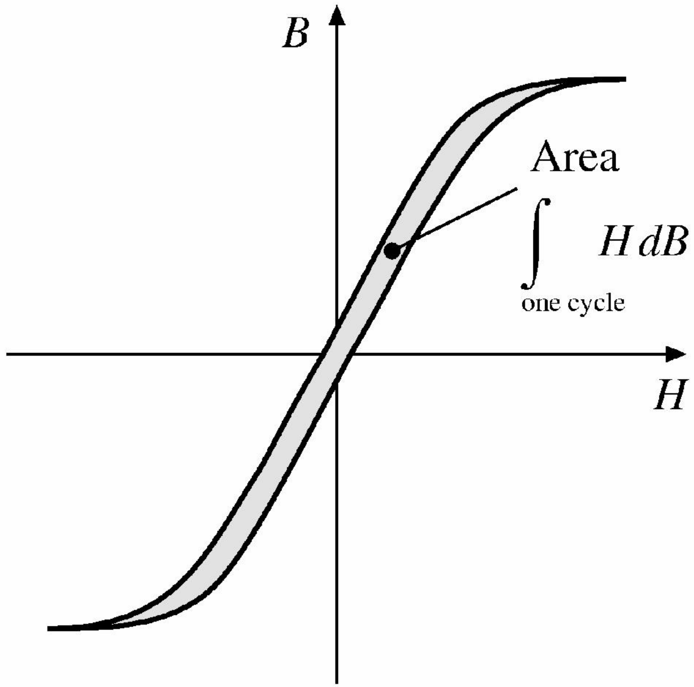
Substitute into integral:

$$W = \int_{one\ cycle} \left(nA_c \frac{dB(t)}{dt}\right)\left(\frac{H(t)\ell_m}{n}\right)dt$$

$$= \left(A_c \ell_m\right) \int_{one\ cycle} H\,dB$$

Hysteresis loss

$$W = \left(A_c \ell_m\right) \int_{\text{one cycle}} H\,dB$$



The term $A_c \ell_m$ is the volume of the core, while the integral is the area of the B–H loop.

(energy lost per cycle) = (core volume) (area of B–H loop)

$$P_H = \left(f\right)\left(A_c \ell_m\right) \int_{\text{one cycle}} H\,dB$$

Hysteresis loss is directly proportional to applied frequency

Faraday’s law – induce voltage from a time-varying magnetic field

1. Induced voltage magnitude and polarity

$$e_{ind} = -\frac{d\phi}{dt}$$

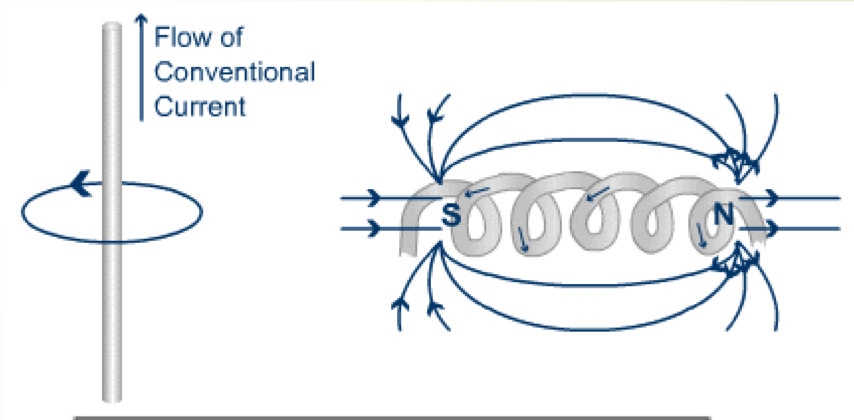
$$e_{ind} = \sum_{i=1}^{N} e_i$$

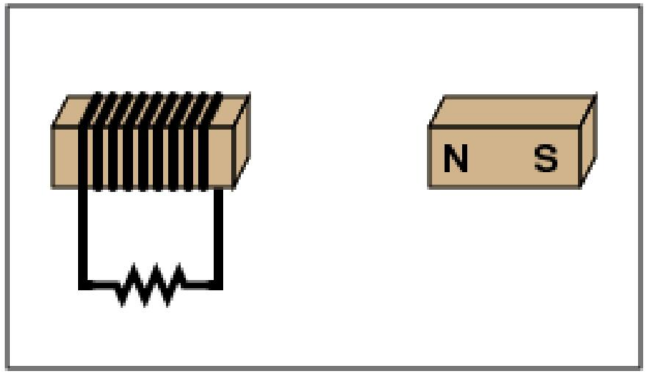
$$e_{ind} = -N\frac{d\phi}{dt}$$

e_{ind} = voltage induced in the coil

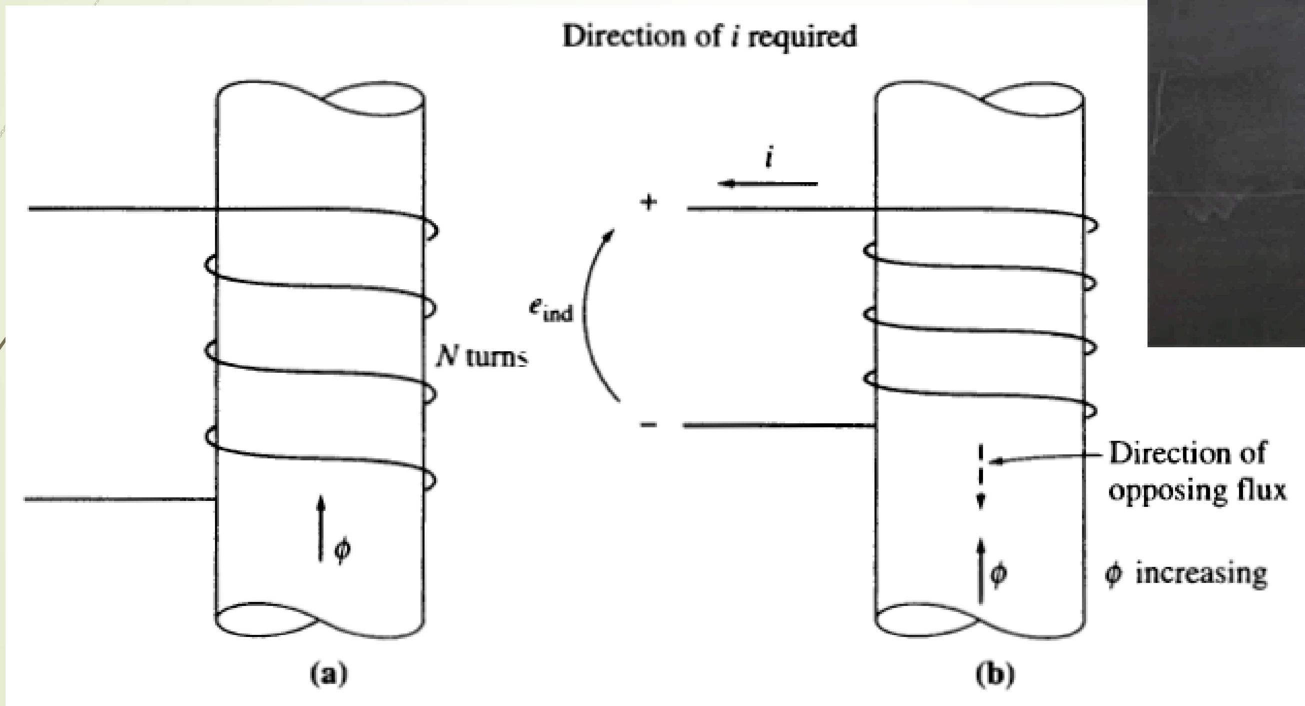
N = number of turns of wire in coil

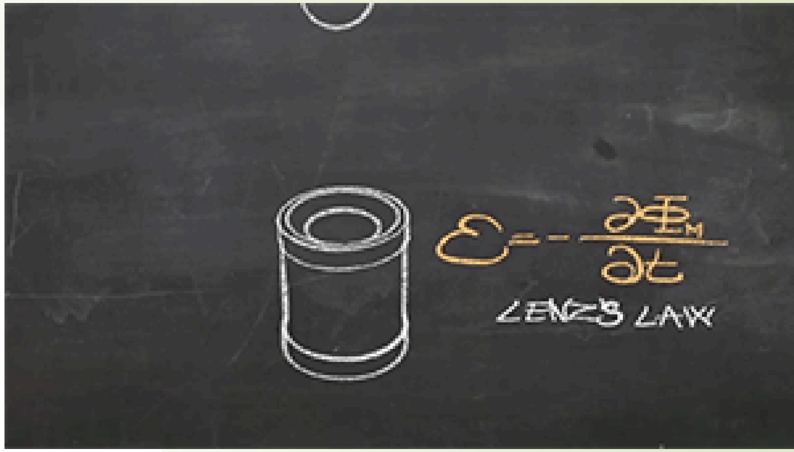
ϕ = flux passing through coil





The induced voltage polarity – Lenz's law





Flux and Flux Linkage

The term in parentheses in Equation (1–40) is called *the flux linkage* λ of the coil, and Faraday's law can be rewritten in terms of flux linkage as

$$e_{\text{ind}} = \frac{d\lambda}{dt} \qquad (1\text{–}41)$$

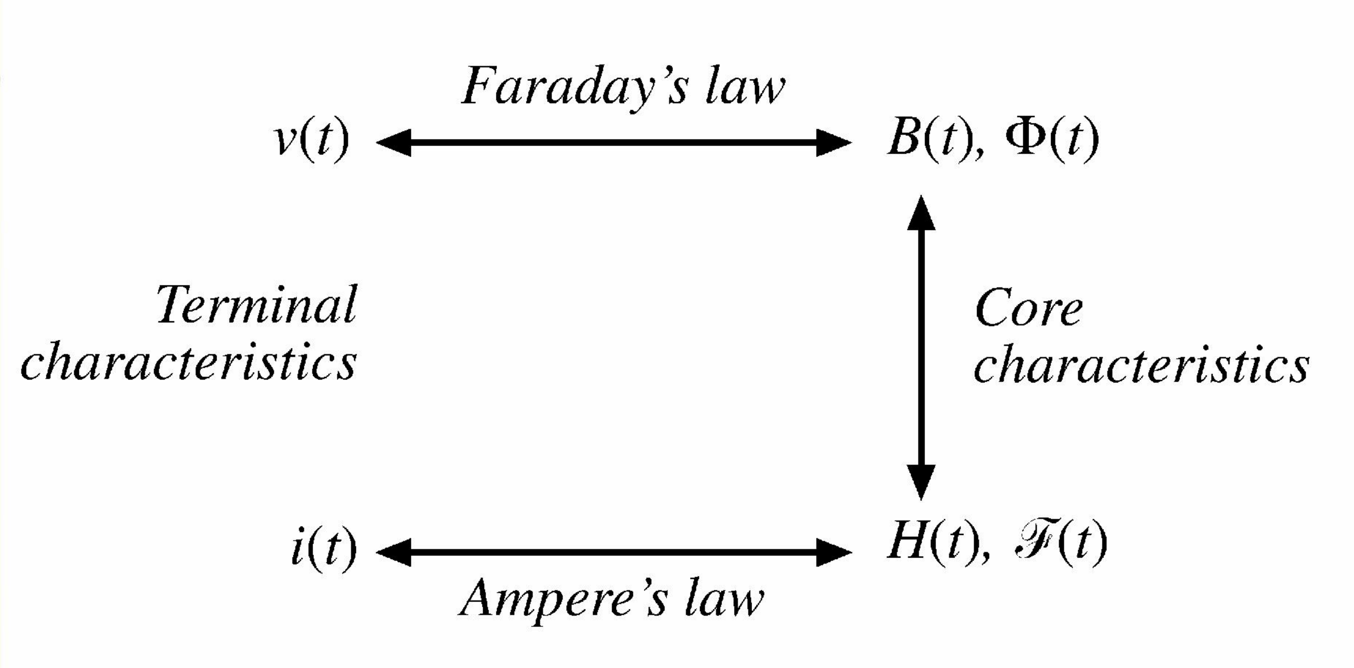
where

$$\lambda = \sum_{i=1}^{N} \phi_i \qquad (1\text{–}42)$$

The units of flux linkage are weber-turns.

Relationship between electric-magnetic variables

- Magnetic field: Ampere's law
- Magnetic flux: magnetic material, hysteresis characteristics
- Transformer: Faraday's law, Len's law



Example 1-6

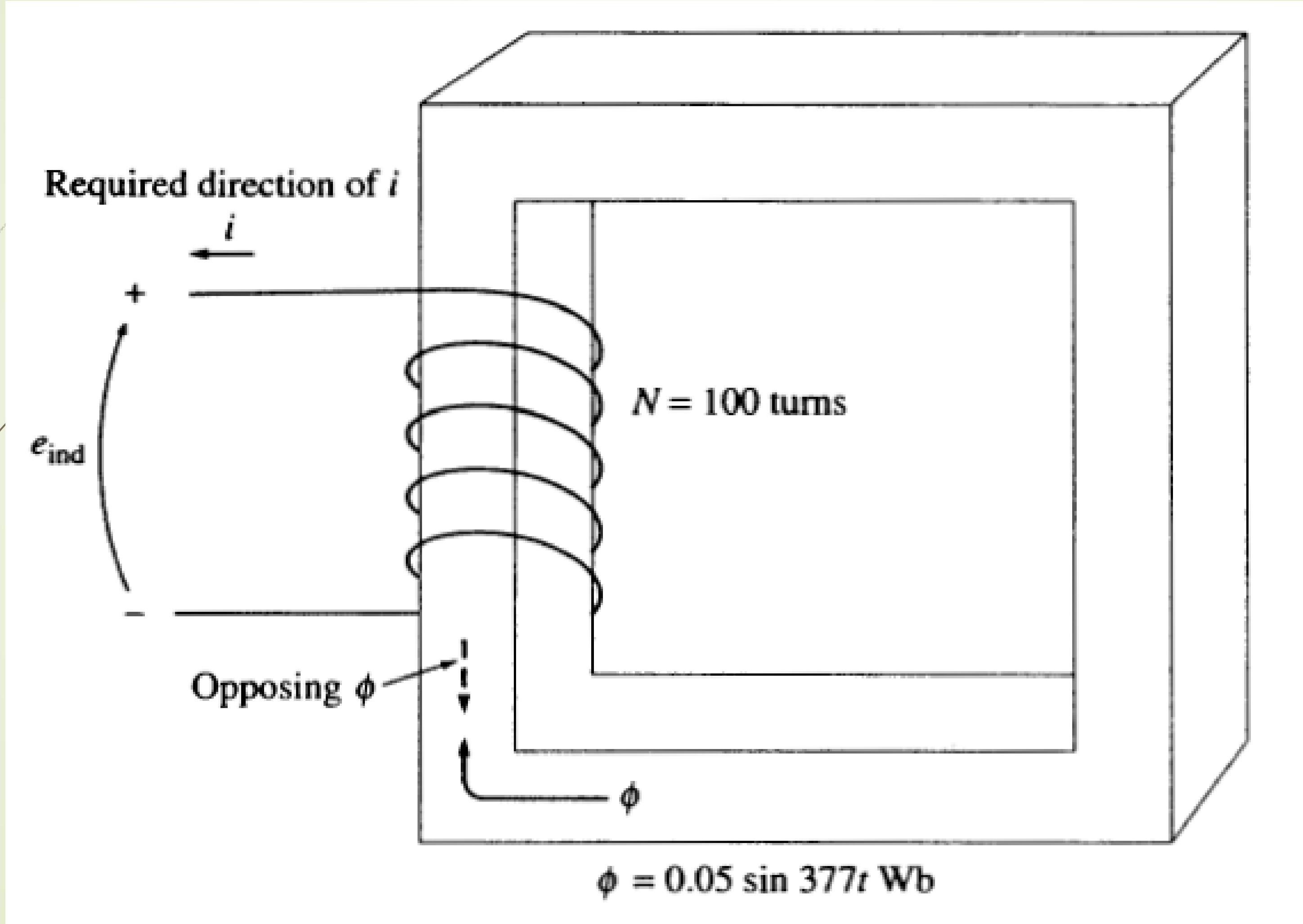
Example 1–6. Figure 1–15 shows a coil of wire wrapped around an iron core. If the flux in the core is given by the equation

$$\phi = 0.05 \sin 377t \qquad \text{Wb}$$

If there are 100 turns on the core, what voltage is produced at the terminals of the coil? Of what polarity is the voltage during the time when flux is *increasing* in the reference

The core of Example 1–6. Determination of the voltage polarity at the terminals is shown.

direction shown in the figure? Assume that all the magnetic flux stays within the core (i.e., assume that the flux leakage is zero).


Required direction of i
i
+
e_{ind}
–
N = 100 turns
Opposing ϕ
ϕ
ϕ = 0.05 sin 377t Wb

Solution

By the same reasoning as in the discussion on pages 29–30, the direction of the voltage while the flux is increasing in the reference direction must be positive to negative, as shown in Figure 1–15. The *magnitude* of the voltage is given by

$$e_{\text{ind}} = N\frac{d\phi}{dt}$$

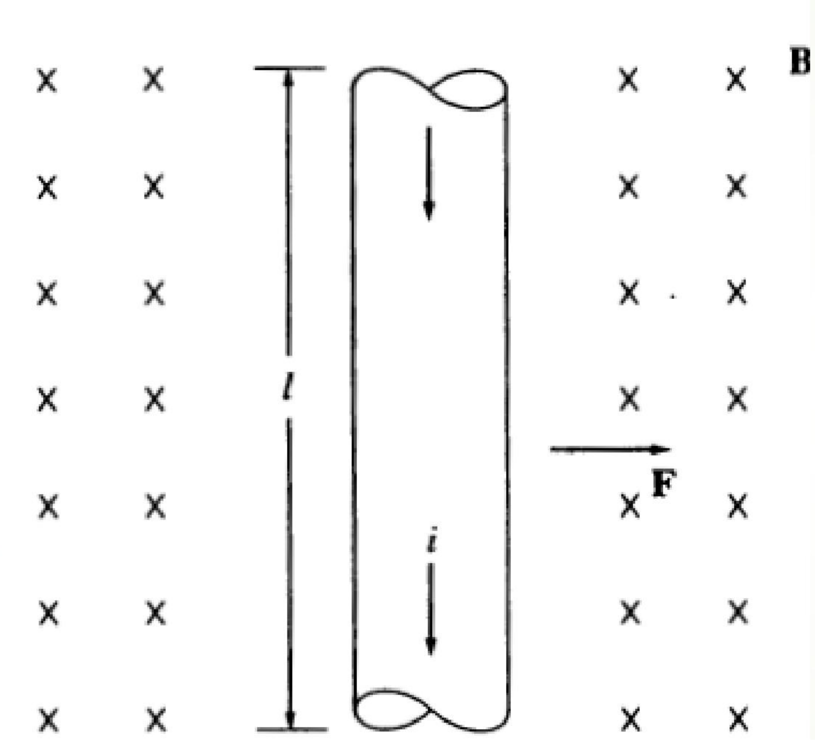
$$= (100 \text{ turns})\frac{d}{dt}(0.05 \sin 377t)$$

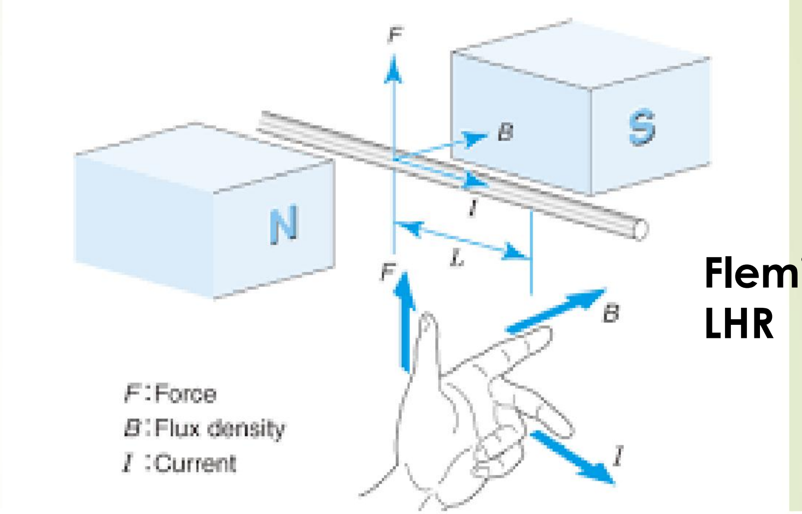
$$= 1885 \cos 377t$$

or alternatively,

$$e_{\text{ind}} = 1885 \sin(377t + 90°) \text{ V}$$

Produce an induced force on a wire

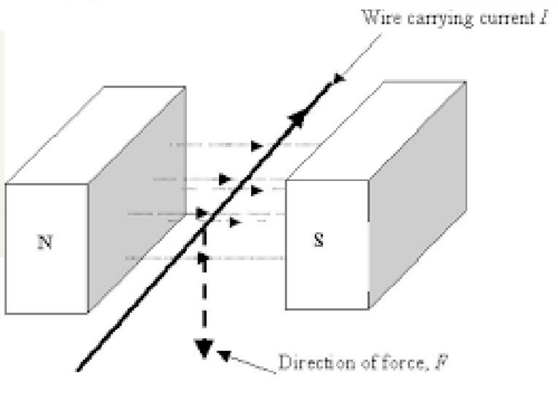




Fleming LHR

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$$

$$F = ilB \sin \theta$$



i = magnitude of current in wire

$\mathbf{l}$ = length of wire, with direction of $\mathbf{l}$ defined to be in the direction of current flow

$\mathbf{B}$ = magnetic flux density vector

Example 1-7

Example 1–7. Figure 1–16 shows a wire carrying a current in the presence of a magnetic field. The magnetic flux density is 0.25 T, directed into the page. If the wire is 1.0 m long and carries 0.5 A of current in the direction from the top of the page to the bottom of the page, what are the magnitude and direction of the force induced on the wire?

Example 1-7

Solution

The direction of the force is given by the right-hand rule as being to the right. The magnitude is given by

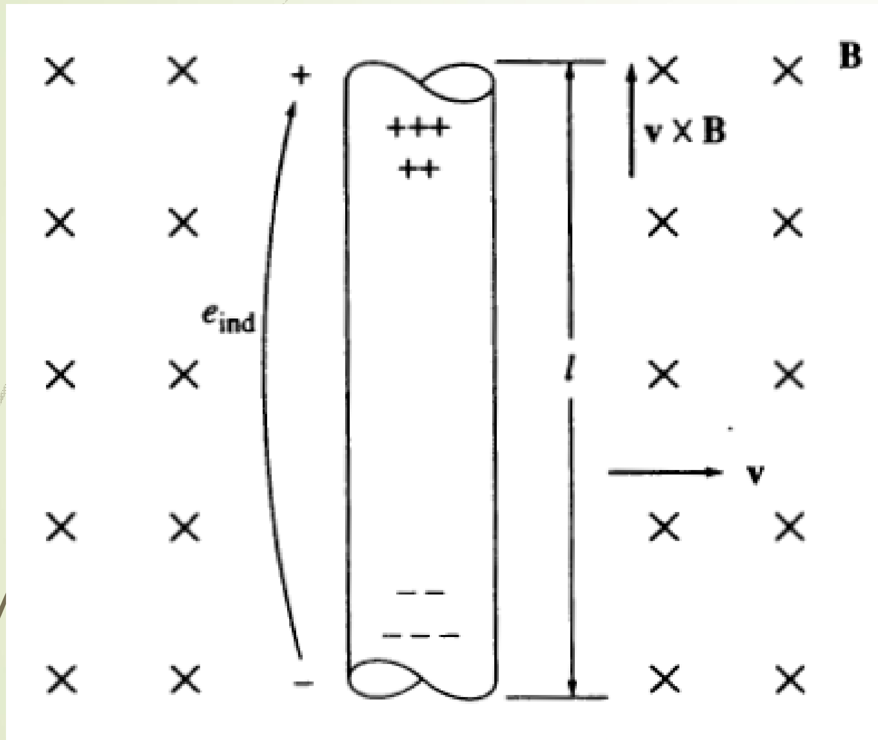
$$F = ilB \sin \theta \tag{1-44}$$

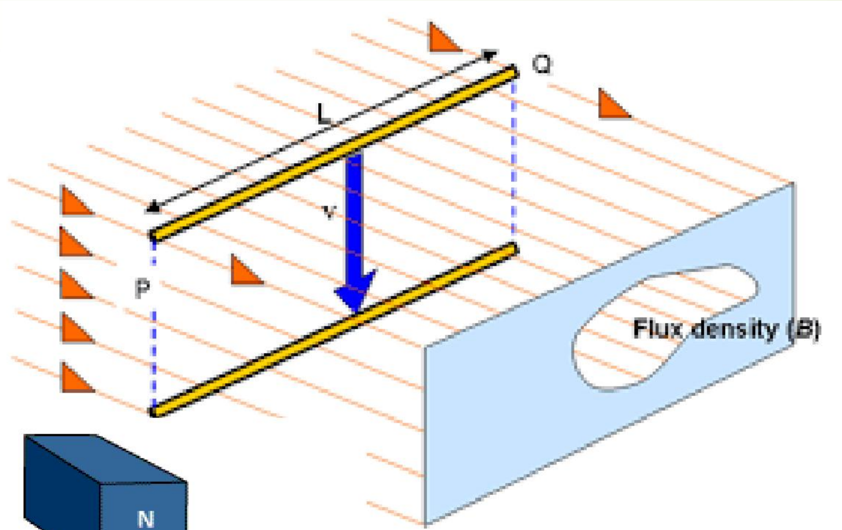
$$= (0.5 \text{ A})(1.0 \text{ m})(0.25 \text{ T}) \sin 90° = 0.125 \text{ N}$$

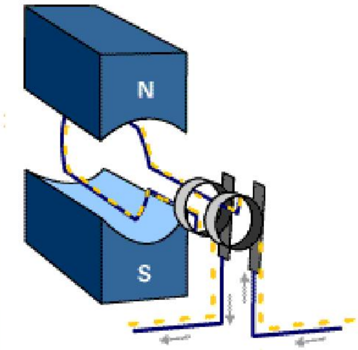
Therefore,

$$\mathbf{F} = 0.125 \text{ N, directed to the right}$$

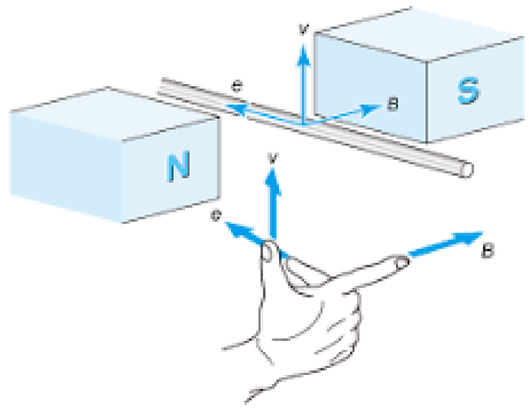
Induced voltage on a conductor







Fleming RHR



$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$$

$\mathbf{v}$ = velocity of the wire

$\mathbf{B}$ = magnetic flux density vector

$\mathbf{l}$ = length of conductor in the magnetic field

Example 1-8

Example 1–8. Figure 1–17 shows a conductor moving with a velocity of 5.0 m/s to the right in the presence of a magnetic field. The flux density is 0.5 T into the page, and the wire is 1.0 m in length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?

Solution

The direction of the quantity $\mathbf{v} \times \mathbf{B}$ in this example is up. Therefore, the voltage on the conductor will be built up positive at the top with respect to the bottom of the wire. The direction of vector $\mathbf{l}$ is up, so that it makes the smallest angle with respect to the vector $\mathbf{v} \times \mathbf{B}$.

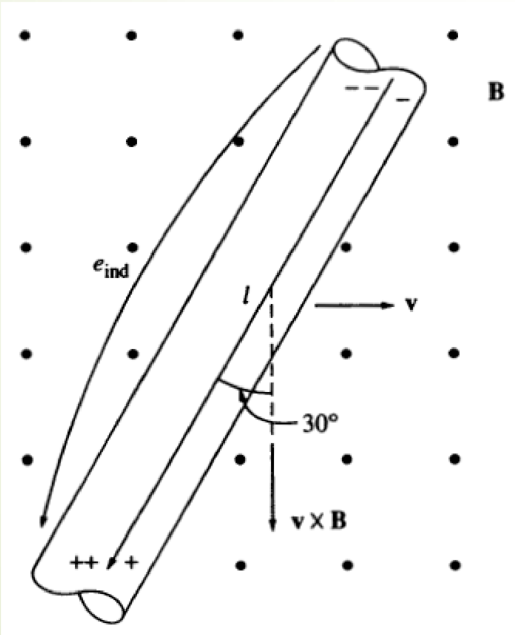
Since $\mathbf{v}$ is perpendicular to $\mathbf{B}$ and since $\mathbf{v} \times \mathbf{B}$ is parallel to $\mathbf{l}$, the magnitude of the induced voltage reduces to

$$
\begin{aligned}
e_{ind} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \qquad (1\text{–}45)\\
&= (vB \sin 90^\circ)\, l \cos 0^\circ \\
&= vBl \\
&= (5.0 \text{ m/s})(0.5 \text{ T})(1.0 \text{ m}) \\
&= 2.5 \text{ V}
\end{aligned}
$$

Thus the induced voltage is 2.5 V, positive at the top of the wire.

Example 1-9

Example 1–9. Figure 1–18 shows a conductor moving with a velocity of 10 m/s to the right in a magnetic field. The flux density is 0.5 T, out of the page, and the wire is 1.0 m in length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?



Solution

The direction of the quantity **v** × **B** is down. The wire is not oriented on an up-down line, so choose the direction of l as shown to make the smallest possible angle with the direction of **v** × **B**. The voltage is positive at the bottom of the wire with respect to the top of the wire. The magnitude of the voltage is

$$\begin{aligned} e_{ind} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \qquad (1\text{–}45) \\ &= (vB \sin 90^\circ)\, l \cos 30^\circ \\ &= (10.0 \text{ m/s})(0.5 \text{ T})(1.0 \text{ m}) \cos 30^\circ \\ &= 4.33 \text{ V} \end{aligned}$$

The linear DC machine – a simple example

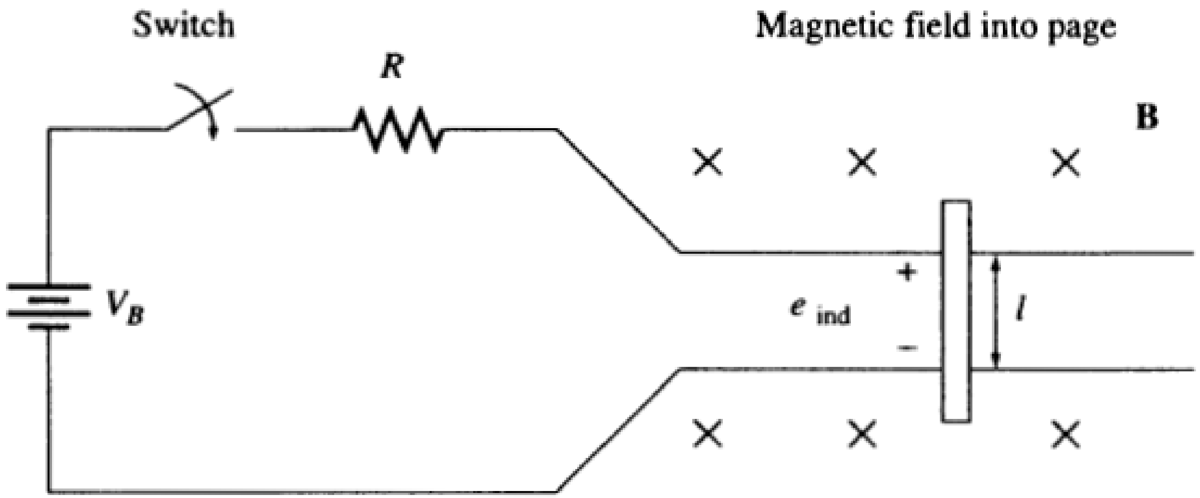


FIGURE 1–19
A linear dc machine. The magnetic field points into the page.

A *linear dc machine* is about the simplest and easiest-to-understand version of a dc machine, yet it operates according to the same principles and exhibits the same behavior as real generators and motors. It thus serves as a good starting point in the study of machines.

A linear dc machine is shown in Figure 1–19. It consists of a battery and a resistance connected through a switch to a pair of smooth, frictionless rails. Along the bed of this "railroad track" is a constant, uniform-density magnetic field directed into the page. A bar of conducting metal is lying across the tracks.

How does such a strange device behave? Its behavior can be determined from an application of four basic equations to the machine. These equations are

1. The equation for the force on a wire in the presence of a magnetic field:

$$\boxed{\mathbf{F} = i(\mathbf{l} \times \mathbf{B})} \qquad (1\text{–}43)$$

where $\mathbf{F}$ = force on wire

i = magnitude of current in wire

$\mathbf{l}$ = length of wire, with direction of $\mathbf{l}$ defined to be in the direction of current flow

$\mathbf{B}$ = magnetic flux density vector

2. The equation for the voltage induced on a wire moving in a magnetic field:

$$\boxed{e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}} \tag{1–45}$$

where e_{ind} = voltage induced in wire
$\mathbf{v}$ = velocity of the wire
$\mathbf{B}$ = magnetic flux density vector
$\mathbf{l}$ = length of conductor in the magnetic field

3. Kirchhoff's voltage law for this machine. From Figure 1–19 this law gives

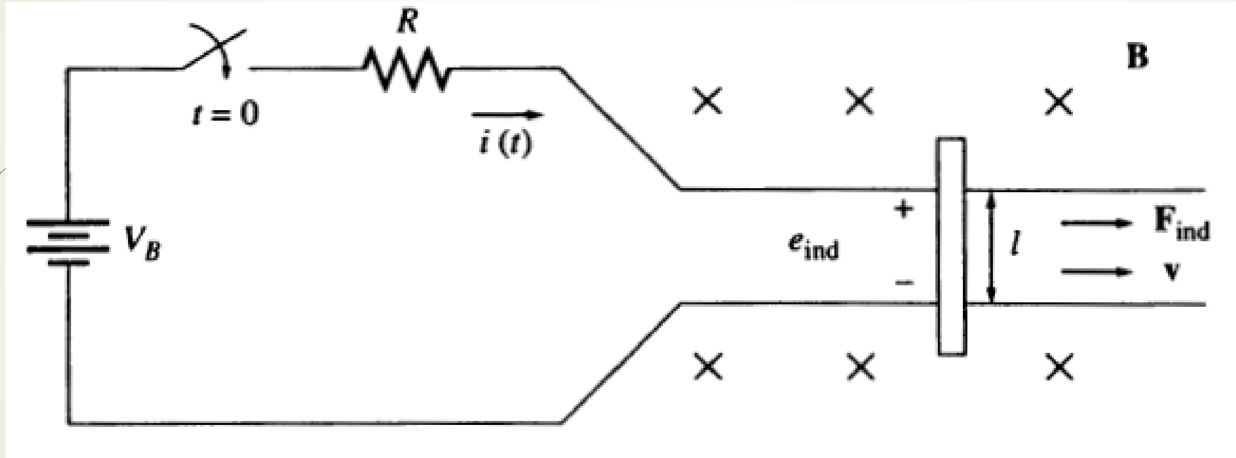
$$V_B - iR - e_{ind} = 0$$

$$\boxed{V_B = e_{ind} + iR = 0} \tag{1–46}$$

4. Newton's law for the bar across the tracks:

$$\boxed{F_{net} = ma} \tag{1–7}$$

Starting a linear DC machine



Starting a linear DC machine

1. Curren $i = \frac{V_B - e_{ind}}{R}$
2. Induced force $F_{\text{ind}} = ilB$ to the right
3. Induced voltage $e_{\text{ind}} = vBl$ positive upward

$$i\downarrow = \frac{V_B - e_{\text{ind}}\uparrow}{R}$$

Starting a linear DC machine

The result of this action is that eventually the bar will reach a constant steady-state speed where the net force on the bar is zero. This will occur when e_{ind} has risen all the way up to equal the voltage V_B. At that time, the bar will be moving at a speed given by

$$V_B = e_{ind} = v_{ss}Bl$$

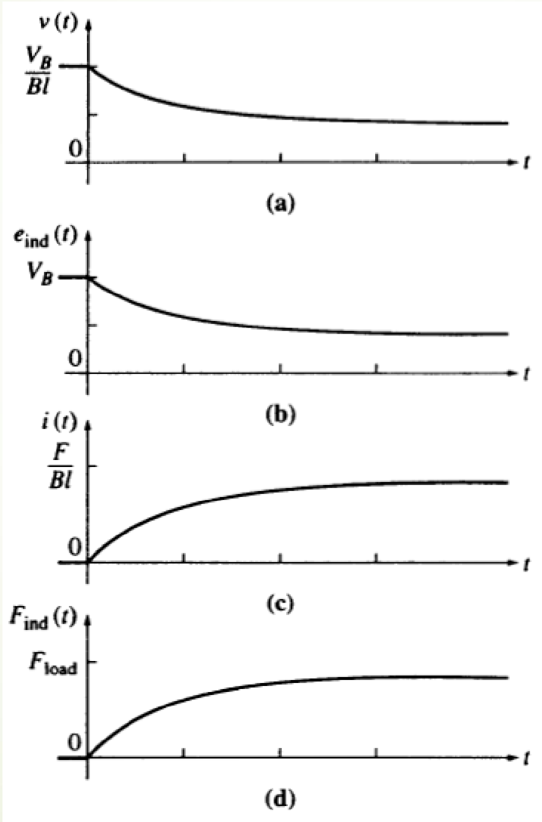
$$v_{ss} = \frac{V_B}{Bl} \tag{1–50}$$

The bar will continue to coast along at this no-load speed forever unless some external force disturbs it. When the motor is started, the velocity v, induced voltage e_{ind}, current i, and induced force F_{ind} are as sketched in Figure 1–21.

Summarize of a dc machine starting

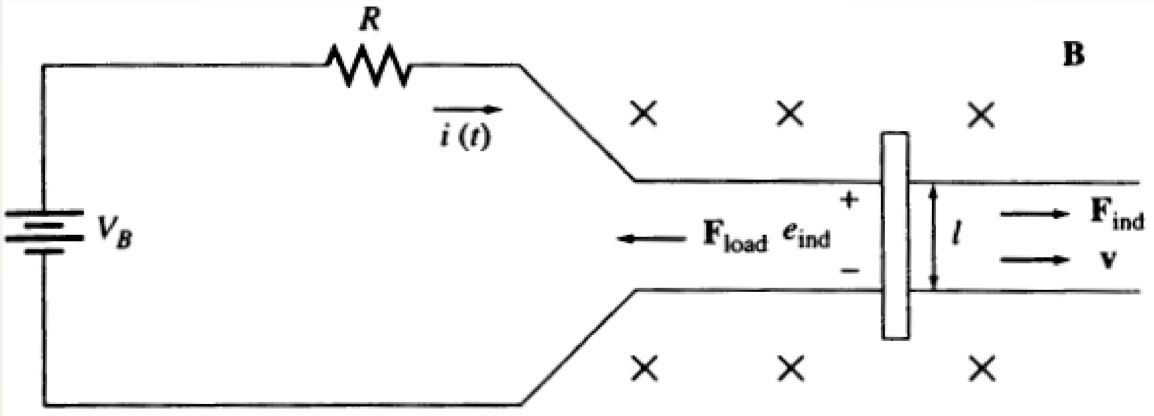
1. Closing the switch produces a current flow $i = V_B/R$.
2. The current flow produces a force on the bar given by $F = ilB$.
3. The bar accelerates to the right, producing an induced voltage e_{ind} as it speeds up.
4. This induced voltage reduces the current flow $i = (V_B - e_{ind}\uparrow)/R$.
5. The induced force is thus decreased ($F = i\downarrow lB$) until eventually $F = 0$. At that point, $e_{ind} = V_B$, $i = 0$, and the bar moves at a constant no-load speed $v_{ss} = V_B/Bl$.

DC linear machine operates at no-load condition



Linear dc motor

- While the load is applied



- The conversion power between mechanical and electrical

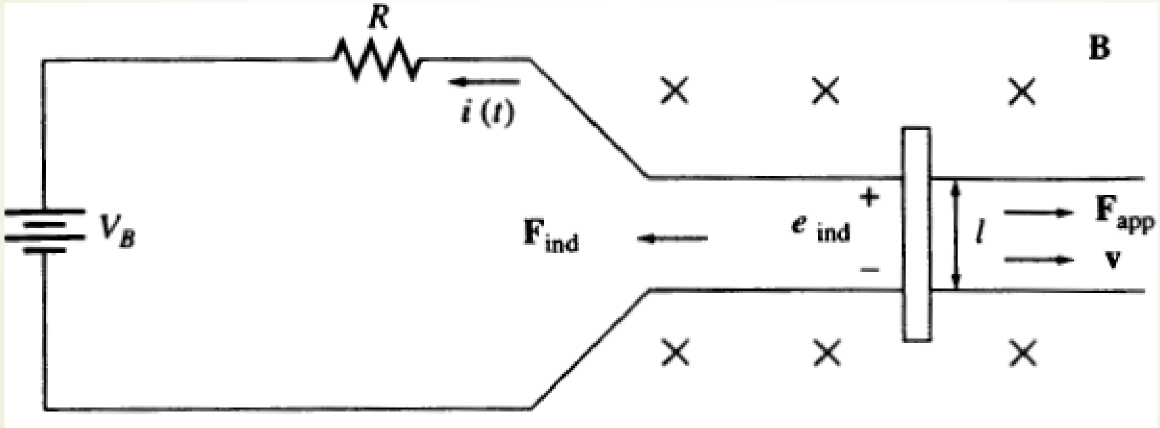
$$P_{conv} = e_{ind} i = F_{ind} v$$

Summarize of a dc motor operation

1. A force $\mathbf{F}_{load}$ is applied opposite to the direction of motion, which causes a net force $\mathbf{F}_{net}$ opposite to the direction of motion.
2. The resulting acceleration $a = F_{net}/m$ is negative, so the bar slows down ($v\downarrow$).
3. The voltage $e_{ind} = v\downarrow Bl$ falls, and so $i = (V_B - e_{ind}\downarrow)/R$ increases.
4. The induced force $F_{ind} = i\uparrow lB$ increases until $|\mathbf{F}_{ind}| = |\mathbf{F}_{load}|$ at a lower speed v.
5. An amount of electric power equal to $e_{ind}i$ is now being converted to mechanical power equal to $F_{ind}v$, and the machine is acting as a motor.

Linear dc generator

- While the external force is applied on the moving direction

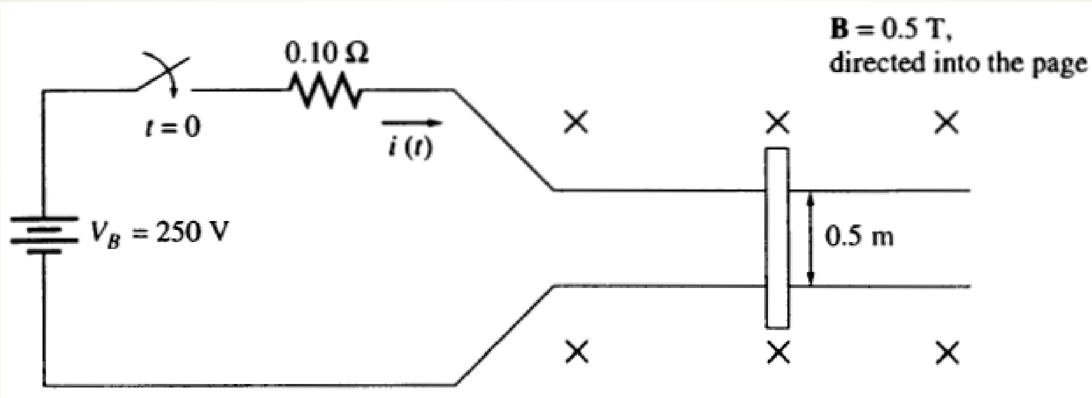


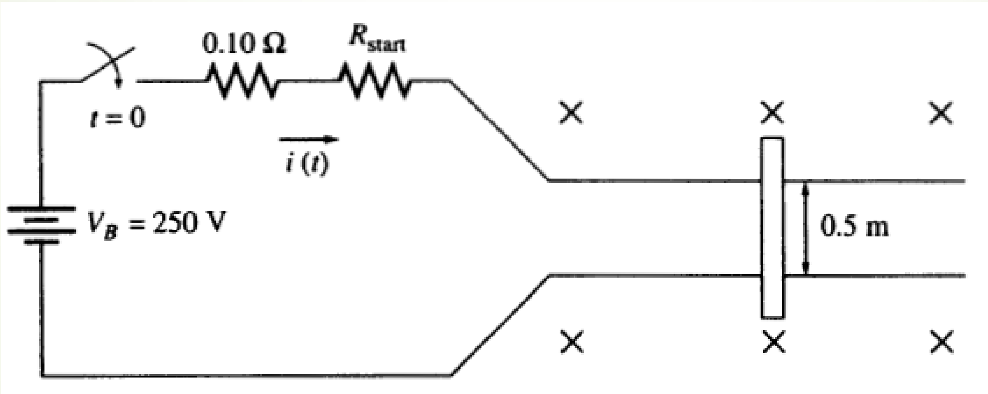
$$P_{conv} = \tau_{ind}\omega$$

Summarize of a dc generator operation

1. A force $\mathbf{F}_{app}$ is applied in the direction of motion; $\mathbf{F}_{net}$ is in the direction of motion.
2. Acceleration $a = F_{net}/m$ is positive, so the bar speeds up ($v\uparrow$).
3. The voltage $e_{ind} = v\uparrow Bl$ increases, and so $i = (e_{ind}\uparrow - V_B)/R$ increases.
4. The induced force $F_{ind} = i\uparrow lB$ increases until $|\mathbf{F}_{ind}| = |\mathbf{F}_{load}|$ at a higher speed v.
5. An amount of mechanical power equal to $F_{ind}v$ is now being converted to electric power $e_{ind}i$, and the machine is acting as a generator.

Starting problem of dc linear machine





Example 1-10

Example 1–10. The linear dc machine shown in Figure 1–27a has a battery voltage of 120 V, an internal resistance of 0.3 Ω, and a magnetic flux density of 0.1 T.

(a) What is this machine's maximum starting current? What is its steady-state velocity at no load?

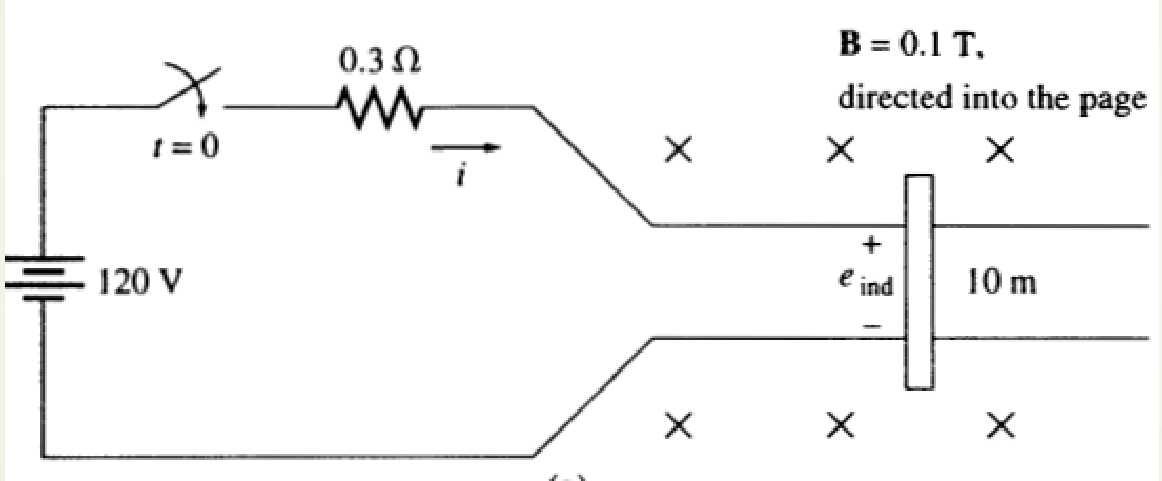
(b) Suppose that a 30-N force pointing to the right were applied to the bar. What would the steady-state speed be? How much power would the bar be producing or consuming? How much power would the battery be producing or consuming? Explain the difference between these two figures. Is this machine acting as a motor or as a generator?

(c) Now suppose a 30-N force pointing to the left were applied to the bar. What would the new steady-state speed be? Is this machine a motor or a generator now?

(d) Assume that a force pointing to the left is applied to the bar. Calculate speed of the bar as a function of the force for values from 0 N to 50 N in 10-N steps. Plot the velocity of the bar versus the applied force.

(e) Assume that the bar is unloaded and that it suddenly runs into a region where the magnetic field is weakened to 0.08 T. How fast will the bar go now?

Example 1-10



(a)

Solution

(a) At starting conditions, the velocity of the bar is 0, so $e_{ind} = 0$. Therefore,

$$i = \frac{V_B - e_{ind}}{R} = \frac{120 \text{ V} - 0 \text{ V}}{0.3 \ \Omega} = 400 \text{ A}$$

When the machine reaches steady state, $\mathbf{F}_{ind} = 0$ and $i = 0$. Therefore,

$$VB = e_{ind} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl}$$

$$= \frac{120 \text{ V}}{(0.1 \text{ T})(10 \text{ m})} = 120 \text{ m/s}$$

(*b*) Refer to Figure 1–27b. If a 30-N force to the right is applied to the bar, the final steady state will occur when the induced force $\mathbf{F}_{ind}$ is equal and opposite to the applied force $\mathbf{F}_{app}$, so that the net force on the bar is zero:

$$F_{app} = F_{ind} = ilB$$

Therefore,

$$i = \frac{F_{ind}}{lB} = \frac{30\ \text{N}}{(10\text{m})(0.1\ \text{T})}$$

$$= 30\ \text{A} \quad \text{flowing up through the bar}$$

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B + iR$$

$$= 120\ \text{V} + (30\text{A})(0.3\ \Omega) = 129\ \text{V}$$

and the final steady-state speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$

$$= \frac{129\ \text{V}}{(0.1\ \text{T})(10\ \text{m})} = 129\ \text{m/s}$$

The bar is *producing* $P = (129\ \text{V})(30\ \text{A}) = 3870$ W of power, and the battery is *consuming* $P = (120\ \text{V})(30\ \text{A}) = 3600$ W. The difference between these two numbers is the 270 W of losses in the resistor. This machine is acting as a *generator.*

(c) Refer to Figure 1–25c. This time, the force is applied to the left, and the induced force is to the right. At steady state,

$$F_{app} = F_{ind} = ilB$$

$$i = \frac{F_{ind}}{lB} = \frac{30\text{ N}}{(10\text{ m})(0.1\text{ T})}$$

$$= 30\text{ A} \quad \text{flowing down through the bar}$$

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B - iR$$

$$= 120\text{ V} - (30\text{ A})(0.3\ \Omega) = 111\text{ V}$$

and the final speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$

$$= \frac{111\text{ V}}{(0.1\text{ T})(10\text{ m})} = 111\text{ m/s}$$

This machine is now acting as a *motor,* converting electric energy from the battery into mechanical energy of motion on the bar.

(*e*) If the bar is initially unloaded, then $e_{ind} = V_B$. If the bar suddenly hits a region of weaker magnetic field, a transient will occur. Once the transient is over, though, e_{ind} will again equal V_B.