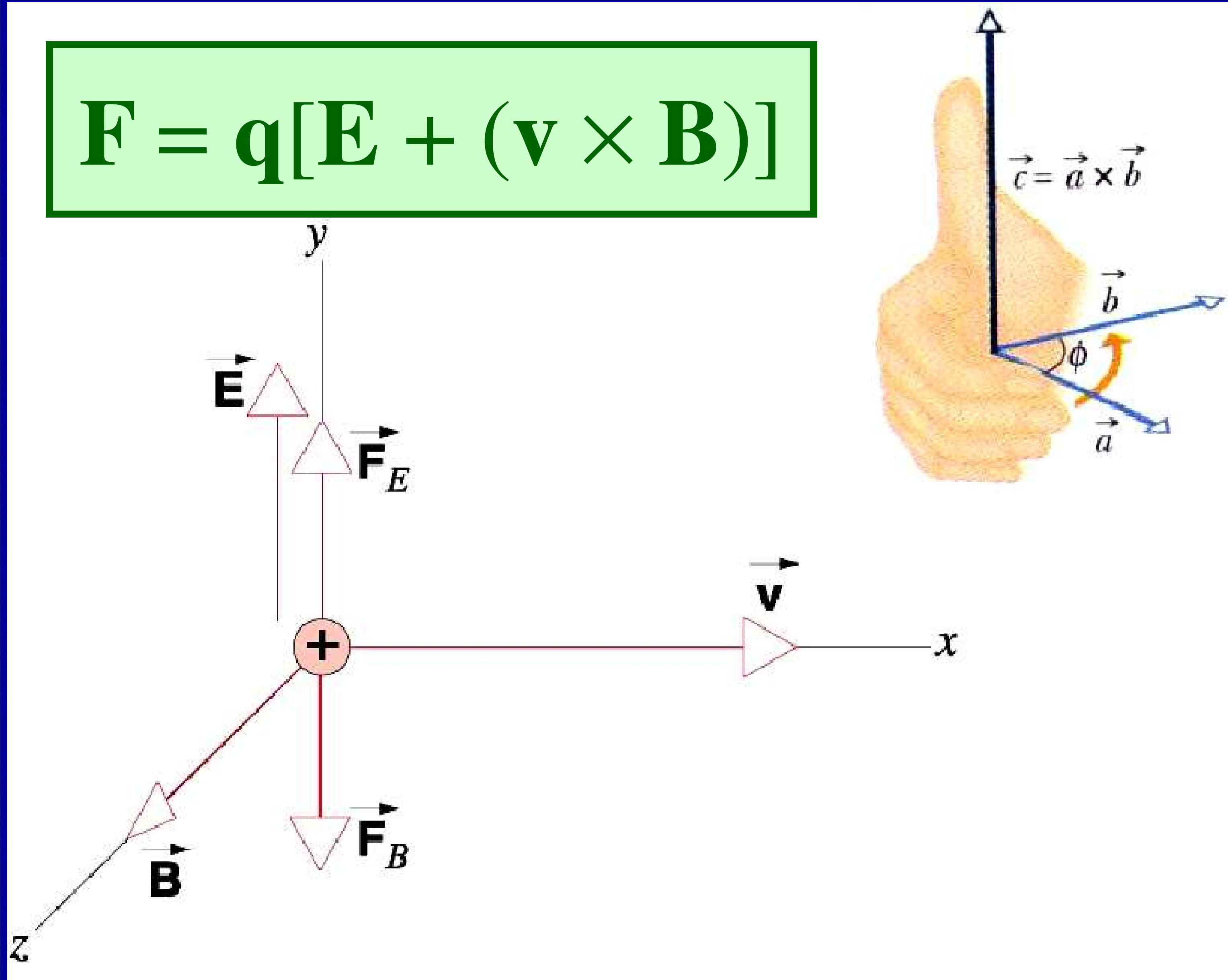
The Classical Hall effect

Reminder: The Lorentz Force

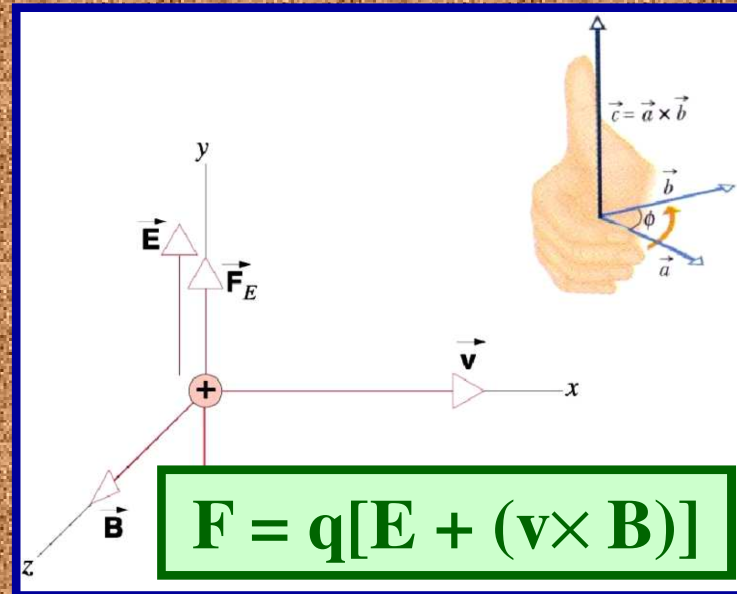
$$\mathbf{F} = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$



The Lorentz Force: Review

Crossed E & B Fields

- In the configuration shown here, can have ***several practical uses!***



F = q[E + (v× B)]

1. **Velocity Filters:** Undeflected trajectories of charged particles. Choose **E & B** so that the particles have a desired **v = (E/B)**
2. **Cyclotron Motion:**

$$F_B = ma_r \Rightarrow qvB = (mv^2/r)$$

Crossed E & B Fields

- **Cyclotron Motion:**

$$F_B = ma_r \Rightarrow qvB = (mv^2/r)$$

$\Rightarrow$ **Orbit Radius:**

$$r = [(mv)/(|q|B)] = [p/(q|B|])$$

$\Rightarrow$ **A Momentum (p) Filter!!**

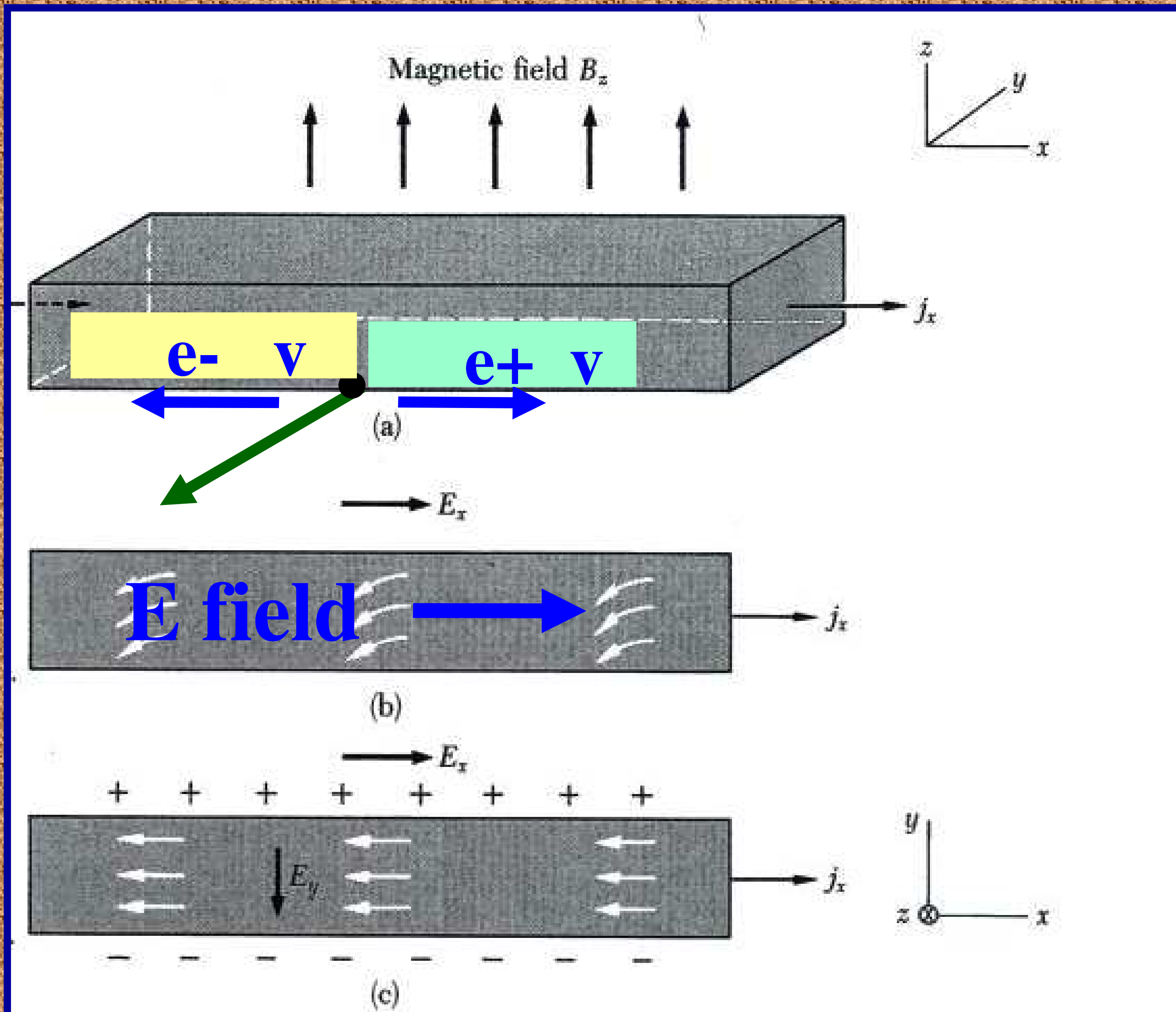
Orbit Frequency: $\omega = 2\pi f = (|q|B)/m$

$\Rightarrow$ **A Mass Measurement Method!**

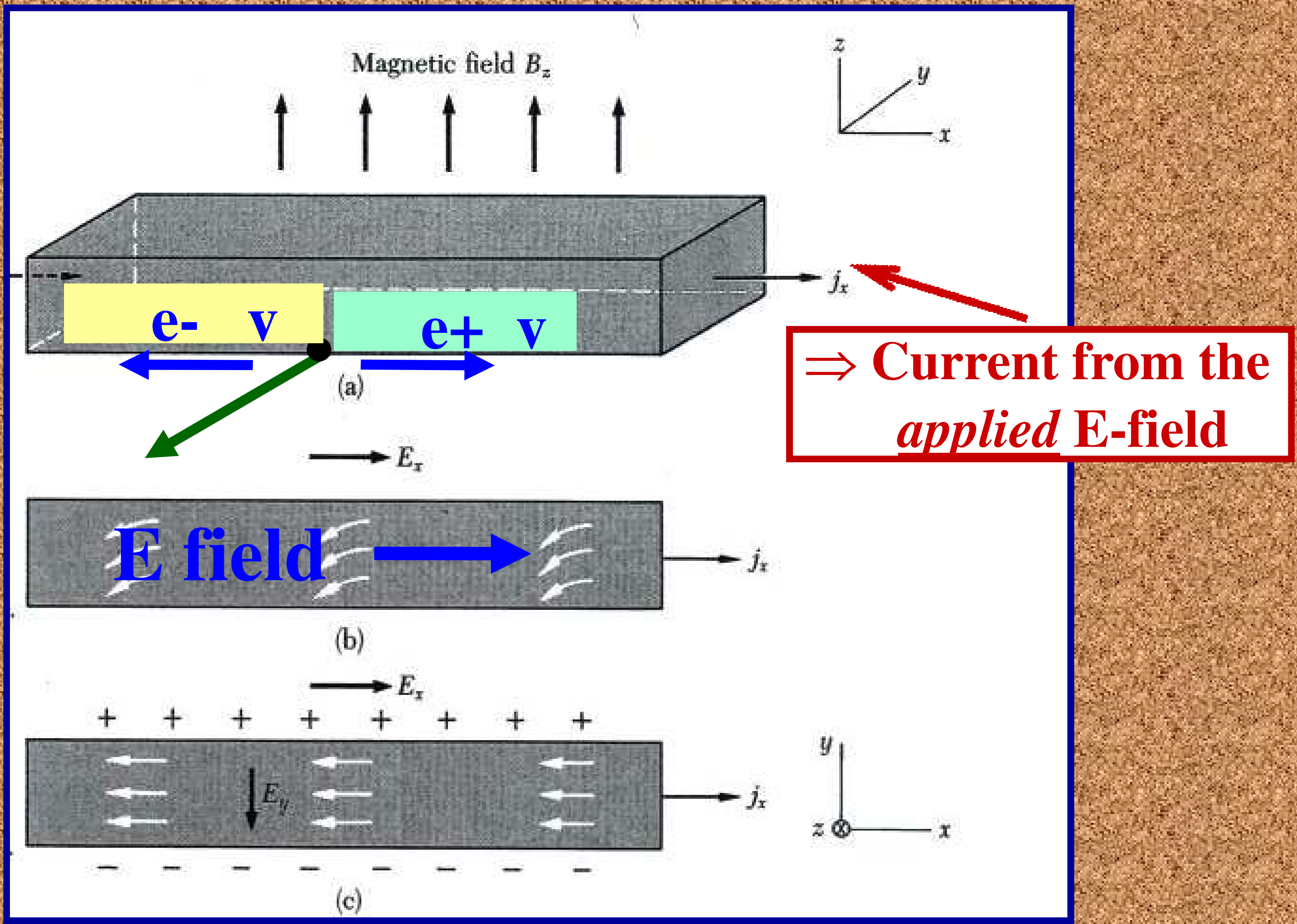
$\Rightarrow$ **Orbit Energy:**

$$K = (½)mv^2 = (q^2B^2r^2)/2m$$

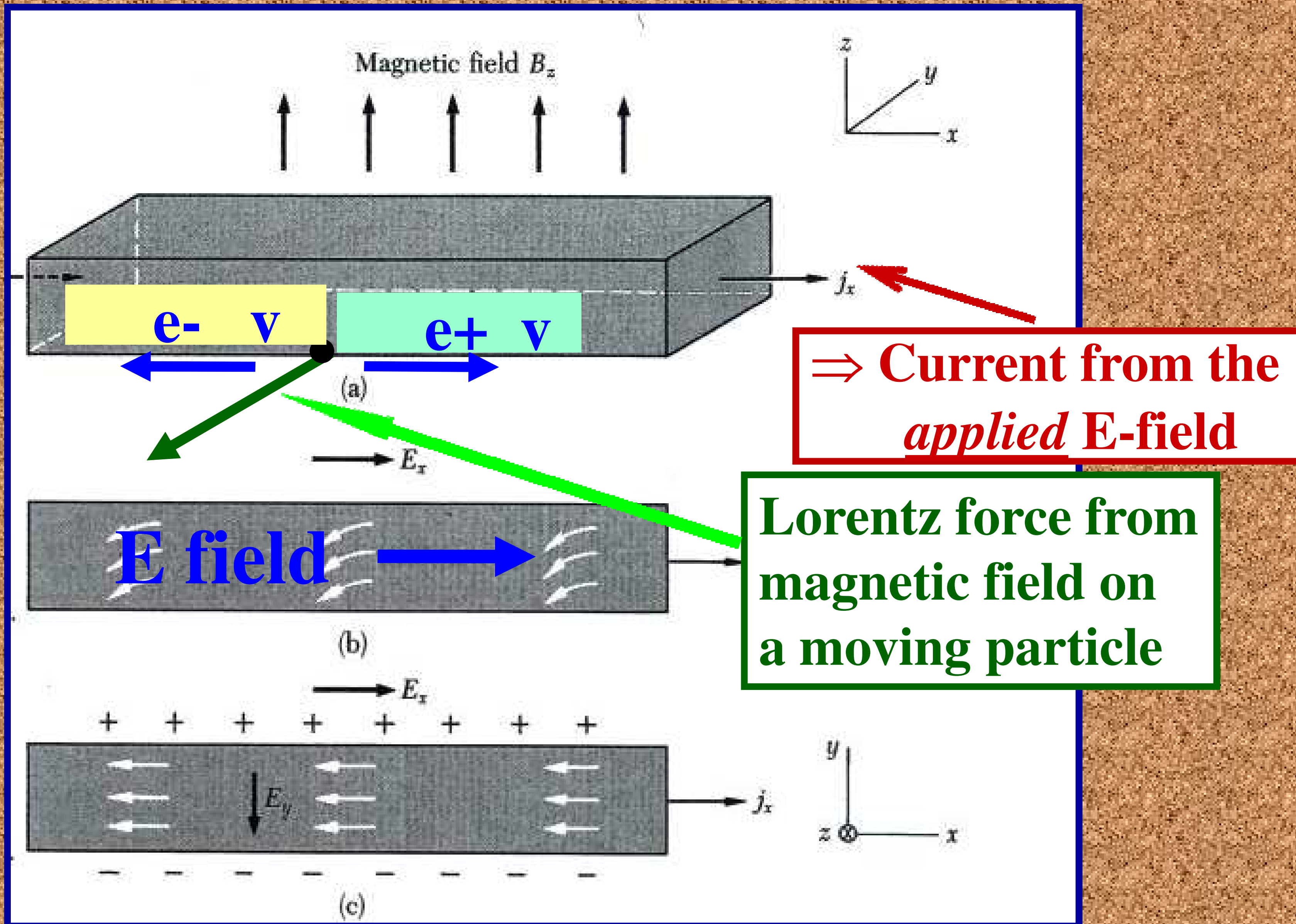
Standard Hall Effect Experiment



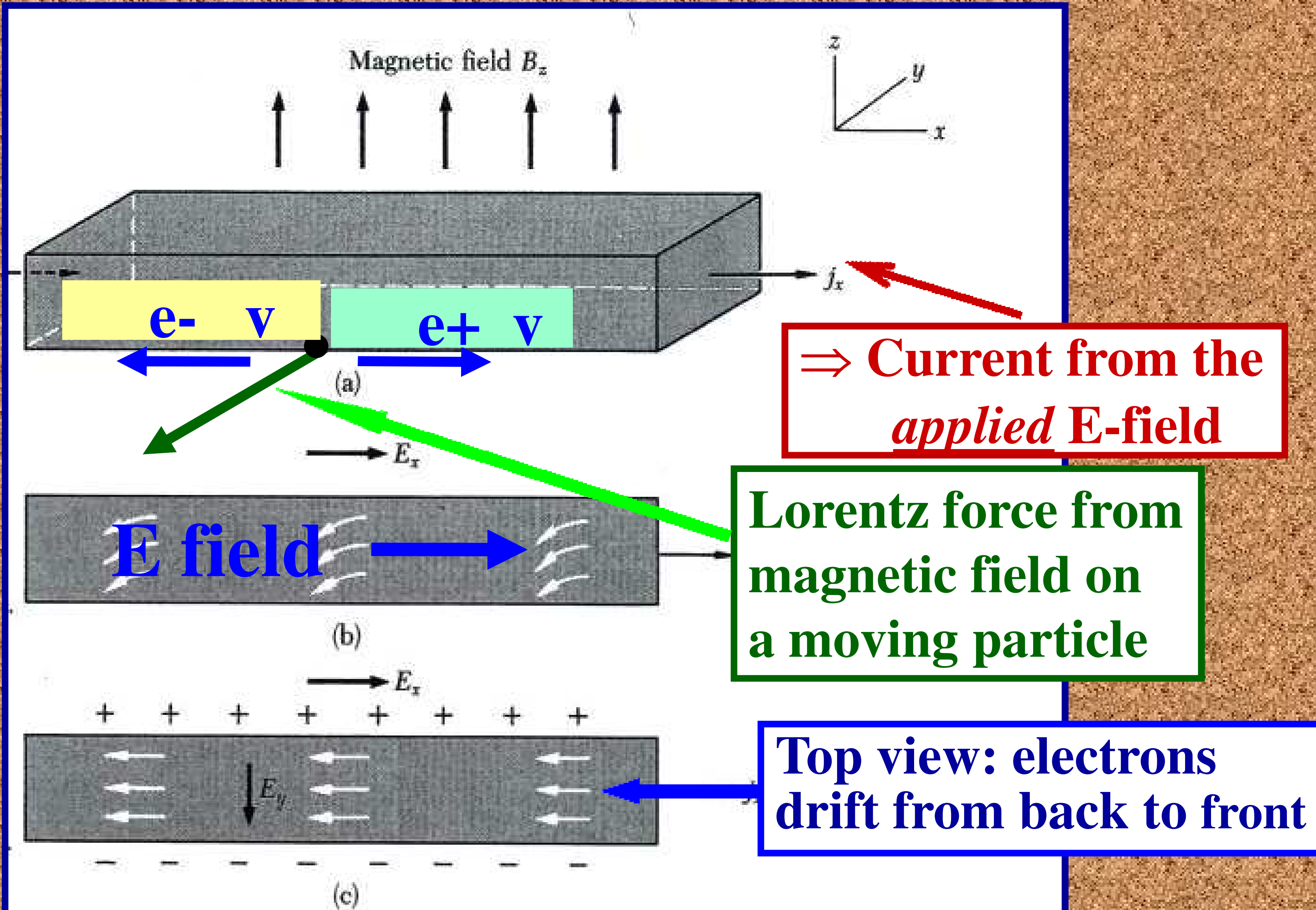
Standard Hall Effect Experiment



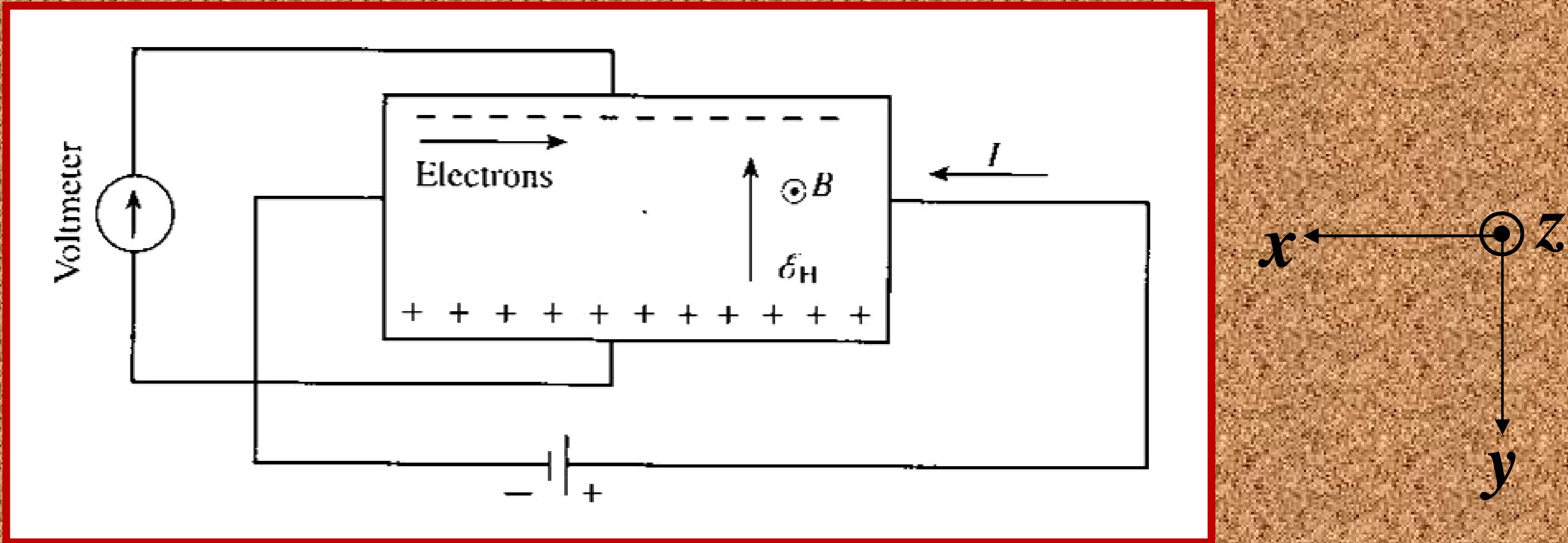
Standard Hall Effect Experiment



Standard Hall Effect Experiment



The Hall Effect



Lorentz force: $-e\vec{v}\times\vec{B} \propto \hat{y}$

Balance equation: $-eE_{\mathrm{H}} = -evB$

$$E_H = R_H JB, \quad R_H = -\frac{1}{ne}$$

R_{H} is independent of τ and m

→ An excellent method for determining n

The Hall Effect: A more formal derivation

$$\frac{d\vec{v}}{dt} = -\frac{\vec{v}}{\tau} - \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B}), \ \vec{J} = ne\vec{v}, \ \vec{v} = (v_x, v_y, 0), \ \vec{E} = (E_x, E_y, 0), \ \vec{B} = (0,0,B)$$

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \tilde{\rho} \begin{pmatrix} J_x \\ J_y \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} J_x \\ J_y \end{pmatrix} = \tilde{\sigma} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\tilde{\rho} = \frac{1}{\sigma_0} \begin{pmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{pmatrix}, \quad \tilde{\sigma} = \frac{\sigma_0}{1 + (\omega_c \tau)^2} \begin{pmatrix} 1 & -\omega_c \tau \\ \omega_c \tau & 1 \end{pmatrix}$$

magneto-resistivity tensor

magneto-conductivity tensor

$J_y = 0$ ⇒

$$E_x = E_L = \frac{1}{\sigma_0} J_x = \rho_0 J_x$$

$$E_y = E_H = -\frac{\omega_c \tau}{\sigma_0} J_x = R_H B J_x, \ R_H = -\frac{1}{ne}$$

Charge Density in the Drude Model

ρ_m [kg/m^3]: mass density

A [kg]: atomic mass (mass of one mole)

➡ ρ_m/A moles atoms per m^3

➡ $N_A\rho_m/A$ atoms per m^3, $N_A = 6.02 \times 10^{23}$

➡ $n = N_A\rho_m Z/A$ electrons per m^3,
Z: # of valence electrons

For Li, $\rho_m = 0.542 \times 10^3$, $A = 6.941 \times 10^{-3}$, $Z = 1$

➡ $n = 4.70 \times 10^{28}$ m^{-3}

Comparison with Experiment

For Li, $\rho_m = 0.542 \times 10^3$, $A = 6.941 \times 10^{-3}$, Z = 1

good

$n = 4.70 \times 10^{28}\ \mathrm{m^{-3}}$

$R_H = -1.33 \times 10^{-10}\ \mathrm{m^3/C}$

$R_H(\mathrm{exp}) = -1.7 \times 10^{-10}\ \mathrm{m^3/C}$

For Zn, $\rho_m = 7.13 \times 10^3$, $A = 65.38 \times 10^{-3}$, Z = 2

bad

$n = 1.31 \times 10^{29}\ \mathrm{m^{-3}}$

$R_H = -4.77 \times 10^{-11}\ \mathrm{m^3/C}$

$R_H(\mathrm{exp}) = +3 \times 10^{-11}\ \mathrm{m^3/C}$

Positive Hall coefficient!

Cyclotron Frequency and the Hall Angle

Newtonian equation of motion in *E* and *B*:

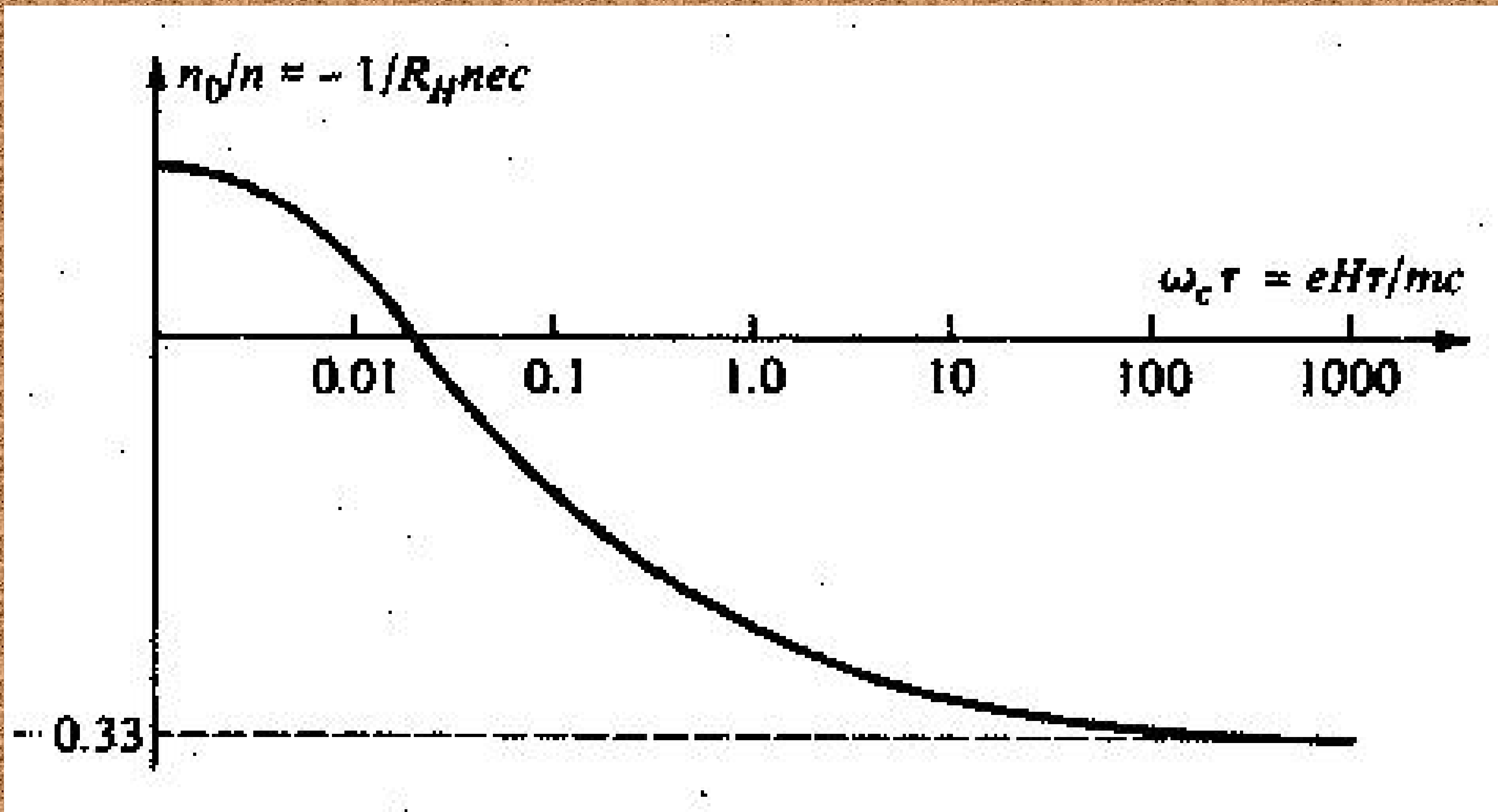
$$\frac{d\vec{p}}{dt} = -e\left(\vec{E} + \frac{\vec{p}}{m}\times\vec{B}\right) - \frac{\vec{p}}{\tau}$$

steady state ⇒

$$\begin{cases} \sigma_0 E_z = \omega_c \tau J_y + J_z \\ \sigma_0 E_y = -\omega_c \tau J_z + J_y \end{cases}$$

$$\omega_c = \frac{eB}{m}, \quad \tan\phi = \omega_c \tau$$

Deviation from the Classical Hall Effect



How Difficult is $\omega_c \tau > 1$?

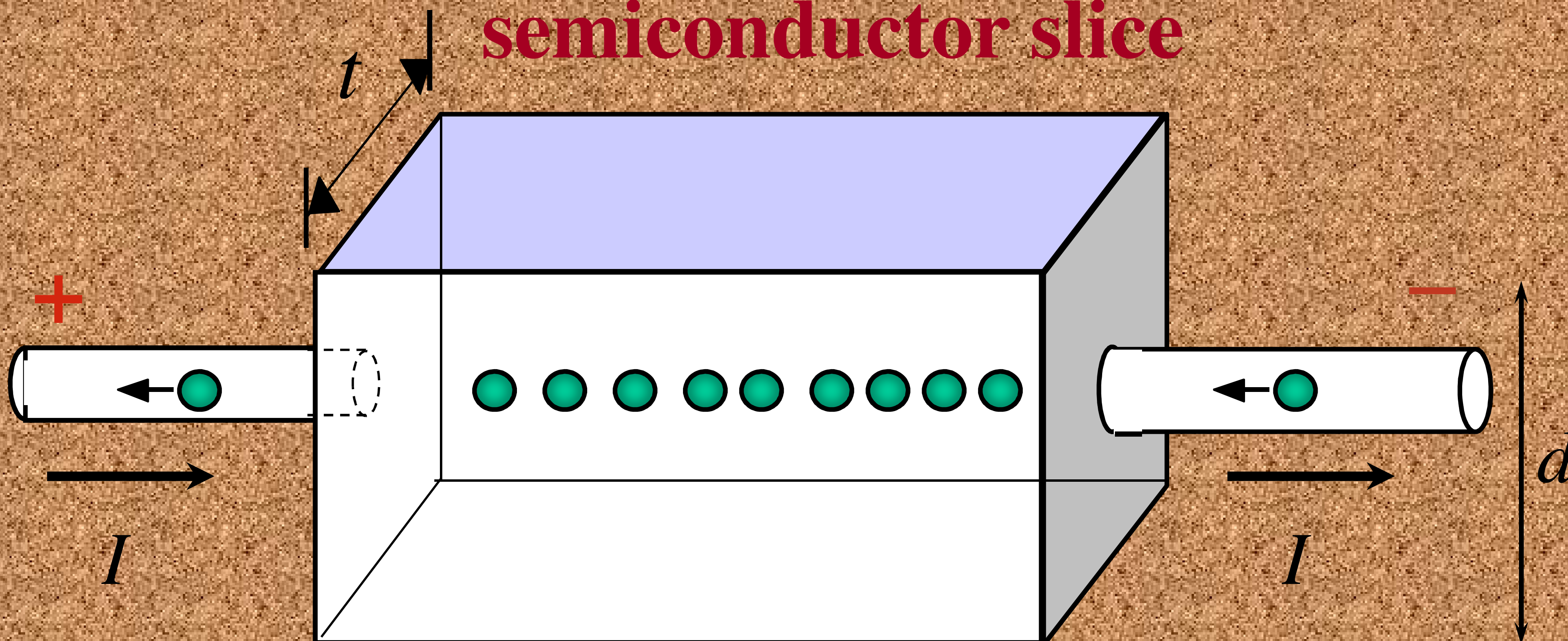
$$\omega_c \tau = \frac{eB}{m^*}\tau = B\left(\frac{e\tau}{m^*}\right) = B\mu_e > 1$$

μ_e = 10000 cm^2/Vs → *B* > 1 Tesla

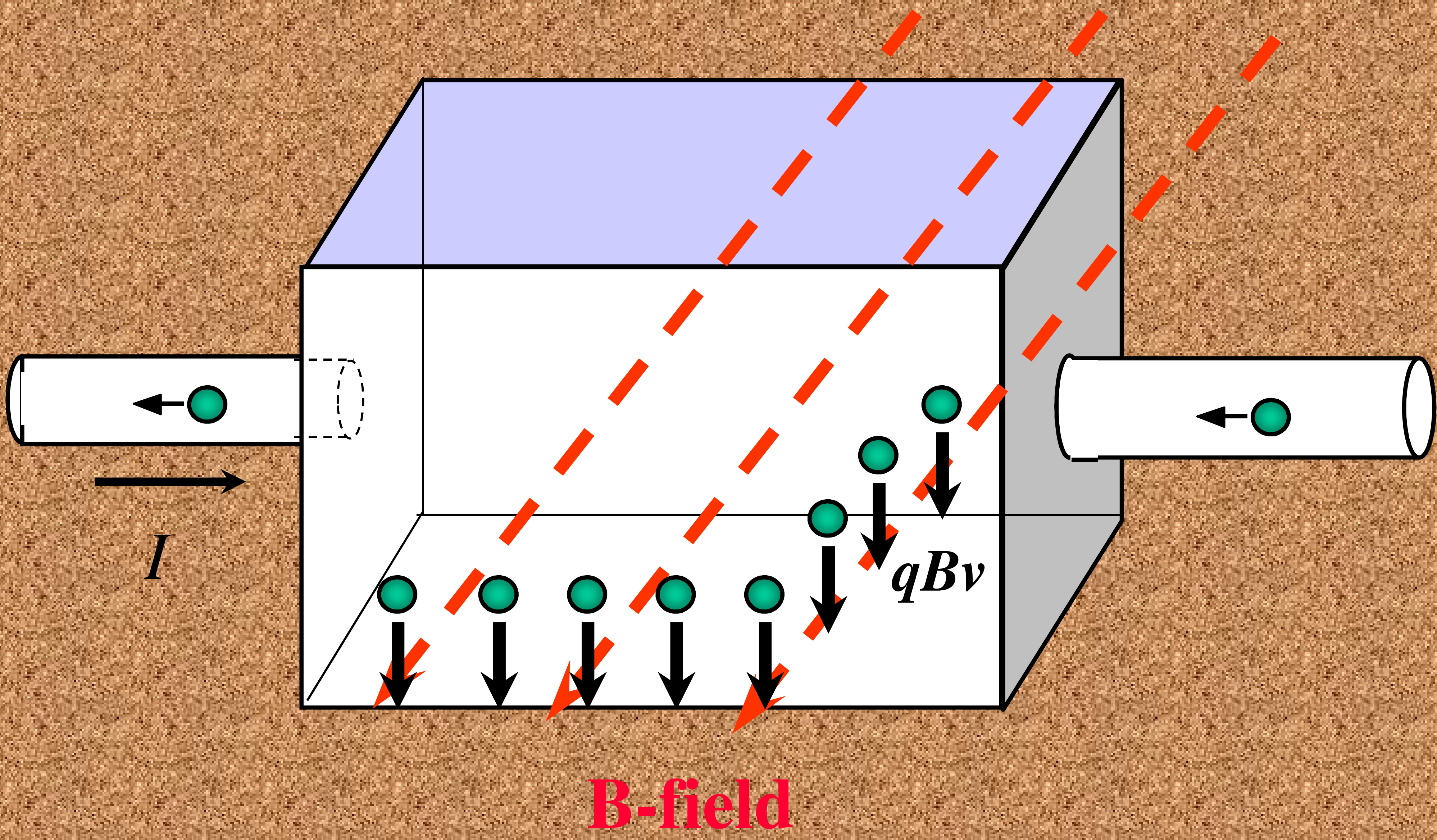
μ_e = 1000 cm^2/Vs → *B* > 10 Tesla

μ_e = 100 cm^2/Vs → *B* > 100 Tesla

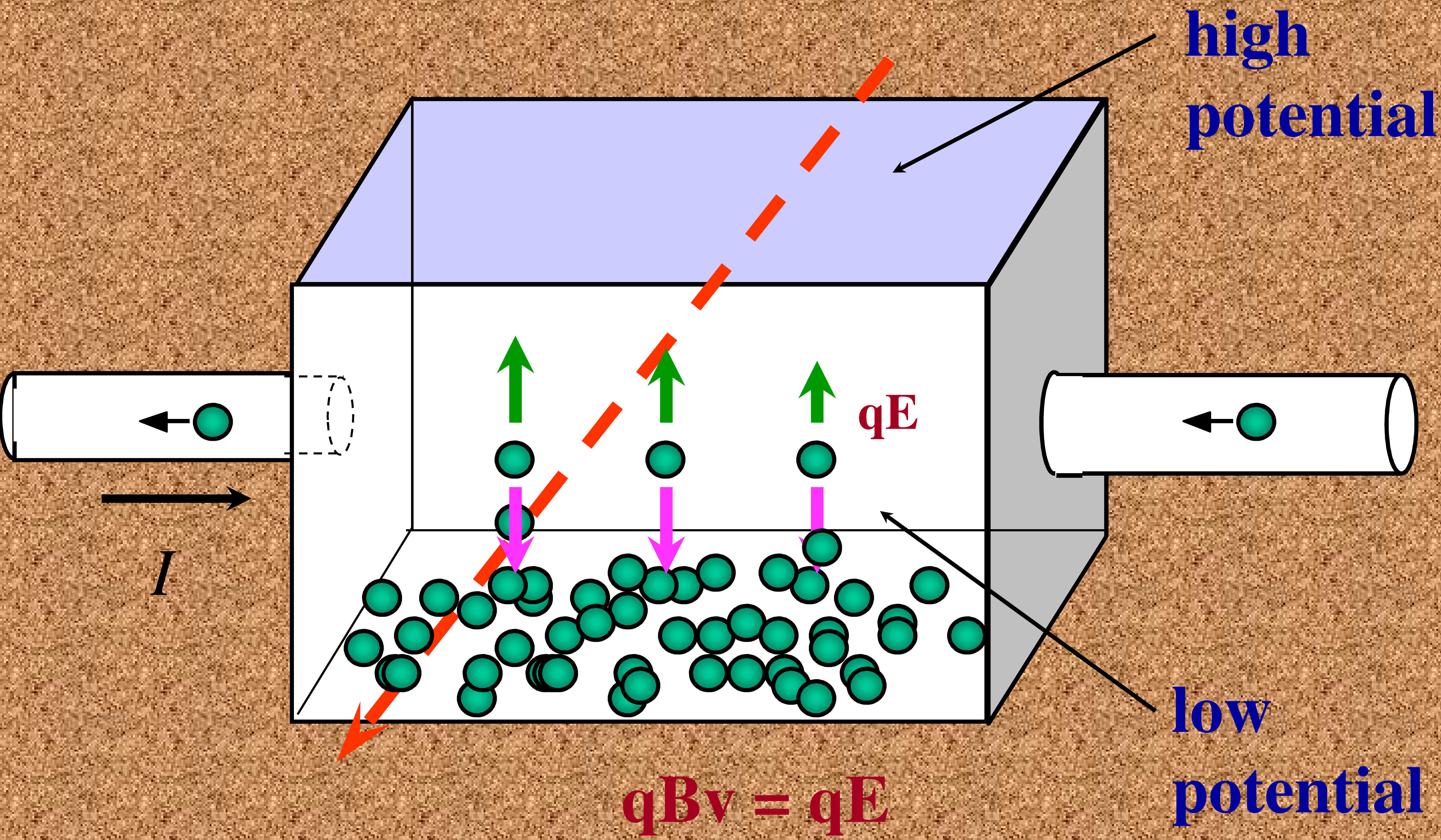
Electrons flowing without a magnetic field



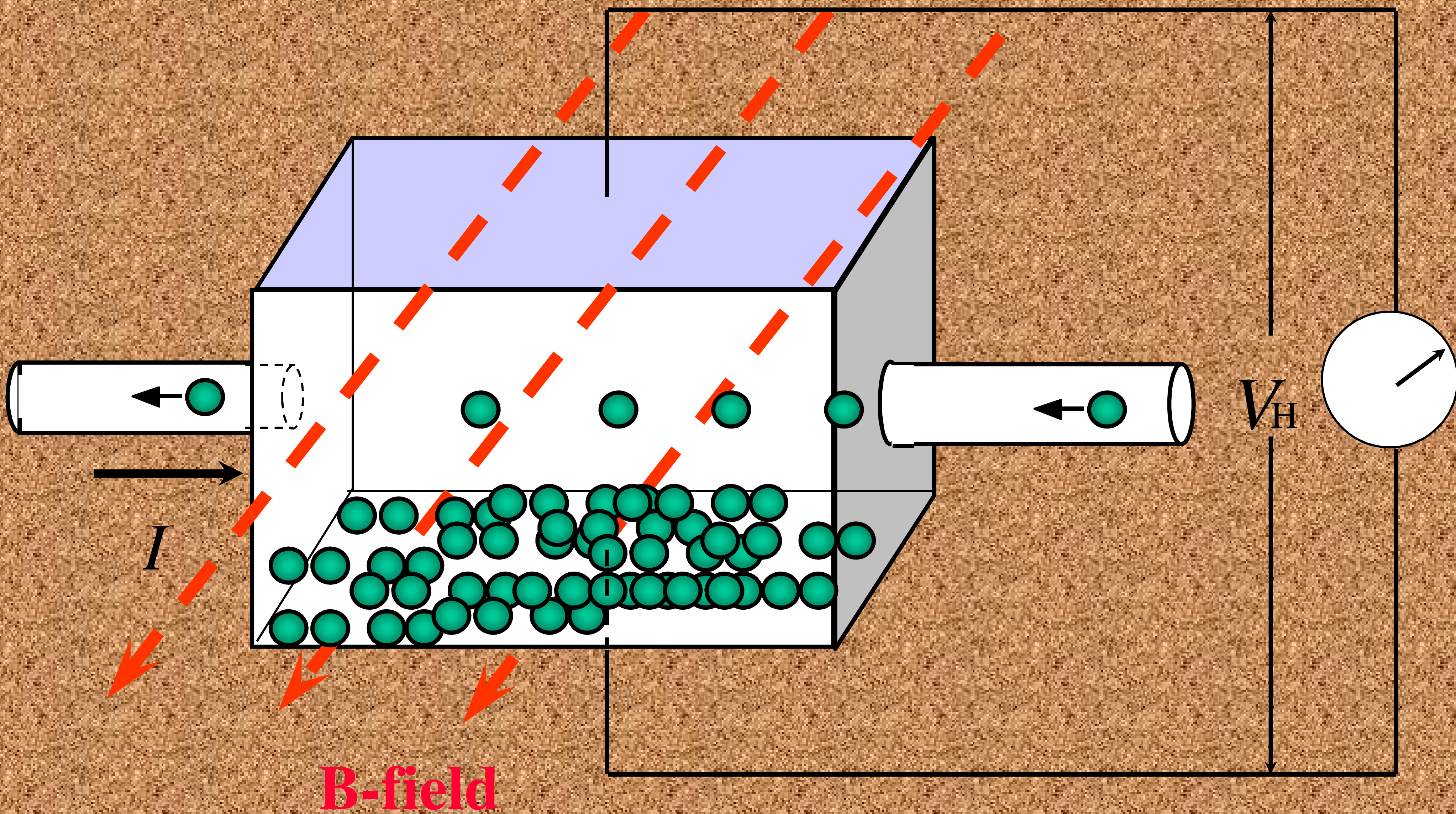
When the magnetic field is turned on ..



As time goes by...



Finally...



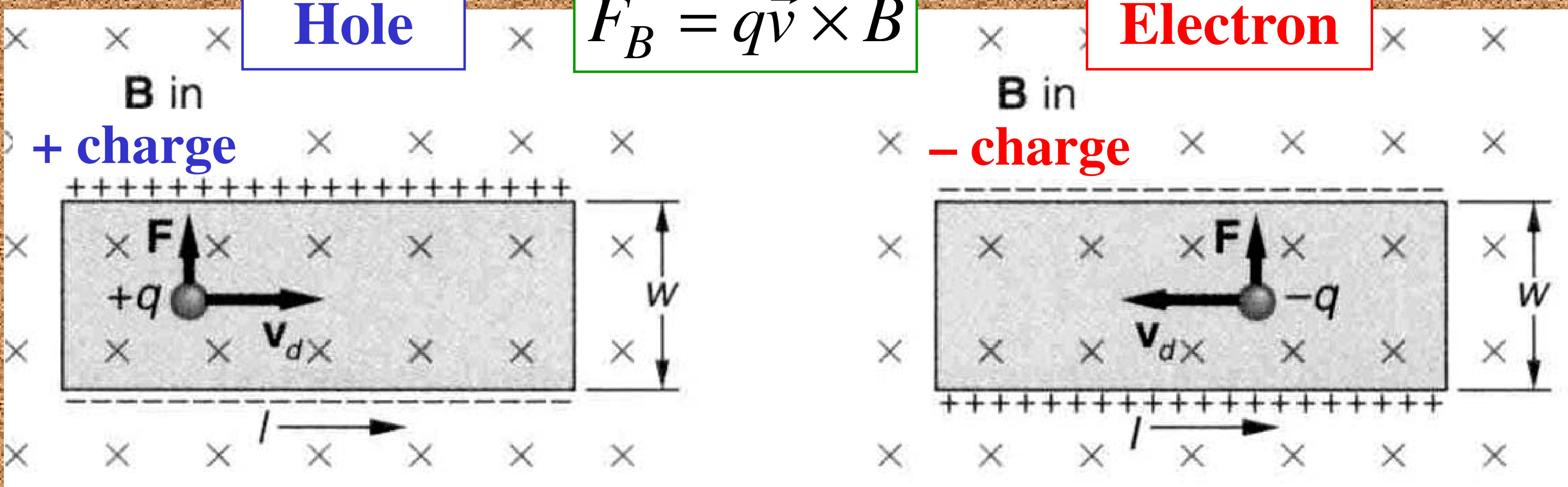
Semiconductors: Charge Carrier Density via Hall Effect

- ***Why is the Hall Effect useful?*** It can determine the **carrier type** (electron vs. hole) & the **carrier density n** for a semiconductor.
- **How?** Place the semiconductor into external **B** field, push current along one axis, & measure the induced Hall voltage V_H along the perpendicular axis. The following can be derived:

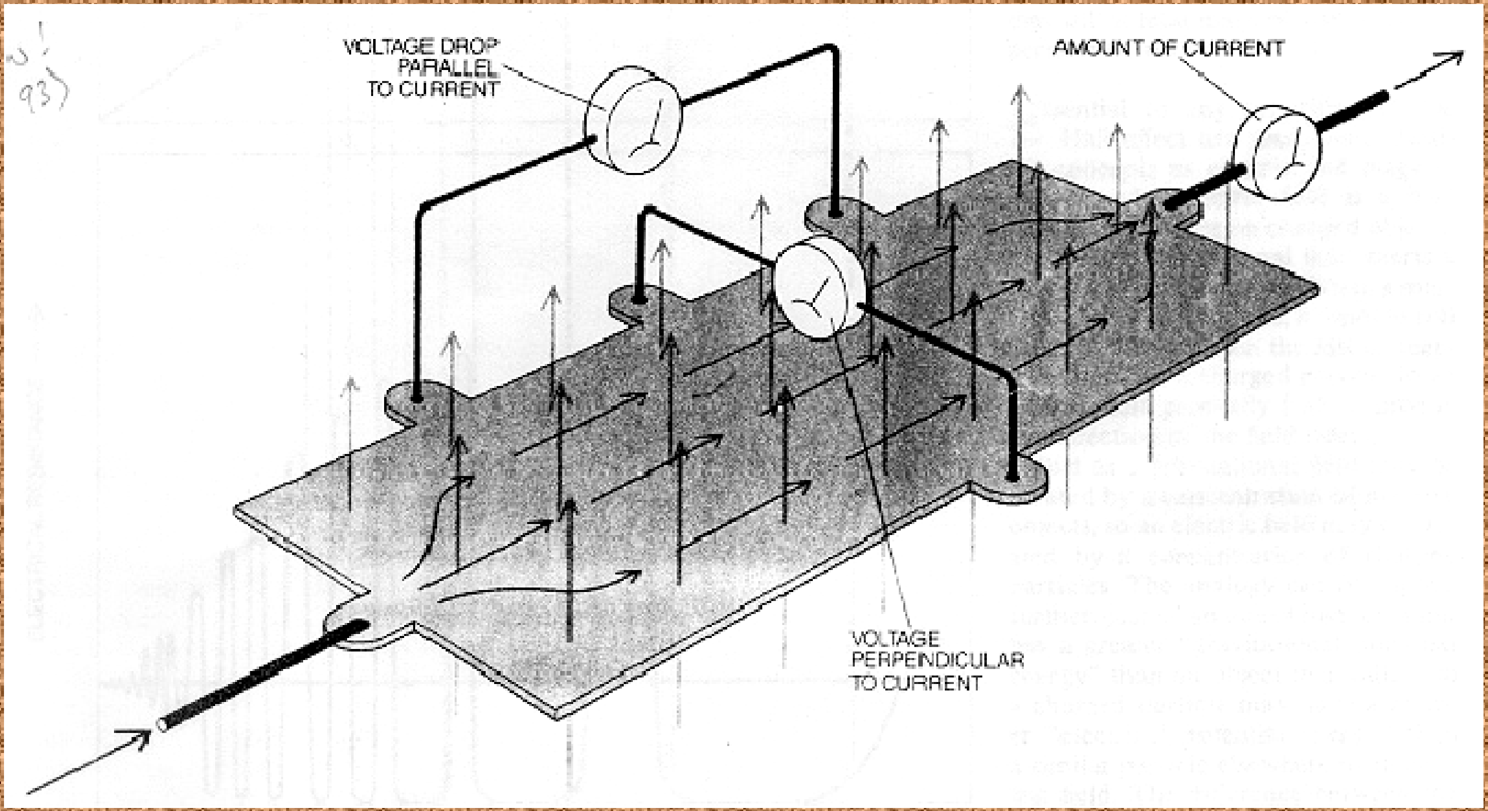
$$n = [(IB)/(qwV_H)]$$

Derived from the Lorentz force $F_E = qE = F_B = (qvB)$.

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

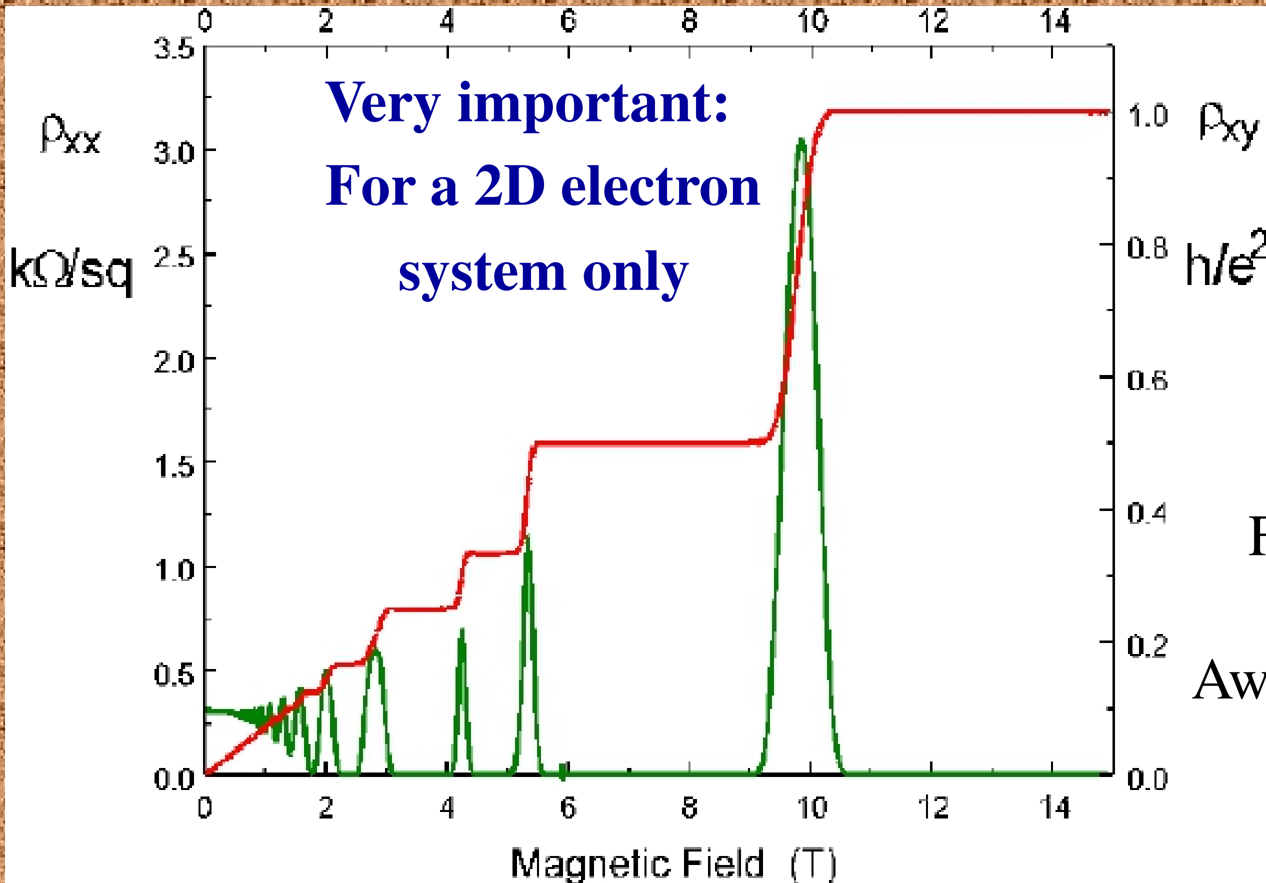


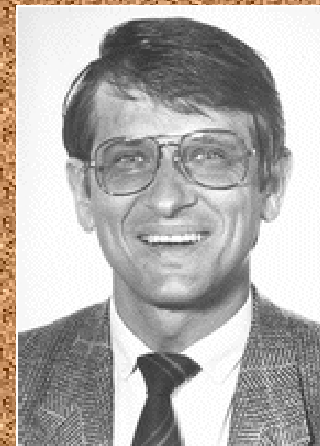
The 2Dimensional Hall effect



The surface current density $s_x = v_x \sigma q$, (σ = surface charge density) Again, $R_H = 1/\sigma e$. But, now: $R_{xy} = V_y / i_x = R_H B_z$ since $s_x = i_x / l_y$. & $E_y = V_y / l_y$. That is, R_{xy} does ***NOT*** depend on the sample shape of the sample. This is a very important aspect of the **Quantum Hall Effect (QHE)**

The Integer Quantum Hall Effect





First observed in 1980 by
Klaus von Klitzing
Awarded the 1985 Nobel Prize.

The Hall Conductance is ***quantized*** in units of **e^2/h,** or
The Hall Resistance $R_{xy} = h/(ie^2)$ where **i** is an integer.
The ***quantum of conductance*** **h/e^2** is now known as the
"Klitzing" !!
Has been measured to 1 part in 10^8

The *Fractional* Quantum Hall effect

The Royal Swedish Academy of Sciences

awarded ***The 1998 Nobel Prize in Physics***

jointly to Robert B. Laughlin (Stanford),

Horst L. Störmer (Columbia & Bell Labs) & Daniel C. Tsui, (Princeton)

The 3 researchers were awarded ***the Nobel Prize*** for discovering that electrons acting together in strong magnetic fields can form new types of "particles", with charges that are fractions of an electron charge.

Citation: ***"For their discovery of a new form of quantum fluid with fractionally charged excitations."***

Störmer & Tsui made the discovery in 1982 in an experiment using extremely high magnetic fields very low temperatures. Within a year Laughlin had succeeded in explaining their result. His theory showed that electrons in high magnetic fields & low temperatures can condense to form a quantum fluid similar to the quantum fluids that occur in superconductivity & liquid helium. Such fluids are important because events in a drop of quantum fluid can give deep insight into the inner structure & dynamics of matter. Their contributions were another breakthrough in the understanding of quantum physics & to development of new theoretical concepts of significance in many branches of modern physics.