- Equation of motion of an electron with an applied electric and magnetic field.

$$m_e \frac{d\vec{v}}{dt} = -e\vec{E} - e\vec{v} \times \vec{B}$$

- This is just Newton's law for particles of mass m_e and charge (-e).
- The use of the classical equation of motion of a particle to describe the behaviour of electrons in plane wave states, which extend throughout the crystal. A particle-like entity can be obtained by superposing the plane wave states to form a wavepacket.

- The velocity of the wavepacket is the group velocity of the waves. Thus

$$\vec{v} = \frac{d\omega}{d\vec{k}} = \frac{1}{\hbar}\frac{dE}{d\vec{k}} = \frac{\hbar\vec{k}}{m_e} = \frac{\vec{p}}{m_e}$$

$$E = \hbar\omega = \frac{\hbar^2 k^2}{2m_e}$$

$$p = \hbar k$$

- So one can use equation of mdv/dt

$$m_e\left(\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau}\right) = -e\vec{E} - e\vec{v}\times\vec{B} \qquad (*)$$

τ = mean free time between collisions. An electron loses all its energy in time τ

- In the absence of a magnetic field, the applied E results a constant acceleration but this will not cause a continuous increase in current. Since electrons suffer collisions with
  - phonons
  - electrons

- The additional term $m_e\left(\frac{\vec{v}}{\tau}\right)$ cause the velocity v to decay exponentially with a time constant τ when the applied E is removed.

The Electrical Conductivty

- In the presence of DC field only, eq.(*) has the steady state solution

$$\vec{v} = -\underbrace{\frac{e\tau}{m_e}}_{\text{a constant of proportionality (mobility)}}\vec{E}$$

$$\mu_e = \frac{e\tau}{m_e}$$

Mobility for electron

- Mobility determines how fast the charge carriers move with an E.

- Electrical current density, J

$$J = n(-e)v \qquad \vec{v} = -\frac{e\tau}{m_e}\vec{E} \qquad n = \frac{N}{V}$$

- Where n is the electron density and v is drift velocity. Hence

$$\vec{J} = \frac{ne^2\tau}{m_e}\vec{E} \qquad \sigma = \frac{ne^2\tau}{m_e}$$

Electrical conductivity

Ohm's law

$$\vec{J} = \sigma\vec{E}$$

Electrical Resistivity and Resistance

$$\rho = \frac{1}{\sigma} \qquad R = \frac{\rho L}{A}$$

Collisions

- In a perfect crystal; the collisions of electrons are with thermally excited lattice vibrations (scattering of an electron by a phonon).
- This electron-phonon scattering gives a temperature dependent $\tau_{ph}(T)$ collision time which tends to infinity as T $\rightarrow$0.

- In real metal, the electrons also collide with impurity atoms, vacancies and other imperfections, this result in a finite scattering time τ_0 even at T=0.

- The total scattering rate for a slightly imperfect crystal at finite temperature;

$$\frac{1}{\tau} = \underbrace{\frac{1}{\tau_{ph}(T)}}_{\text{Due to phonon}} + \underbrace{\frac{1}{\tau_0}}_{\text{Due to imperfections}}$$

- So the total resistivity ρ,

$$\rho = \frac{m_e}{ne^2\tau} = \frac{m_e}{ne^2\tau_{ph}(T)} + \frac{m_e}{ne^2\tau_0} = \rho_I(T) + \rho_0$$

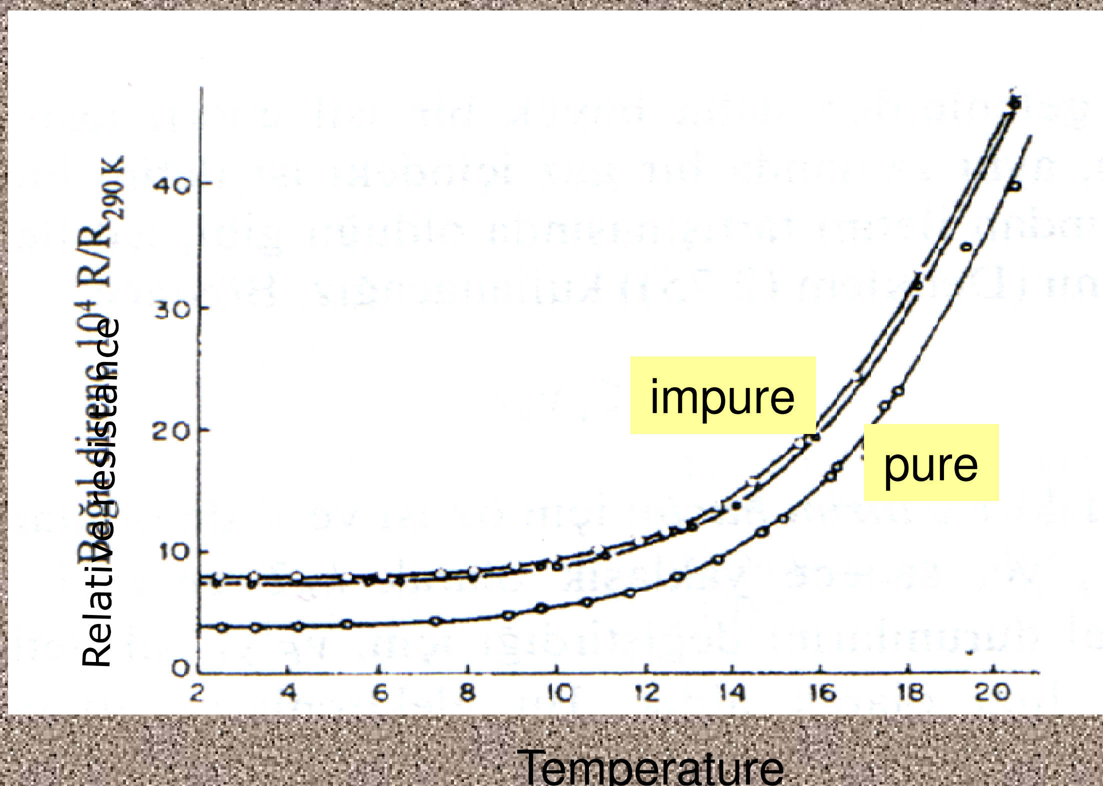
Ideal resistivity ($\rho_I(T)$) — Residual resistivity (ρ_0)

This is known as Mattheisen’s rule and illustrated in following figure for sodium specimen of different purity.

Residual resistance ratio

Residual resistance ratio = room temp. resistivity/ residual resistivity

and it can be as high as 10^6 for highly purified single crystals.



σ

Collision time

$$\sigma(RT)_{sodium} = 2.0x10^{7}(\Omega - m)^{-1} \qquad \sigma_{residual_{pureNa}} = 5.3x10^{10}(\Omega - m)^{-1}$$

τ can be found by taking

$$m_e = m$$

$$n = 2.7x10^{28} m^{-3}$$

$$\tau = \frac{m\sigma}{ne^2} \approx 2.6x10^{-14} s \quad \text{at RT}$$

$$\approx 7.0x10^{-11} s \quad \text{at T=0}$$

Taking $v_F = 1.1x10^{6} m/s$; and $l = v_F \tau$

$$l(RT) = 29nm$$

$$l(T=0) = 77\mu m$$

These mean free paths are much longer than the interatomic distances, confirming that the free electrons do not collide with the atoms themselves.

Thermal conductivity, K

Due to the heat tranport by the conduction electrons

$$K_{metals} >> K_{non-metals}$$

Electrons coming from a hotter region of the metal carry more thermal energy than those from a cooler region, resulting in a net flow of heat. The thermal conductivity

$K = \frac{1}{3} C_V v_F l$ where C_V is the specific heat per unit volume

v_F is the mean speed of electrons responsible for thermal conductivity since only electron states within about $k_B T$ of ε_F change their occupation as the temperature varies.

l is the mean free path; $l = v_F \tau$ and Fermi energy $\varepsilon_F = \frac{1}{2} m_e v_F^2$

$$K = \frac{1}{3} C_V v_F^2 \tau = \frac{1}{3} \frac{\pi^2}{2} \frac{N}{V} k_B \left(\frac{T}{T_F}\right) \frac{2}{m_e} \varepsilon_F \tau = \frac{\pi^2 n k_B^2 T \tau}{3 m_e}$$ where $C_v = \frac{\pi^2}{2} N k_B \left(\frac{T}{T_F}\right)$

Wiedemann-Franz law

$$\sigma = \frac{ne^2\tau}{m_e} \qquad K = \frac{\pi^2 n k_B^2 T\tau}{3m_e}$$

The ratio of the electrical and thermal conductivities is independent of the electron gas parameters;

Lorentz number

$$\frac{K}{\sigma T} = \frac{\pi^2}{3}\left(\frac{k_{\mathbf{B}}}{e}\right)^2 = 2.45x10^{-8} W\Omega K^{-2}$$

$$L = \frac{K}{\sigma T} = 2.23x10^{-8} W\Omega K^{-2}$$

For copper at 0 C