Heat Capacity of Electron Gas

By definition, the heat capacity (at constant volume) of the electron gas is given by

$$C_V = \frac{dU}{dT}$$

where U is the total energy of the gas. For a gas of N electrons, each with average energy <E>, the total energy is given by

$$U = N\langle E \rangle$$

Heat Capacity of Electron Gas

Total energy

$$U = N\langle E\rangle = \int_0^\infty E\, n(E)dE$$

$$= V\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2}\int_0^\infty \frac{E^{3/2}dE}{e^{(E-E_F)/kT}+1}$$

In general, this integral must be done numerically. However, for $T \ll T_F$, we can use a reasonable approximation.

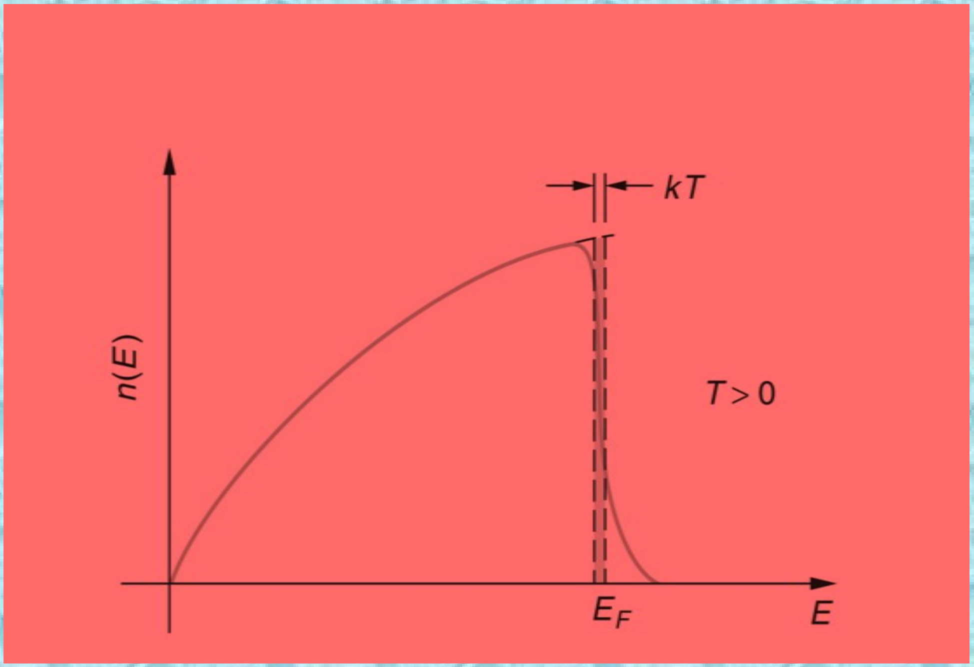
Heat Capacity of Electron Gas

At **T= 0**, the total energy of the electron gas is

$$U = N\langle E \rangle = N\left(\frac{3}{5}E_F\right)$$

For **0 < T << T_F**, only a small fraction **kT/E_F** of the electrons can be excited to higher energy states

Moreover, the energy of each is increased by roughly **kT**



Heat Capacity of Electron Gas

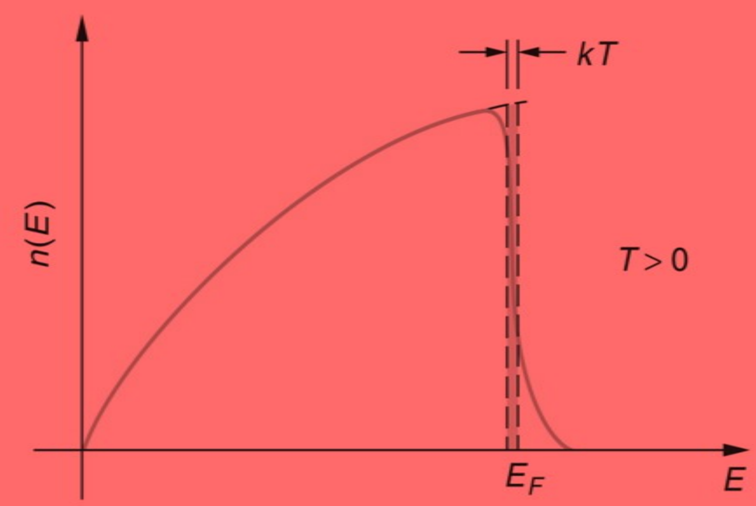
Therefore, the total energy can be written as

$$U = \frac{3}{5} N E_F + \alpha \left(\frac{kT}{E_F} \right) NkT$$

where $\alpha = \pi^2/4$, as first shown by Sommerfeld

The heat capacity of the electron gas is predicted to be

$$C_V = \frac{dU}{dT} = \frac{\pi^2}{2} Nk \frac{T}{T_F}$$



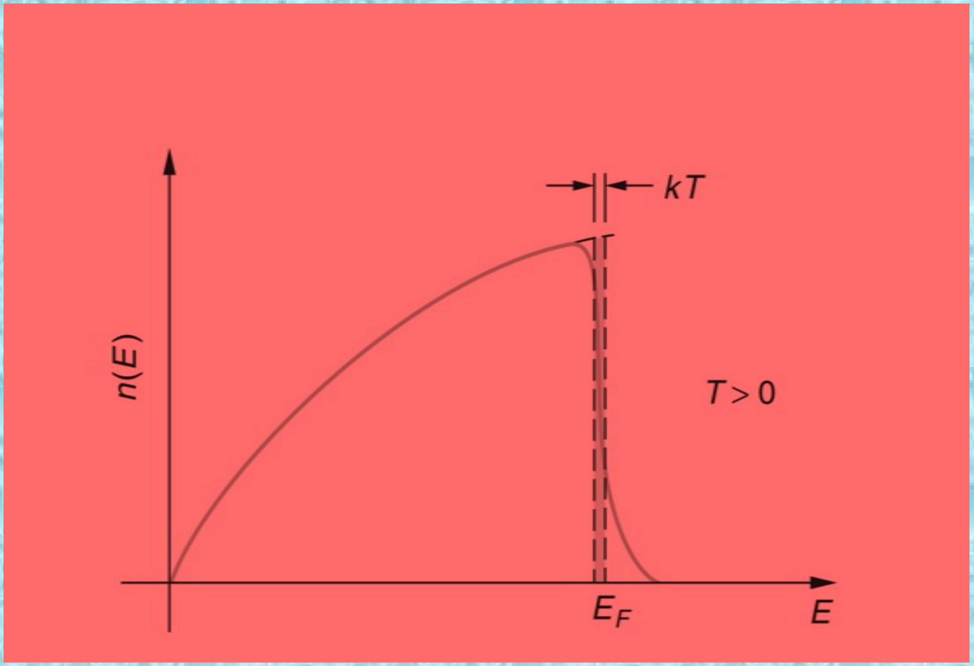
Heat Capacity of Electron Gas

Consider 1 mole of copper. In this case Nk = R

$$C_V = \frac{\pi^2}{2} R \frac{T}{T_F}$$

For copper, **T_F = 89,000 K**. Therefore, even at room temperature, **T = 300 K**, the contribution of the electron gas to the heat capacity of copper is small:

C_V = 0.018 R



Summary

- The heat capacity of the electron gas is small compared with that of the ions