Example 1:

Find the quantity (q) from the given function at which AC is minimum.

$$AC = q^2 - 5q + 8$$

$$\frac{dAC}{dq} = Ac' = 2q - 5$$

$$\frac{d^2AC}{dq^2} = AC'' = 2 > 0$$

Putting

$$AC' = 0$$

$$2q - 5 = 0$$

$$2q = 5$$

$$q = \frac{5}{2} = 2.5$$

As AC' = 0 and AC'' > 0. Thus at q = 2.5 both the conditions of minima are satisfied.
Putting the values of q = 2.5 in AC function.

$$AC = (2.5)^2 - 5(2.5) + 8$$

$$= 6.25 - 12.5 + 8$$

$$= 14.25 - 12.5$$

$$AC = 1.75$$

Example 2:

Find the quantity (Q) at which AC is minimum from the given function.

$$AC = 40 - 6q + q^2$$

$$\frac{dAC}{dq} = AC' = 0 - 6 + 2q = -6 + 2q$$

$$\frac{d^2AC}{dq^2} = AC'' = 2 > 0$$

Putting

$$AC' = 0$$

$$-6 + 2q = 0$$

$$2q = 6$$

$$q = 3$$

As AC' = 0 and AC'' = 2 > 0 both the conditions of minima are satisfied.

At $Q = 3$

$$AC = 40 - 6(3) + (3)^2$$
$$= 40 - 18 + 9$$
$$= 49 - 18$$
$$AC = 31$$

Example 3:

(i) Find quantity at which AC is minimum from the given function.
(ii) Prove that: AC = MC at that quantity.

$$C = 25Q - 5Q^2 + Q^3$$

(i) We derive Ac function from the above total cost function (C).

$$AC = \frac{C}{Q} = \frac{25Q - 5Q^2 + Q^2}{Q}$$
$$= \frac{Q(25 - 5Q + Q^2)}{Q}$$
$$AC = 25 - 5Q + Q^2$$

Ist derivative of AC function

$$\frac{dAC}{dQ} = AC' = -5 + 2Q = -5 + 2Q$$

Putting AC' equal to zero

$$-5 + 2Q = 0$$
$$2Q = 5$$
$$Q = \frac{5}{2}$$
$$Q = 2.5$$

Test under 2^{nd} derivative of AC function

$$AC'' = 2 > 0$$

MC function $MC = \frac{dc}{dQ} = \frac{d}{dQ}(25Q - 5Q^2 + Q^3)$

$$= 25 - 10Q + 3Q^2$$

MC at $Q = 2.5$

$$MC = 25 - 10(2.5) + 3(2.5)^2$$
$$= 25 - 25 + 18.75$$
$$MC = 18.75$$

AC at Q = 2.5, $AC = 25 - 5Q + Q^2$

$$= 25 - 5(2.5) + (2.5)^2$$
$$= 25 - 12.5 + 6.25$$
$$= 31.25 - 12.5$$
$$AC = 18.75$$

Thus, MC = AC at Q = 2.5

Example 4:

(i) Find the quantity at which MC = minimum.
(ii) Calculate value of MC also from the give cost function

$$C = Q^3 - 2Q^2 + 9Q$$

MC function $\quad MC = \dfrac{dc}{dQ} = 3Q^2 - 4Q + 9$

Ist derivative of MC function

$$MC' = 6Q - 4$$

Putting it equal to zero

$$6Q - 4 = 0$$

$$6Q = 4$$

$$Q = \frac{6}{4} = \frac{2}{3} \text{ or } 0.666$$

2nd derivative of MC function

$$MC'' = 6 > 0$$

To find MC, we put $Q = \frac{2}{3}$ in MC function

$$MC = 3\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) + 9 = 3\left(\frac{4}{9}\right) - 4\left(\frac{2}{3}\right) + 9$$

$$MC = \frac{4}{3} - \frac{8}{3} + 9$$

$$MC = \frac{4 - 8 + 27}{3} = \frac{23}{3} = 7\frac{2}{3} = 7.66$$

Thus $\quad MC = 7.66$

Example 7:

Following Total Cost function is given

$$C = Q^3 - 12Q^2 + 60Q$$

(i) Find the quantity at which Average Cost (AC) is minimum.

(ii) Prove that; at that quantity Ac = MC

Solution:

From the given cost function we find Average Cost function.

$$C = Q^3 - 12Q^2 + 60Q$$

(i) As

$$AC = \frac{Tc}{Q}$$

$$AC = \frac{Q^3 - 12Q + 60Q}{Q}$$

$$AC = \frac{Q^3}{Q} - \frac{12Q^2}{Q} + \frac{60Q}{Q}$$

$$AC = Q^2 - 12Q + 60$$

$$MC = \frac{dc}{dq} = \frac{d}{dq}(Q^3 - 12Q^2 + 60Q)$$

$$= 3Q^2 - 24Q + 60$$

We take first derivative of the Average Cost function

$$AC = Q^2 - 12Q + 60$$

$$AC' = 2Q - 12$$

Putting Ist derivative equal to zero

$$2Q - 12 = 0$$

$$2Q = 12$$

$$Q = 6$$

Thus, Q = 6 is the quantity where AC is minimum.

$$AC = Q^2 - 12Q + 60$$

(ii) Putting the value of Q = 6 in AC function given above.

$$AC = (6)^2 - 12(6) + 60$$

$$AC = 36 - 72 + 60$$

$$AC = 96 - 72$$

$$AC = [24]$$

Putting the value of Q = 6 in the marginal cost function.

$$MC = 3Q^2 - 24Q + 60$$

$$MC = 3(6)^2 - 24(6) + 60$$

$$= 3(36) - 144 + 60$$

$$= 108 - 144 + 60$$

$$= 168 - 144$$

$$MC = [24]$$

Thus, $MC = AC$