EXPONENTIAL FUNCTIONS

Exponential functions and logarithm functions are important in both theory and practice. In this unit we look at the graphs of exponential and logarithm functions, and see how they are related.

In addition to linear, quadratic, rational, and radical functions, there are **exponential functions**. Exponential functions have the form $f(x) = b^x$, where $b > 0$ and $b \neq 1$. Just as in any exponential expression, b is called the **base** and x is called the **exponent**.

Exponential functions

Exponential functions have the form:

$$\boxed{f(x) = b^x}$$

where b is the **base** and x is the **exponent** (or **power**).

If b is greater than 1, the function continuously increases in value as x increases. A special property of exponential functions is that the **slope** of the function also continuously increases as x increases.

It is common to write exponential functions using the carat (^), which means "raised to the power". Computer programing uses the ^ sign, as do some calculators.

Other calculators have a button labeled x^y which is equivalent to the ^ symbol.

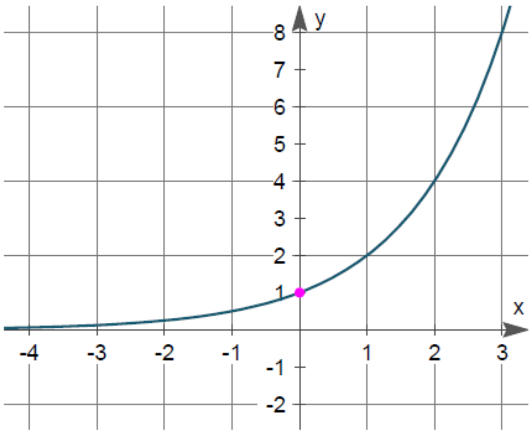
Example of an Exponential Function

Consider the function $f(x) = 2^x$.

In this case, we have an exponential function with base 2. Some typical values for this function would be:

x	-2	-1	0	1	2	3
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Here is the graph of $y = 2^x$.



Graph of $y = 2^x$.

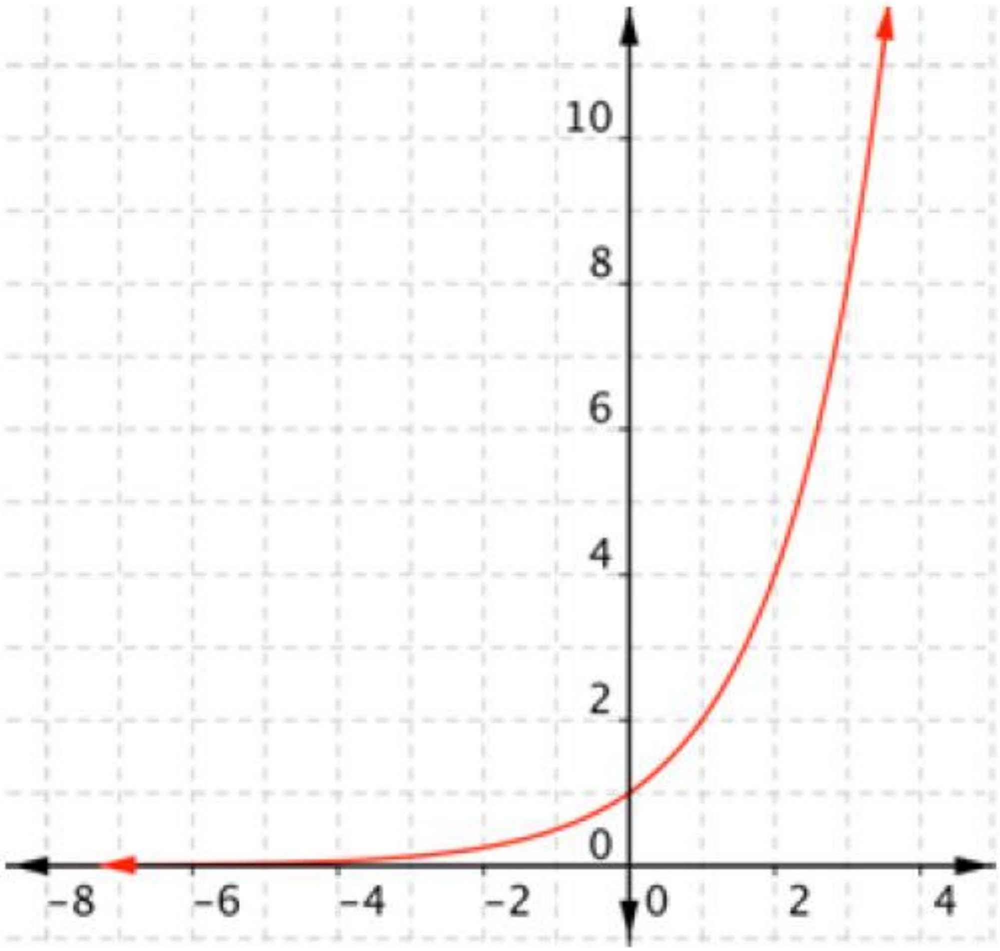
- As x increases, y also increases.
- As x increases, the slope of the graph also increases.
- The curve passes through $(0, 1)$. All exponential curves of the form $f(x) = b^x$ pass through $(0, 1)$, if $b > 0$.
- The curve does not pass through the x-axis. It just gets closer and closer to the x-axis as we take smaller and smaller x-values.

An example of an exponential function is the growth of bacteria. Some bacteria double every hour. If you start with 1 bacterium and it doubles every hour, you will have 2^x bacteria after x hours. This can be written as $f(x) = 2^x$.

Before you start,	$f(0) = 2^0 = 1$
After 1 hour	$f(1) = 2^1 = 2$
In 2 hours	$f(2) = 2^2 = 4$
In 3 hours	$f(3) = 2^3 = 8$

and so on.

With the definition $f(x) = b^x$ and the restrictions that $b > 0$ and that $b \neq 1$, the domain of an exponential function is the set of all real numbers. The range is the set of all positive real numbers. The following graph shows $f(x) = 2^x$.



Exponential Growth

The exponential function has a graph that gets very close to the *x*-axis as the graph extends to the left (as *x* becomes more negative), but never really touches the *x*-axis. Knowing the general shape of the graphs of exponential functions is helpful for graphing specific exponential equations or functions.

Making a table of values is also helpful, because you can use the table to place the curve of the graph more accurately. One thing to remember is that if a base has a negative exponent, then take the reciprocal of the base to make the exponent positive. For example, $4^{-2} = \left(\frac{4}{1}\right)^{-2} = \left(\frac{1}{4}\right)^{2}$.

Example

Problem **Make a table of values for $f(x) = 3^x$.**

x	$f(x)$

Make a "T" to start the table with two columns. Label the columns x and $f(x)$.

x	$f(x)$
−2	
−1	
0	
1	
2	

Choose several values for x and put them as separate rows in the x column.

SOLUTION

x	$f(x)$
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9

Evaluate the function for each value of x, and write the result in the $f(x)$ column next to the x value you used. For example, when

$x = -2$, $f(x) = 3^{-2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$, so

$\frac{1}{9}$ goes in the $f(x)$ column next to

−2 in the x column. $f(1) = 3^1 = 3$, so 3 goes in the $f(x)$ column next to 1 in the x column.

Note that your table of values may be different from someone else's, if you chose different numbers for x.

Now that you have a table of values, you can use these values to help you draw both the shape and location of the function. Connect the points as best you can to make a smooth curve (not a series of straight lines). This shows that all of the points on the curve are part of this function.

Example

Problem **Graph $f(x) = 3^x$.**

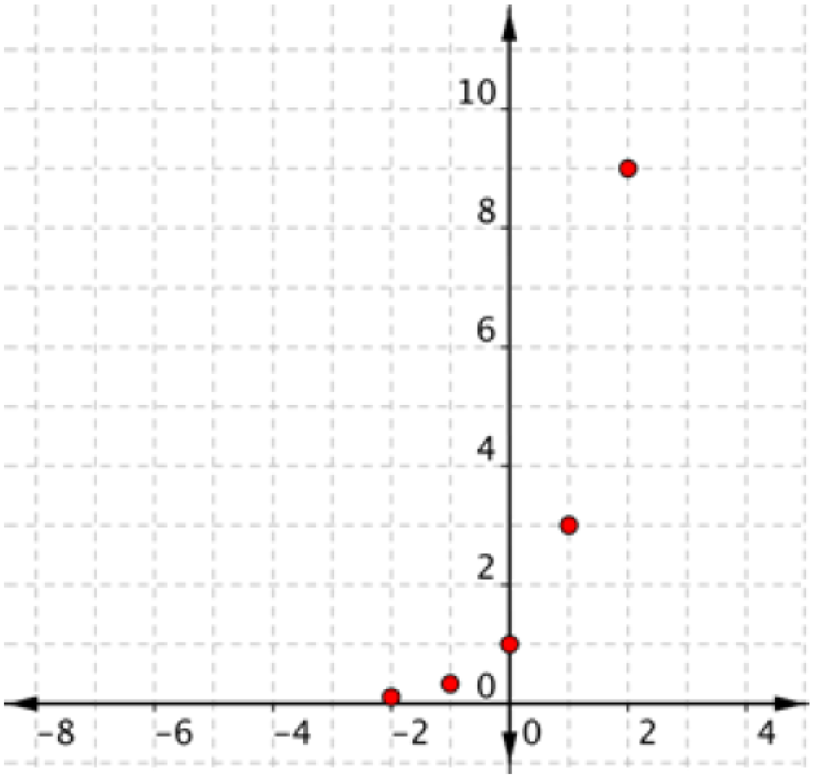
x	$f(x)$
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9

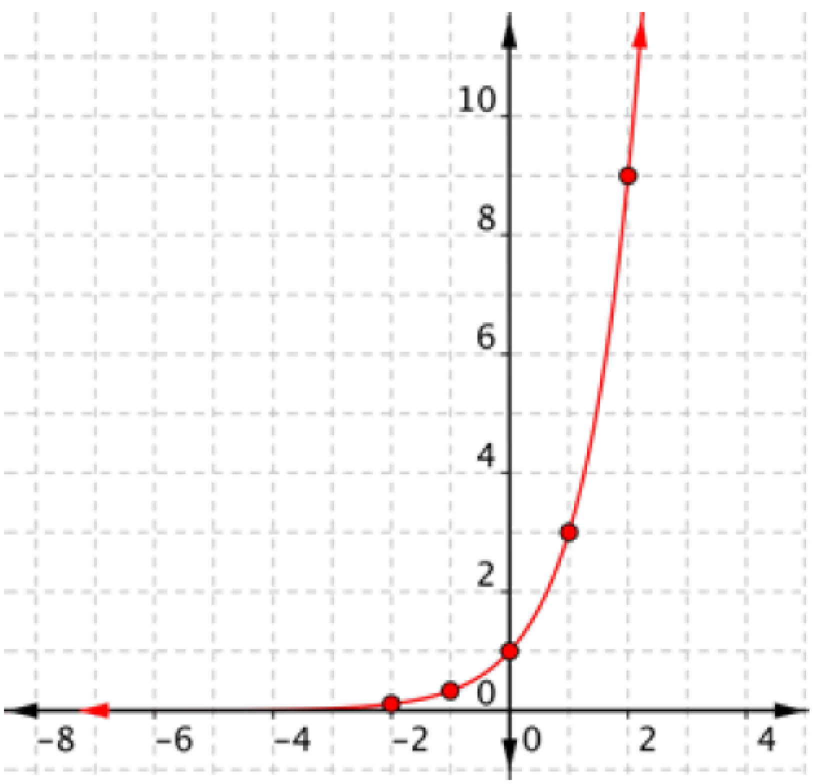
Start with a table of values, like the one in the example above.

x	$f(x)$	point
−2	$\frac{1}{9}$	$(-2, \frac{1}{9})$
−1	$\frac{1}{3}$	$(-1, \frac{1}{3})$
0	1	(0, 1)
1	3	(1, 3)
2	9	(2, 9)

If you think of $f(x)$ as y, each row forms an ordered pair that you can plot on a coordinate grid.

Plot the points.





Connect the points as best you can, using a *smooth curve* (not a series of straight lines). Use the shape of an exponential graph to help you: this graph gets very close to the *x*- axis on the left, but never really touches the *x*-axis, and gets steeper and steeper on the right.

This is an example of **exponential growth**. As x increases, $f(x)$ "grows" more quickly.