# Electrodynamics_I

## PHYS_303

### Text Book:

[0] David J. Griffiths, Introduction to Electrodynamics, 3rd Edition, Pearson Prentice Hall, India, 1999.

### Recommended Books:

[1] David K. Cheng, Field and Wave Electromagnetics, 2nd Edition,

[2] Jack Vanderlinde, Classical Electromagnetic Theory

[3] Bhag S. Guru, Hüseyin R. Hiziroğlu, Electromagnetic Field Theory_ Fundamentals.

[4] John R. Reitz, Frederick J. Milford, Robert W. Christy, Foundations of Electromagnetic Theory.

[5] Paul Lorrain, Dale R. Corson, Electromagnetic Fields and Waves.

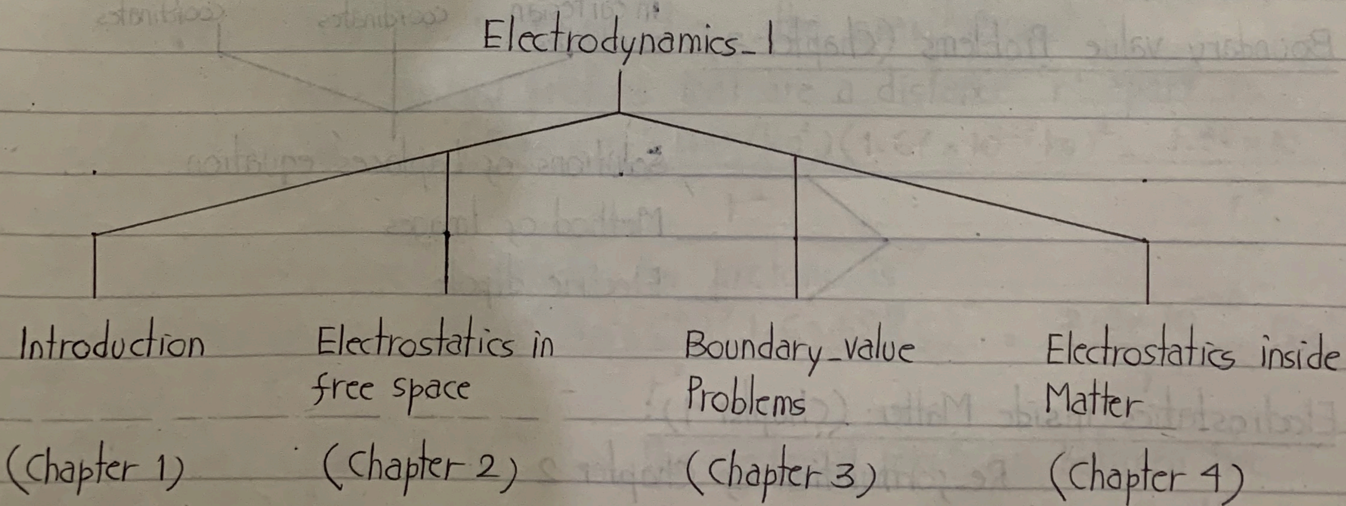
[6] Hans C. Ohanian, Classical Electrodynamics.

[7] Clayton R. Paul, Keith W. Whites, Syed A. Nasar, Introduction to Electromagnetic Fields.

[8] Yung K. Lim, Introduction to Classical Electrodynamics.

[9] Fawwaz T. Ulaby, Fundamentals of Applied Electromagnetics.

[10] John D. Jackson, Classical Electrodynamics.



## Introduction (Chapter 1):

- Differential Calculus
  - The Gradient
  - The Divergence
  - The Curl
- Integral Calculus
  - Gradient theorem
  - Divergence theorem
  - Curl theorem
- Orthogonal coordinate system
  - Cartesian coordinates $(x, y, z)$
  - Cylindrical coordinates $(r, \phi, z)$
  - Spherical coordinates $(r, \theta, \phi)$

## Electrostatics in free space (Chapter 2):

- Electric force
- Electric field
- Electric potential
- Electric energy

For single point charge; discrete charge distribution; Continuous charge distribution

- Induced charges, Conductors, Capacitors
  - Parallel-plate capacitor
  - Cylindrical Capacitor
  - Spherical capacitor

## Boundary-value Problems (Chapter 3):

- in cartesian coordinates
- in cylindrical coordinates
- in spherical coordinates

- Solutions of Laplace's equation
- Method of Images
- Electric dipole

## Electrostatics inside Matter (Chapter 4):

Re-formulation of Chapter 2.

## Electrodynamics :

In this course, we will study/investigate the nature of electromagnetic interaction — one of the four known forces/interactions of nature.

I list them in the order of decreasing strength :

1. Strong Nuclear Force :
   - binds protons and neutrons together to form nuclei.
   - ★ extremely short range but is hundred times more powerful than electrical force.

2. Electromagnetic Force :
   - binds electrons and nuclei together to form atoms.
   - binds atoms together to form molecules (gases, liquids, solids, ...)

3. Weak Nuclear Force :
   - responsible for radioactivity (e.g., $\beta$- decay)
   - ★ short range but are far weaker than electromagnetic ones.

4. Gravitational Force :
   - binds matter together to form stars, planets, solar system, galaxies, etc.
   - ★ feeble as compared to all of the others.

The proton mass is $1.67\times10^{-27}$ kg, the gravitational attraction between two protons that are a distance 'r' apart is

$$F_{grav} = \frac{G m_1 m_2}{r^2} = \frac{(6.67\times10^{-11}\ Nm^2/kg^2)(1.67\times10^{-27}\ kg)^2}{r^2} = \frac{1.86\times10^{-64}\ Nm^2}{r^2}$$

The electrical repulsion between the protons is

$$F_{elec.} = \frac{k\, q_1 q_2}{r^2} = \frac{(9\times10^{9}\ Nm^2/C^2)(1.6\times10^{-19}\ C)^2}{r^2} = \frac{2.3\times10^{-28}\ Nm^2}{r^2}$$

Therefore, $\frac{F_{elec.}}{F_{grav.}} \cong 10^{36}$ $\Rightarrow$ The electrical repulsion between two protons is $10^{36}$ times as large as their gravitational attraction.

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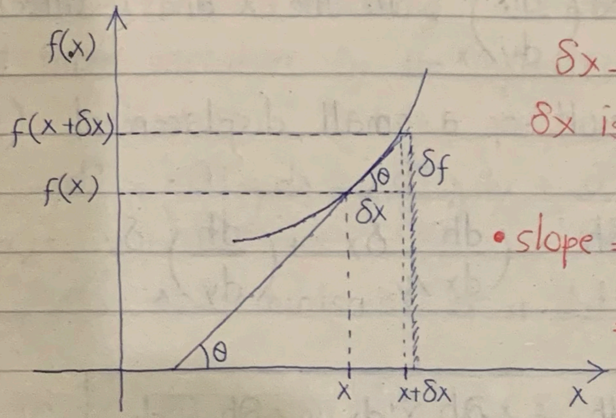
- large +ve slope ⇒ function is increasing rapidly.
- 〃 −ve 〃 ⇒ 〃 〃 decreasing 〃 .

## Differential Calculus
## "Ordinary" Derivatives :

- +ve slope ⇐ increasing function ⇒ rise from left to right.
- −ve slope ⇐ decreasing function ⇒ fall from left to right.

1-D




$\delta x \to 0$

$\delta x$ is infinitesimally small.

- slope = $\dfrac{\text{rise}}{\text{run}} = \dfrac{\text{Vertical distance}}{\text{horizontal distance}}$

$= \dfrac{y_2 - y_1}{x_2 - x_1}$ $(\equiv m)$

For the increment $\delta x$ shown in the diagram above, $\tan\theta$ is not equal $\dfrac{\delta f}{\delta x}$. But as $\delta x$ is made infinitesimally small, we have the exact relation

$$\tan\theta = \lim_{\delta x \to 0}\left(\frac{\delta f}{\delta x}\right) = \frac{df}{dx}$$

For a tiny step $\delta x$, the change in $f(x)$ is

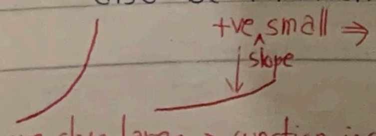
$$\delta f = \left(\frac{df}{dx}\right)\delta x,$$

$x \to x + \delta x$, where $\delta x \to 0$

where the derivative $\dfrac{df}{dx}$ is the <u>SLOPE</u> of the graph of $f(x)$ versus $x$. In other words, If we change $x$ by an amount $\delta x$, then $f(x)$ changes by an amount $\delta f$; the derivative is the proportionality factor. This derivative $\dfrac{df}{dx}$ tells us how rapidly the function $f(x)$ varies when we change the argument $x$ by a tiny amount, $\delta x$. When the function varies slowly with $x$, the derivative is correspondingly small. When $f(x)$ increases rapidly with $x$, the derivative is large.

<u>OR</u>

For a very very small change in $x$ ($\delta x \to 0$), the above relation can also be written as




$$\boxed{df = \left(\frac{df}{dx}\right) dx}$$

⇒ function increases slowly when we change the argument

$x \to x + dx$ (1.1)

+ve slope large ⇒ function increases rapidly when when we change the argument

-ve slope large → function decreases rapidly with the change in the argument.

-ve slope small → " " slowly " " " " " ".

2-D

A scalar function $h(x,y)$ that depends on two coordinates (like the height of a point on a map) may have different rates of change, $\left(\frac{dh}{dx}\right)_y$ and $\left(\frac{dh}{dy}\right)_x$, in the x and y directions. The change in h as a result of a small displacement $(\delta x, \delta y)$ is

$$\delta h = \left(\frac{dh}{dx}\right)_y \delta x + \left(\frac{dh}{dy}\right)_x \delta y \qquad (x,y) \to (x+\delta x, y+\delta y)$$

OR

$$\boxed{dh = \left(\frac{\partial h}{\partial x}\right) dx + \left(\frac{\partial h}{\partial y}\right) dy} \qquad (x,y) \to (x+dx, y+dy)$$

$$(x,y) \to (x+dx, \underset{\substack{\uparrow \\ dy=0}}{y}) \qquad dh = \left(\frac{\partial h}{\partial x}\right) dx + 0 \quad (\because dy=0)$$

3-D

A scalar function $V(x,y,z)$ that depends on three coordinates will change, in a small displacement $(\delta x, \delta y, \delta z)$, according to

$$(x,y,z) \to (x+\delta x, y+\delta y, z+\delta z) \qquad \delta V = \left(\frac{dV}{dx}\right)_{y,z} \delta x + \left(\frac{dV}{dy}\right)_{z,x} \delta y + \left(\frac{dV}{dz}\right)_{x,y} \delta z$$

OR

$$\boxed{dV = \left(\frac{\partial V}{\partial x}\right) dx + \left(\frac{\partial V}{\partial y}\right) dy + \left(\frac{\partial V}{\partial z}\right) dz}$$

NOTE: At any point $(x,y,z)$, a function $V(x,y,z)$ may have different slopes $\left(\frac{dV}{dx}\right)_{y,z}$, $\left(\frac{dV}{dy}\right)_{z,x}$, and $\left(\frac{dV}{dz}\right)_{x,y}$, in the x, y, and z directions.

$$(x,y,z) \to (x+dx, y+dy, \overset{\substack{dz=0 \\ \downarrow}}{z})$$

$$dV = \left(\frac{\partial V}{\partial x}\right) dx + \left(\frac{\partial V}{\partial y}\right) dy + \underset{\substack{\uparrow \\ (\because dz=0)}}{0}$$

## The Gradient:

Suppose, we have a function of three variables – say, the temperature $T(x,y,z)$ in a building. A small change in 'T' when we alter all three variables by the infinitesimal amounts $dx, dy, dz$ is $(x,y,z) \rightarrow (x+dx, y+dy, z+dz)$

$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz.$$

The above equation is reminiscent of a dot product:

$$dT = \left(\frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}\right) \cdot \left(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right),$$

$$dT = (\vec{\nabla} T) \cdot (d\vec{l}), \qquad (1\text{-}2)$$

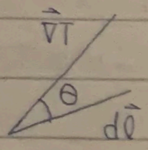
where

$$\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k} = \text{Gradient of } T,$$

and $d\vec{l} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$ = Infinitesimal displacement vector, from $(x,y,z)$ to $(x+dx, y+dy, z+dz)$

$\vec{\nabla} T$ is a vector quantity, with three components. Equation (1.2) is the generalized form of Eq. (1.1).
From the definition of scalar product,

$$dT = \vec{\nabla} T \cdot d\vec{l} = |\vec{\nabla} T|\,|d\vec{l}|\cos\theta,$$




where $\theta$ is the angle between $\vec{\nabla} T$ and $d\vec{l}$.
The dependence on $\cos\theta$ shows that the greatest change in T for a fixed step length is achieved when $\theta = 0°$, i.e., when the step $d\vec{l}$ is taken in the direction of $\vec{\nabla} T$.
The maximum change in T in a step of length $|d\vec{l}|$ is then,

$$dT_{max} = |\vec{\nabla} T|\,|d\vec{l}|.$$

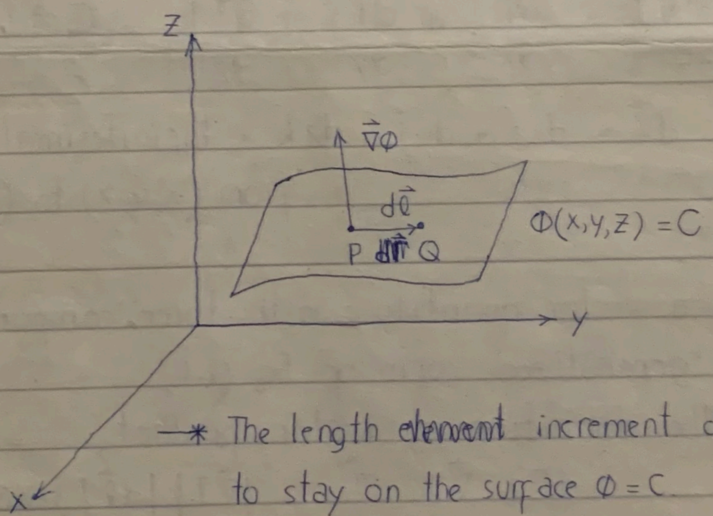
So the magnitude of the vector $\vec{\nabla} T$ is $|\vec{\nabla} T| = \dfrac{dT_{max}}{|d\vec{l}|}$, i.e., the maximum rate of change of T. Thus:

The gradient $\vec{\nabla}T$ points in the direction of maximum increase of the function T.
and
The magnitude $|\vec{\nabla}T|$ gives the slope (rate of increase) along this maximal direction.

OR

We define the vector that represents both the magnitude and the direction of the maximum space rate of increase of a scalar as the gradient of that scalar.

A Geometrical Interpretation:
Consider P and Q to be two points on a surface $\Phi(x,y,z) = C$, where C is constant. If $\Phi(x,y,z)$ is a potential, the surface is an equipotential surface. These points are chosen so that Q is




* The length element increment $d\vec{l}$ has to stay on the surface $\Phi = C$

a distance $d\vec{l}$ from P. Then moving from P to Q, the change in $\Phi(x,y,z) = C$ is given by

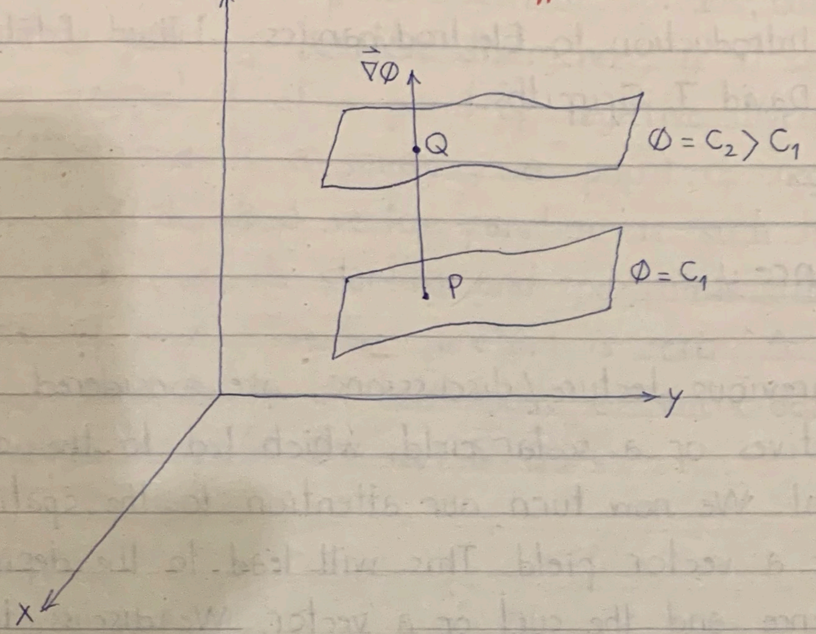
$$d\Phi = (\vec{\nabla}\Phi)\cdot d\vec{l} = 0 \quad \Rightarrow \quad \vec{\nabla}\Phi \perp d\vec{l}$$

as we stay on the surface $\Phi(x,y,z) = C$. This shows that $\vec{\nabla}\Phi$ is perpendicular to $d\vec{l}$. $d\vec{l}$ may have any direction from P as long as it stays on the surface $\Phi = C$, with point Q being restricted to the surface. $\vec{\nabla}\Phi$ is seen as normal to the surface $\Phi$ = constant.

If we now permit $d\vec{l}$ to take us from one surface $\Phi = C_1$ to an adjacent surface $\Phi = C_2$,

$$d\phi_{max.} = C_2 - C_1 = \Delta C = (\vec{\nabla}\phi)\cdot d\vec{l}$$

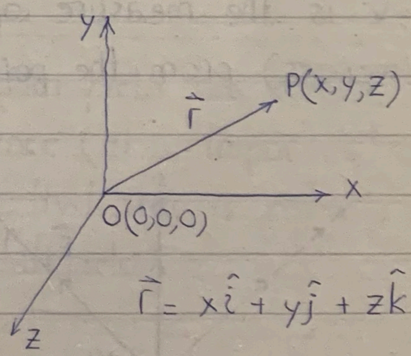
$$\Rightarrow \vec{\nabla}\phi \parallel d\vec{l}$$




For a given $|d\vec{l}|$, the change in the scalar function $\phi$ is maximized by choosing $d\vec{l}$ parallel to $\vec{\nabla}\phi$. This identifies $\vec{\nabla}\phi$ as a vector having the direction of the maximum space rate of change of $\phi$.

Example 1.3 :

Find the gradient of $r = \sqrt{x^2+y^2+z^2}$ (the magnitude of the position vector)




$$\vec{\nabla} r = \frac{\partial r}{\partial x}\hat{i} + \frac{\partial r}{\partial y}\hat{j} + \frac{\partial r}{\partial z}\hat{k}$$

$$= \frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-1/2}(\cancel{2}x)\,\hat{i} + \cdots$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \cdots$$

$$= \frac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}} = \frac{\vec{r}}{|\vec{r}|} = \hat{r} \quad ; \; |\vec{\nabla} r| = \hat{r}\cdot\hat{r} = 1$$

It shows that the distance from origin increases most rapidly in the radial direction, and that its rate of increase in that direction is 1.

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