$2^1 = 2$

$2^2 = 4$

$2^3 = 8 \rightarrow$ Number

Base ← $2^3$ → exponent

$\log_2 8 = 3$

Base

$2^3 = 8$

# Types of Logarithm.

## Common log

$\log_{10} x = \log x$

base 10 — written without base

## Natural log.

Log with base 'e'

$\log_e x$ $\quad e \simeq 2.71828$

$\log_e x = \ln x$

**Example:**

$\log_{10} 1000 = 3$

power کو base (10) پر لے جا کر log کو غائب کر دیتا ہے۔

e.g., $10^3 = 1000$

The log is exponent

$$a^{x} = y \iff \log_a y = x$$

Base

e.g.,

log is exponent

$$2^{5} = 32 \text{ means } \log_2 32 = 5$$

Base

## ⇒ Some Rules or Properties for Logarithm.

1) $\log_e = 2.303 \log_{10}$

2) $\log(a \times b) = \log a + \log b$

3) $\log \frac{a}{b} = \log a - b \log b$

4) $\log_a b = b \log a$

5) $\log_{10} 10 = 1 = \log_e e$ or $\ln e = \log_e e = 1$

# Logarithmic Functions

Logarithmic function is a function of the form which is a y equals to the log of x, base b or log base 'b' of x.

Mathematically.

$$y = \log_b x$$

$$y = \log_{10} 2 = 0.30$$

$$y = \log_{10} 3 = 0.477$$

$y = \log_2 8$ ii) the value of base is 2 & x is 8 so

$y = ?$

(4)

NOW $\log_2 8 = ?$

∴ [redacted]

[redacted] = [redacted]

$(2)^3 = 8$

$\log_2 8 = 3$ so $y = 3$

→ Use of logarithmic Function:

a) For solving exponential equations

b) In Chemistry measure of acidity and alkalinity (pH measurements)

$pH = -\log [H^+]$

$pOH = -\log [\bar{O}H]$

* Find pH of 0.001M $HNO_3$.

$[H^+] = 0.001M = 10^{-3}M.$

$pH = -\log [H^+]$

$= -\log 10^{-3}$

$= -3 (-\log 10)$ ∴ $\log 10 = 1$

$= 3$

* If we want to convert $\ln x$ / $\log_e x$ (Natural log) into common log i.e. $\log_{10} x$ e.g.:

$$\log_e x = 2.303 \log_{10} x$$

We have to use **Base change log rule** which is:

$$\log_a b = \frac{\log_c b}{\log_c a}$$

→ Base change

If we want to convert $\log_e x$ to $\log_{10} x$

so,

$$\log_e x = \frac{\log_{10} x}{\log_{10} e}$$

$\therefore b = x$
$a = e$

$$= \frac{\log_{10} x}{\log_{10} 2.7182} = \frac{1}{0.4342} = 2.303$$

$\log_e x = 2.303 \log_{10} x$

$\log_e x$ → $\ln x$

$\log_{10} x$ → common log

# Differentiation

Rate of change of function with respect to one of its variable.

## Rules for Differentiation.

### Rule #01

$y = 2 \rightarrow$ constant no.

$$\frac{dy}{dx} = f'(x) = 0$$

### Rule #02

$y = 1x^1$

$$\frac{dy}{dx} = 1 \times 1\, x^{1-1} \Rightarrow 1x^0 \qquad \because x^0 = 1$$

$$1(1) = 1$$

So

$$\frac{dy}{dx} = x = 1$$

* $y = 2x^1$

$$\frac{dy}{dx} = 2 \times 1\, x^{1-1}$$

$$= 2x^0$$

$$= 2(1) = 2$$

Direct

$$\frac{dy}{dx} = 2x$$

$$= 2(1)$$

$$= 2$$

★ $y = 5x^2$

$$\frac{dy}{dx} = 5 \times 2\, x^{2-1}$$

$$= 10x^1 \quad \text{or} \quad 10x$$

★ $y = f'(x) = 5x^2 + 2x + 5$

$$= 10x + 2(1) + 0$$

$$= 10x + 2$$

## Rule # 03

## Product Rule

$u \quad v$

$f(x) = (x^2 - 5)(x - x^3)$

derivative of $u$ ← $uv = u'v + uv'$ → derivative of $v$

$f'(x) = 2x(x - x^3) + (x^2 - 5)(1 - 3x^2)$

$= 2x^2 - 2x^4 + x^2 - 3x^4 - 5 + 15x^2$

$= 2x^2 + x^2 + 15x^2 - 2x^4 - 3x^4 - 5$

$f'(x) = -5x^4 + 18x^2 - 5$

# Rule #04

## Quotient Rule

$$y = f'(x) = \frac{10-x}{x^2+1} \quad \begin{matrix} \to u \\ \to v \end{matrix}$$

$$\frac{u}{v} = \frac{u'v - uv'}{v^2}$$

$$f'(x) = \frac{(-1)(x^2+1) - (10-x)(2x)}{(x^2+1)^2}$$

$$= \frac{-x^2(x^2+1) - (10-x)(2x)}{(x^2+1)^2}$$

$$= \frac{-x^2-1 - (20x-2x^2)}{(x^2+1)^2}$$

$$= \frac{-x^2-1-20x+2x^2}{(x^2+1)^2}$$

$$\boxed{f'(x) = \frac{x^2-20x-1}{(x^2+1)^2}}$$

# Rule # 05

## Power Function.

$$y = f(x) = (7x^4 - 5x - 9)^3$$

$$(x)^3 \Rightarrow 3x^{3-1} = 3x^2$$

$$f(x) = 3(7x^4 - 5x - 9)^2 \cdot (28x^3 - 5)$$

$$= (84x^3 - 15)(7x^4 - 5x - 9)^2$$

* $$f(x) = \sqrt[3]{x^3 - 2x + 5}$$

$$\therefore \sqrt{} = \frac{1}{2}$$

$$= (x^3 - 2x + 5)^{1/3}$$

$$\sqrt[3]{} = \frac{1}{3}$$

$$= \frac{1}{3}(x^3 - 2x + 5)^{1/3 - 1} \cdot (3x^2 - 2)$$

$$\sqrt[4]{} = \frac{1}{4}$$

$$= \frac{1}{3}(x^3 - 2x + 5)^{-2/3} \cdot (3x^2 - 2)$$

$$\vdots$$

$$\sqrt[n]{} = \frac{1}{n}$$

$$f(x) = \frac{3x^2 - 2}{3(x^3 - 2x + 5)^{2/3}}$$

→ Attempt following questions by applying rules of differentiation.

1) $y = 10x^2$

2) $y = 20x^{-4} + 9$

3) $y = 6x^3 - 9x + 4$

4) $y = 2t^4 - 10t^2 + 13t$

5) $y = \frac{6}{x^2} + 2x$

6) $t = (4t^2 - t)(t^3 - 8t^2 + 12)$

7) $y = \frac{6x^2}{2 - x}$

# Attempt following Questions.

* Find the value of 'a'

1 $\log_a 8' = 4$

2 $\log_a 64 = 1$

3 $\log_a 36 = 2$

* Find the value of 'x'

4 $\log_5 (x-7) = 1$

$x-1$

5 $\log(x+5) + \log(x-5) = 4\log 2 + 2\log 3$

6 $\log(75/16) - 2\log(5/9) + \log(32/243)$ in terms of $\log 2$

7 $\log_{10}(x-10) = 1$