Applications of Propositional Logic

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Bi-conditional statement

- p if and only if q ($p \leftrightarrow q$)
- $p \leftrightarrow q$ has the same truth value as $(p \rightarrow q) \wedge (q \rightarrow p)$
- Example:

P="You can take the flight"

Q="You buy a ticket." Then $p \leftrightarrow q$ is the statement

"You can take the flight if and only if you buy a ticket."

The Truth Table for the Bi-conditional $p \leftrightarrow q$.		
p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Tables of Compound Propositions

- Example: Construct the truth table of the compound proposition

  $(p \vee \neg q) \longrightarrow (p \wedge q)$.

The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators

Precedence of Logical Operators.	
Operator	Precedence
¬	1
∧ ∨	2 3
→ ↔	4 5

Translating English Sentences

- Example: How can this English sentence be translated into a logical expression?

  "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

- Solution: Let q, r, and s represent "You can ride the roller coaster," "You are under 4 feet tall," and "You are older than 16 years old." The sentence can be translated into:

  $$(r \wedge \neg s) \rightarrow \neg q.$$

Translating English Sentences

- **EXAMPLE**
- “You can access the Internet from campus only if you are a computer science major or you are not a freshman.”
- ***Solution***
- In particular, we let *a*, *c*, and *f* represent “You can access the Internet from campus,” “You are a computer science major,” and “You are a freshman,” respectively

$$a \to (c \vee \neg f).$$

Logic and Bit Operations

- Bit operation – replace true by 1 and false by 0 in logical operations.

Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Bit string

- Example: Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of the bit string 01 1011 0110 and 11 0001 1101.

Solution:

01 1011 0110

11 0001 1101

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11 1011 1111 bitwise *OR*

01 0001 0100 bitwise *AND*

10 1010 1011 bitwise *XOR*

Logic Circuits

- 1938 by Claude Shannon

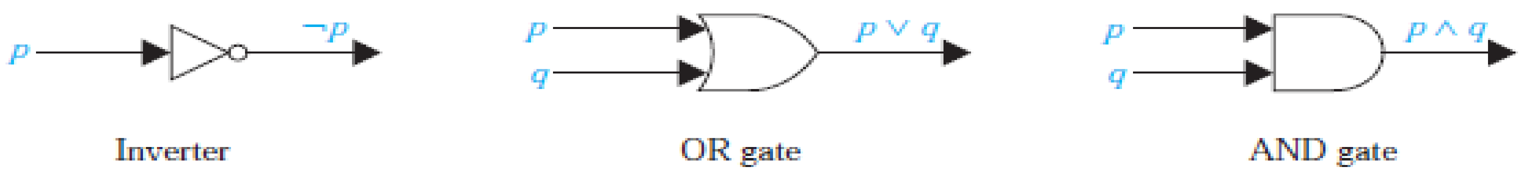

FIGURE 1 **Basic logic gates.**

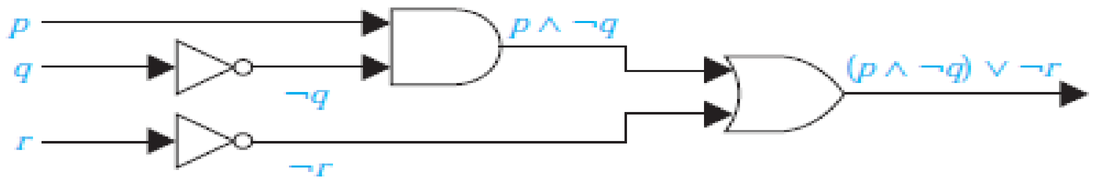

FIGURE 2 **A combinatorial circuit.**

Propositional Equivalences

- Tautology
- Contradiction
- contingency

Examples of a Tautology and a Contradiction.			
p	$\neg p$	$p \lor \neg p$	$p \land \neg p$
T	F	T	F
F	T	T	F

Propositional Equivalences

- The notation $p \equiv q$ denotes that p and q are logically equivalent.

- Compound propositions that have the same truth values in all possible cases are called logically equivalent.
- Example: Show that $\neg p \vee q$ and $p \rightarrow q$ are logically equivalent.

Truth Tables for $\neg p \vee q$ and $p \rightarrow q$.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Propositional Equivalences

TABLE 5 A Demonstration That $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ Are Logically Equivalent.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

De Morgan laws

TABLE 2 De Morgan's Laws.

$\neg(p \land q) \equiv \neg p \lor \neg q$
$\neg(p \lor q) \equiv \neg p \land \neg q$

TABLE 3 Truth Tables for $\neg(p \lor q)$ and $\neg p \land \neg q$.

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T