

$$\lambda' - \lambda = 4\pi \lambda_e \sin^2 \theta / 2$$

$$\lambda_e = \frac{h}{m c}$$

→ change in wavelength  $\Delta\lambda = \lambda' - \lambda$  depends on the angle of scattering angle  $\theta$  and do not on the frequency.

→ This is what happens in particle collisions establishing the fact that radiations ~~beave~~ behaves like a beam of particle.

## Wave Aspect of Particles:

### a) de-Broglie's Hypothesis: Matter Waves:

→ A photon of frequency  $\omega$  has a momentum

$$p = \frac{E}{c} = \frac{h\omega}{c}$$

or in terms of wavelength.



$$p = h k = \frac{2\pi h}{\lambda}, \quad k = \frac{2\pi}{\lambda}$$

The formula.  $\lambda = \frac{2\pi h}{p} = \frac{h}{p}$  - photon wave-length

$$\lambda = \frac{h}{p} = \frac{h}{mv} - \text{de-Broglie}$$

applies to material particles as well as photons.

→ Experimental Confirmation of de-Broglie's Hypothesis:

Davisson-Germer Experiment:

→ used X-ray diffraction method to study the wave behaviour of a particle.

→ An  $e^-$  beam is scattered at the surface of nickel crystal - as a result diffraction patterns are obtained.



→ The diffraction patterns are exactly similar to those obtained in X-ray diffraction

The condition is:

$$\frac{nh}{d\sqrt{\lambda m e}} = \sqrt{V} \sin \alpha$$

→ Heisenberg's Uncertainty Principle

→ Classical concept: position and momentum are independent of each other and can be determined exactly at the same time.

→ Quantum Theory: It is impossible to know the exact position and momentum of a particle at the same time.

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

If

then  $\Delta x = 0$  — position exactly known.  
 $\Delta p_x = \infty$



→ The uncertainty in the position of an  $e^-$  is of the order of wavelength of light i.e.  $\Delta x \sim \lambda$

→ Uncertainty in the momentum of an  $e^-$  is the order of momentum of photon, when a photon of momentum  $\frac{2\pi h}{\lambda}$  hits the  $e^-$ . i.e.  $\Delta p \sim \frac{2\pi h}{\lambda}$

Since,  $\Delta x \Delta p \sim \lambda \cdot \frac{2\pi h}{\lambda}$

$$\Delta x \Delta p \sim 2\pi h$$

$$\boxed{\Delta x \Delta p \geq h}$$

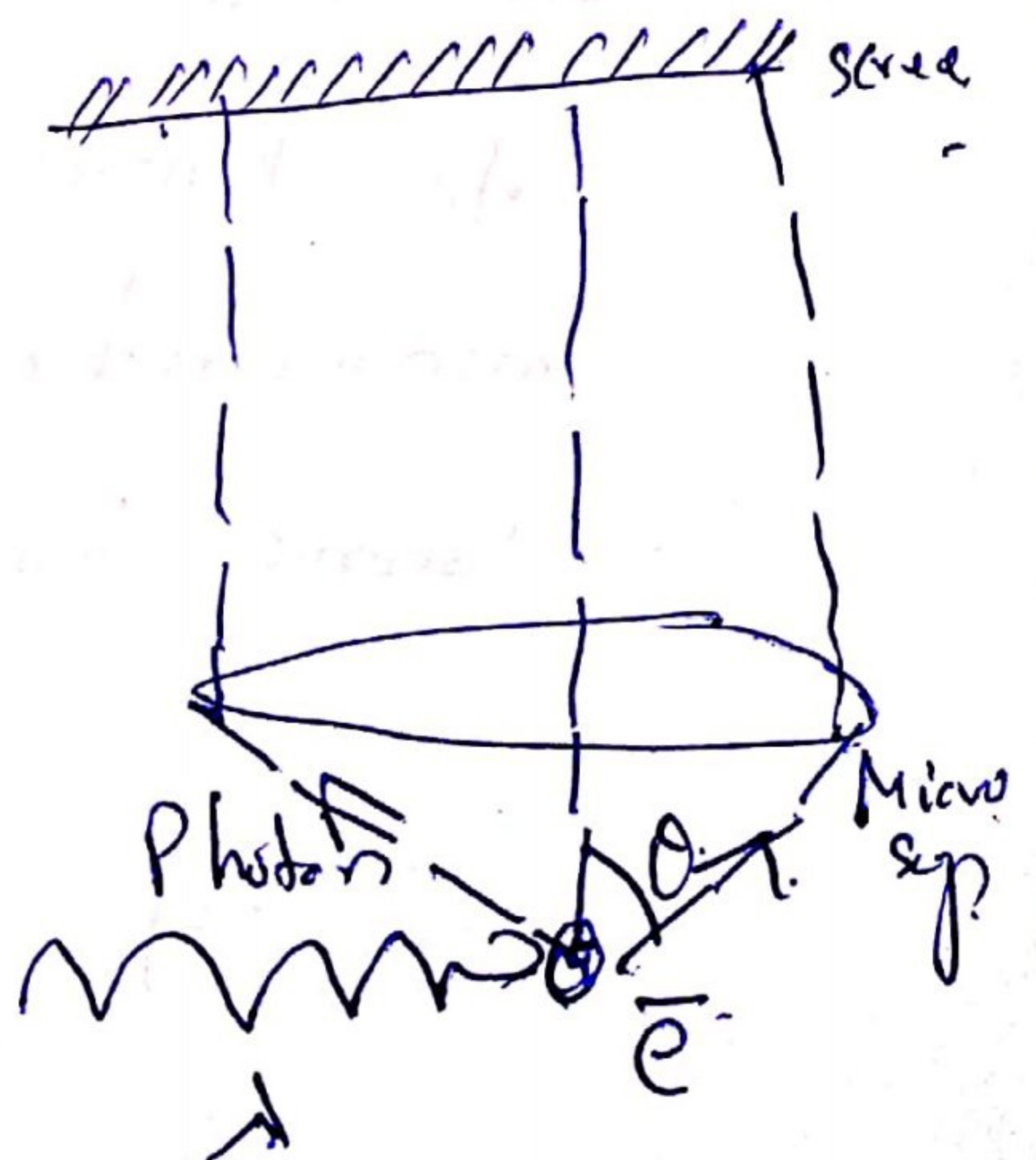
More precise. Picture the Heisenberg

microscope,

The change in position,

$$\Delta x \approx \frac{\lambda}{\sin \alpha}$$

$$\Delta p \approx \frac{h}{\lambda} \sin \alpha$$





Hence,

$$\Delta x \Delta p_x \sim h = 2\pi\hbar$$

$$\boxed{\Delta x \Delta p_x \gtrsim \hbar}$$

Wave fn.  $\Psi$

"A matter wave is represented by a complex variable quantity  $\Psi(x,t)$  called the wavefn."

Normalized to unity:

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$$

$|\Psi(x,t)|^2 \xrightarrow{\text{vanish, when } x \rightarrow \pm\infty}$   
 $\downarrow$   
called norm.  
 $\downarrow$   
probability.

Schrodinger wave Equation:

Basic Equ. of Quantum mechanics which describes propagation of material waves.



$$E = \frac{p^2}{2m} \quad \text{Classical. Equ. of motion}$$

→ In quantum mechanics, particle is describe by wavefn

$$\psi(x, t) = A e^{i(kx - \omega t)}$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = i\hbar \frac{\partial \psi}{\partial t}$$

↓  
time-dependent S.W.E

1D. - free particle

$$\hat{H} \psi = E \psi$$

$$\hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}, \quad E = i\hbar \frac{\partial}{\partial t}$$

classical Hamiltonian.  $\leftarrow \frac{p^2}{2m}$

$$p \rightarrow -i\hbar \frac{\partial}{\partial x}$$

$$E \rightarrow i\hbar \frac{\partial}{\partial t}$$