Particle in a potential

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t).$$

Inner product

$$\phi, \psi \in L^2(\mathbb{R}^3).$$

$$(\phi, \psi) = \int_{-\infty}^{\infty} \phi^*(x,t)\,\psi(x,t)\,dx$$

$$(\phi, \psi) = (\psi, \phi) = 0$$

↓

orthogonality.

$$(\phi, \psi) = (\psi, \phi) = 1$$

↓

Normalized

Operator.

An operator is a transformation which takes any function in a function space in the same space.

$\hat{A} : L^2(\mathbb{R}^3) \longrightarrow L^2(\mathbb{R}^3)$

Commutator:

$[\hat{A}, \hat{B}] = 0$ — takes simultaneous eigenfns.

But in general

$[\hat{A}, \hat{B}] \neq 0.$

Eigenvalues.

$$\hat{A}\,\psi = \phi$$

But.

$$\hat{A}\psi = \lambda\,\psi$$

λ — eigenvalue, ψ — eigenfn.

$$\frac{d}{dx}\left(e^{\alpha x}\right) = \alpha\left(e^{\alpha x}\right)$$

$$\frac{d^2}{dx^2}\,\underline{\sin 4x} = -16.\,\underline{\sin 4x}.$$

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Degenerate Eigenvalues

$$\frac{d^2}{dx^2} \cos ax = -a^2 \cos ax$$

$$\frac{d^2}{dx} \sin ax = -a^2 \sin ax$$

$-a^2 \rightarrow$ degenerate eigenvalues.

Simultaneous Eigenfns.

$$\hat{A}\psi = \lambda\psi$$
$$\hat{B}\psi = \mu\psi$$

$\downarrow$

compatible operators

$\downarrow$

~~are~~ commute with each other

$$[\hat{A}, \hat{B}] = 0$$

$\hat{H}$ermitian operator:

$$\hat{A}^{+} = \hat{A}$$

$\Downarrow$

eigenvalues are real!

2nd

Many Body problems.

Let us assume a ~~sy~~ situation, where we would like to describe the properties of some well defined collections of atoms. One can think of an isolated molecule ~~of~~ or the atoms defining the crystal of an intersting mineral. The most fundmental things, that one would like to know about these atoms is their energy and more importantly, How their energy changes if we move the ~~nucleus~~ atoms around. To define where an atom is, we need to define both where its nucleus and where the atoms are. A key observation is the quantum mechanical approach to solve this issue.

The many body Problem is a general term which contain a lot of Physical Problems, ~~which~~ ~~to~~ ~~to~~ to found ~~give~~ provide Properties of the microscopic Particles systems — made of a large number of interacting Particles.

→ Microscopic — ~~at~~ the nono-scale or less than, especially when you are dealing with an electron systems, the quantum mechanics give the accurate description.

→ Large number of atoms means more than 5 upto infinity (not practially acceptable), Homogeneous systems, Periodic system (ordered systems)