Many-body situation.

If a body of mass "m_1" moving in a force field, which is uncoupled. (Their motion ~~with~~ will not be changed by the presence of other balls).

By knowing the force on the ball, at every point in space and time one can calculate trajectory of the body. So this is the single-body problem moving in an external field. by using Newtons Law.

It does not matter that the field is varying as long as the variation does not depend upon the position or velocity of the ball.

Now take the other body "m_2" and attached is with "m_1" by spring. There is no force b/w the balls, naturally, But if some one stretched or compressed the spring, so mass "m_1" will exert a force on "m_2" and vice versa. So now, one can not solve the trajectories of the ball A and B respectively. But the motions of the balls is still described perfect well by Newtonian mechanics.

But the motion of m_1 is linked with m_2

Their equation of motions are Coupled and must be solved simultaneously.

It is true that the balls still holds the Newton's Law, but the solution of the equation will not be easily calculate, because of the Coupled motion. This thing leads to many-body problem.

However, it can be that if the ~~balls~~ masses are not Connected by springs, then what will be happen?

The answer is, ~~bodyies~~ bodies are like the electrons moving as a single Particles untill they collide with each other. Now the problem is easy, first we will solve the equation of motion for a single particles individually. Thus we dont need to solve any equation simultaneously. untill such times as they instantaneously scatter off each other.

But, this is not correct 100% that to describe these particles simply as single particles ~~that~~ because, in this case we are ignoring the effect of interaction and scattering b/w them when they collide to each other — so these are the quasi-particles.

The appropriate Hamiltonian.

A solid can be described as a collection of heavy, positively-charged particles (nuclei) and lighter, negatively-charged particles (electrons). Each nucleus has a charge of "Z e", where "Z" is the atomic number and "e" is the electronic charge. A system with N nuclei thus leads to a problem of N+ZN interacting particles. This is a many-body problem, and demands a quantum mechanical approach because of the small mass of the particles — called quantum many-body problem

(2)

The exact many particle Hamiltonian for this many body system can be written as.

$$\hat{H}_{Tot} = \hat{T}_e(\vec{r}_i) + \hat{T}_N(\vec{R}_\lambda) + \hat{V}_{ee}(\bar{r}_i, \bar{r}_j) + \hat{V}_{NN}(\hat{R}_\lambda, \hat{R}_\sigma) + \hat{V}_{eN}(\hat{r}_i, \vec{R}_\lambda) \quad (1)$$

where.

$\hat{T}_e(\vec{r}_i) = -\frac{\hbar^2}{2} \sum_i \frac{\nabla^2_{\bar{r}_i}}{m_e}$ is the kinetic energy operator for the electron, & m_e is the mass of the electron at position $\bar{r}_i$

$\hat{T}_N(\vec{R}_\lambda) = -\frac{\hbar^2}{2} \sum_\lambda \frac{\nabla^2_{\bar{R}_\lambda}}{M_N}$ – kinetic energy operator for the nucleus. M_N – mass at $\vec{R}_\lambda$

$\hat{V}_{ee}(\bar{r}_i, \bar{r}_j) = +\frac{1}{8\pi\epsilon_0} \sum_{i \neq j} \frac{e^2}{|\bar{r}_i - \bar{r}_j|}$, Coulomb interaction b/w electrons and electrons.

$\hat{V}_{NN} = \frac{1}{8\pi\epsilon_0} \sum_{\lambda \neq \sigma} \frac{e^2 Z_\lambda Z_\sigma}{|\bar{R}_\lambda - \bar{R}_\sigma|}$, b/w nuclei and nuclei

where.

Z_λ is the charge on nucleus at position $\bar{R}_\lambda$ and Z_σ " " " $\bar{R}_\sigma$

$$V_{eN} = -\frac{1}{4\pi\epsilon_0} \sum_{i,j} \frac{e^2 Z_\lambda}{|R_\lambda - r_i|} \quad \text{— b/w. } \bar{e} \text{ and Nucleus}$$

~~It is important to note that the kinetic and electron-electron terms in our~~

We can not solve this Hamiltonian exactly for the ~~interac~~ interaction particles system, In order to calculate the physically acceptable solutions which are the approximate eigenstates, we will use some approximations.

Born-Oppenheimer Approximation.

In the light of the Born-Oppenheimer approximation, we assume that the nuclei are much heavier, and therefor much slower than electrons. Hence nuclei can stay effectively frozen at fixed positions with only electrons being considered as mobile. The nuclei are deprived from this status, and reduced to a given source of positive charge, they become external to the electron cloud.

After applying this approximation, Now we are dealing with a collection NZ interacting negative particles moving in the (given or external) potential of the nuclei.

Thus as the Consequence of Born-Oppenheimer approximation on equ(1); the kinetic energy of the nuclei is zero and the ~~first~~ 2nd term disappears. The potential $\hat{V}_{NN}(\bar{R}_I)$ term also reduced to a Constant. We are then left with the kinetic energy of the electron gas, the potential energy due to electron-electron interaction and the potential energy of the electrons in the external potential of the nuclei. Therefore, the Hamiltonian operator $\hat{H}$, equ(1) now has only three terms: The kinetic energy of the electrons $(\hat{T}_e)$, the electron-electron interaction $(\hat{V}_{ee})$ and the electron-nuclear $(\hat{V}_{eN})$ interaction.