# INTEGARATION:-

"Reverse of derivatives".

or

"Process of finding anti-derivatives".

**DERIVATIVE**

1) $\frac{d}{dx}x^n = nx^{n-1}$

2) $\frac{d}{dx}\ln x = \frac{1}{x}$

3) $\frac{d}{dx}e^x = e^x$

4) $\frac{d}{dx}\sin x = \cos x$

5) $\frac{d}{dx}\cos x = -\sin x$

6) $\frac{d}{dx}(-\cos x) = \sin x$

**INTEGRATION**

① $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

② $\int \frac{1}{x} dx = \ln x + c$

③ $\int e^x dx = e^x + c$

④ $\int \cos x\, dx = \sin x + c$

⑤ $\int \sin x\, dx = -\cos x + c$

or

⑥ $\int -\sin x\, dx = \cos x + C$

⑦ $\int (u \pm v)\, dx = \int u\, dx \pm \int v\, dx$

⑧ $\int c x^n dx = c\int x^n dx$

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1) $\int 3x^2 dx$

$= 3\int x^2 dx$

$= 3 \frac{x^{2+1}}{2+1}$

$= \cancel{3} \frac{x^3}{\cancel{3}}$

$= x^3 + c$

2) $\int (3x^2+7x) dx$

$= 3\int x^2 dx + 7\int x dx$

$= 3 \frac{x^{2+1}}{2+1} + 7 \frac{x^{1+1}}{1+1}$

$= \cancel{3} \frac{x^3}{\cancel{3}} + 7 \frac{x^2}{2}$

$= x^3 + \frac{7}{2} x^2 + C$

3) $\int (3x^2 + 7x + 100) dx$

$= 3\int x^2 dx + 7\int x dx + 100\int 1 dx$

$= \int 1 dx$ $\qquad \int \frac{1 d\cdot}{x}$ $\qquad \int \frac{dy}{y}$

$= \int x^0 dx$

$= \frac{x^{0+1}}{0+1} = x + c$

$$= x^3 + \frac{7}{2}x^2 + 100x + C$$

| | |
|---|---|
| $y = 6x^2 + 3x + 7$ | $y = 6x^2 + 3x + 100$ |
| Derivative | |
| $\frac{dy}{dx} = 12x + 3$ | $\frac{dy}{dx} = 12x + 3$ |

Differential:

| | |
|---|---|
| $dy = (12x+3)dx$ | $dy = (12x+3)dx$ |
| $\int 1 \cdot dy = \int (12x+3)dx$ | $\int 1 \cdot dy = \int (12x+3)dx$ |
| $\int 1 \cdot dy = 12\int x^{dx} + 3\int 1 \cdot dx$ | $\int 1 \cdot dy = 12\int x^{dx} + 3\int 1 \cdot dx$ |
| $y = 12 \cdot \frac{x^{1+1}}{1+1} + 3x$ | $y = 12\frac{x^{1+1}}{1+1} + 3x$ |
| $y = \cancel{12} \cdot \frac{x^2}{\cancel{2}} + 3x$ | $y = 12\frac{x^2}{2} + 3x$ |
| $y = 6x^2 + 3x + C$ | $y = 6x^2 + 3x + C$ |

# INTEGARATION:

## INDEFINITE INTEGARATION:

Means integauation without any final & initial state

## FIRST THEOREM:

Problem 1: (quotient style)

$$\int \frac{1}{x}\,dx = \ln x + c$$

e.g; $\int \frac{1}{(a-x)^1}\,dx$

$$= -\int \frac{-1}{(a-x)^1}\,dx \qquad \left(\because \frac{d}{dx}(-x) = -1\right)$$

$$= -\ln(a-x) + c$$

e.g; $\int \frac{1}{(a-2x)^1}\,dx$ $\qquad \because \frac{d}{dx}(-2x) = -2$

$$= -\frac{1}{2}\int \frac{-2}{(a-2x)}\,dx$$

$$= \frac{-1}{2} \ln(a - 2x) + c$$

**THEOREM 2**: (product style)

$$\left[ \int \frac{1}{(a-x)^1} dx \quad \text{or} \quad \int (a-x)^{-1} dx \right] \rightarrow \text{theorem } 1$$

e.g; 1) $\int \frac{1}{(a-x)^2} dx$

$$= -\int \frac{-1}{(a-x)^2} dx$$

$$= \underbrace{-\int (a-x)^{-2} (-1)\, dx}_{\text{power rule}}$$

By applying power rul.

$$= -\frac{(a-x)^{-2+1}}{-2+1} + c$$

$$= -\frac{(a-x)^{-1}}{-1} + c$$

$$= 1/_{a-x} + c \quad \text{— Ans.}$$

**2)** $$\int \frac{1}{(a-x)^3}\, dx$$

$$= -\int \frac{-1}{(a-x)^3}\, dx$$

$$= -\int (a-x)^{-3}(-1)\, dx$$

power rul:

$$= -\cancel{\int} \frac{(a-x)^{-3+1}}{-3+1} + c$$

$$= \cancel{-} \frac{(a-x)^{2}}{\cancel{-}2} + c$$

$$= \frac{1}{2(a-x)^2} + c \quad \text{— Ans}$$

# Indefinite integration:

## → Zero order reactions:—

$$A \longrightarrow \text{products}$$

$t = 0$    $a$    $0$

$t = eq$    $a - x$    $x$

$$\frac{dx}{dt} \propto [A]^0$$

$$\frac{dx}{dt} = k[A]^0$$

$\because [A]^0 = 1$

$$\frac{dx}{dt} = k(a-x)^0$$

$$\frac{dx}{dt} = k$$

$$\int dx = k\int dt$$

$$x = kt + C$$

when $t = 0$, $x = 0$
then $C = 0$

$$\boxed{x = kt}$$

# Definite integaration:

Integaration having final & initial state is called definite integaration.

$$\int_0^x dx = k\int_0^t dt$$

$$|x|_0^x = k\,|t|_0^t$$

$$(x-0) = k(t-0)$$

$$\boxed{x = kt}$$

## First Order:

Indefinite integaration:

$$\frac{dx}{dt} = k(a-x)$$

$$\frac{dx}{(a-x)} = k\,dt$$

$$\int \frac{1}{(a-x)}\,dx = k\int dt$$

$$-\int \frac{-1}{(a-x)}\,dx = kt + c$$

$$-\ln(a-x) = kt + C \quad \text{—①}$$

when $t=0$, $x=0$

$$-\ln(a-0) = k(0)+C$$

$$\boxed{-\ln a = C}$$

Put in eq. (1)

$$-\ln(a-x) = kt - \ln a$$

$$-\ln(a-x) + \ln a = kt$$

$$\boxed{\ln \frac{a}{a-x} = kt}$$

Definite integration:

$$-\int_0^x \frac{-1}{a-x}\,dx = k\int_0^t dt$$

$$-\left|\ln(a-x)\right|_0^x = k\,|t|_0^t$$

$$-\left[\ln(a-x) - \ln(a-0)\right] = k(t-0)$$

$$-\ln(a-x) + \ln(a) = kt$$

$$\boxed{\ln \frac{a}{a-x} = kt}$$

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# Second ORDER: when initial conc. are same

$$\frac{dx}{dt} = k[A][B] \quad \text{when}$$

$$\frac{dx}{dt} = k(a-x)(a-x)$$

$$\frac{dx}{dt} = k(a-x)^2$$

$$\int \frac{dx}{(a-x)^2} = k\int dt$$

$$\cancel{\int (a-x)^{-1} dx = k\int dt}$$

$$-\int (a-x)^{-2}(-1)\,dx = k\int dt$$

$$\frac{-(a-x)^{-2+1}}{-2+1} = kt + c$$

$$\frac{+1\,(a-x)^{-1}}{+1} = kt + c$$

$$\boxed{\frac{1}{a-x} = kt + c}$$

# THIRD ORDER: when initial conc. are same

$$\frac{dx}{dt} = k(a-x)^3$$

$$\frac{dx}{(a-x)^3} = k\,dt$$

$$\int \frac{dx}{(a-x)^3} = k\int dt$$

$$-\int (a-x)^{-3}(-1) = k\int dt$$

$$\frac{-(a-x)^{-3+1}}{-3+1} = kt + C$$

$$+\frac{(a-x)^{-2}}{+2} = kt + c$$

$$\boxed{\frac{1}{2(a-x)^2} = kt + C}$$

# TECHNIQUES OF INTEGARATION:

1) Integaration by partial fraction method:

Second order when initial conc. are different.

$$\frac{dx}{dt} = k(a-x)(b-x)$$

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$$\int \frac{dx}{(a-x)(b-x)} = k\int dt \quad -①$$

By partial fraction methode:

$$\frac{1}{(a-x)(b-x)} = \frac{A}{a-x} + \frac{B}{b-x} \quad -②$$

1- if $x = x'$ then $\frac{1}{(a-x')(b-x)}$ this is linear non-repeated partial fraction

2- if $\frac{1}{(a-x')^2(b-x)}$ then it is linear repeated fraction method means $(a-x)(a-x)(b-x)$

3- if $\left(\frac{1}{(a-x^2)(b-x)}\right)$ then it is quadratic non-repeated

4- if $\frac{1}{(a-x^2)^2(b-x)}$ then it is

quadratic repeated fraction method means $(a-x^2)(a-x^2)(b-x)$

Here we will discuss linear partial fraction method

By multiplying eq ② with $(a-x)(b-x)$ on both sides we get.

$$1 = A(b-x) + B(a-x)$$

$\Rightarrow$ when $a-x=0 \Rightarrow x=a$

$$1 = A(b-a) + B(a-a)$$
$$1 = A(b-a) + B(0)$$

$$A = \frac{1}{b-a} = \frac{-1}{a-b} \quad — ③$$

$\Rightarrow$ when $b-x=0 \Rightarrow x=b$

$$1 = A(b-b) + B(a-b)$$
$$1 = A(0) + B(a-b$$
$$B = \frac{1}{a-b} \quad — ④$$

Put ③ & ④ in eq ②.

$$\frac{1}{(a-x)(b-x)} = \frac{-1}{(a-b)(a-x)} + \frac{1}{(a-b)(b-x)}$$

$$\frac{1}{(a-x)(b-x)} = \frac{1}{a-b}\left[\frac{-1}{(a-x)} + \frac{1}{(b-x)}\right]$$

put eq (5) into (1)

$$\frac{1}{(a-b)}\left[\int -\frac{1}{a-x} + \frac{1}{b-x}\right]dx = k\int dt$$

$$\frac{1}{a-b}\left[\int -\frac{1}{(a-x)}dx + \int \frac{1}{b-x}dx\right] = k\int dt$$

$$\frac{1}{a-b}\left[\int -\frac{1}{(a-x)}dx - \int -\frac{1}{b-x}\right] = k\int dt$$

$$\frac{1}{a-b}\left[\ln(a-x) - \ln(b-x)\right] = kt + c$$

$$\boxed{\frac{1}{(a-b)}\left[\ln\frac{(a-x)}{(b-x)}\right] = kt + c}$$

## 3rd ORDER Rex.

when initial conc. is diffrent

$$\frac{dx}{dt} = k(a-x)(b-x)(c-x)$$

$$\frac{dx}{(a-x)(b-x)(c-x)} = k\,dt \quad \text{—(1)}$$

partial fraction method

$$\frac{1}{(a-x)(b-x)(c-x)} = \frac{A}{a-x} + \frac{B}{b-x} + \frac{C}{c-x} \quad \text{—(2)}$$

$$\frac{1}{c}{}^{2}$$

$$1 = A(b-x)(c-x) + B(a-x)(b-x) + C(a-x)(b-x)$$

let $(a-x) = 0 \Rightarrow x = a$

$$1 = A(b-a)(\cancel{a}\,c-a)$$

$$A = \frac{1}{(b-a)(c-a)} = -\frac{b-c}{(a-b)(b-c)(c-a)} \quad -③$$


Let $b-x=0$, $x=b$

$$1 = B(a-b)(c-b)$$

$$B = \frac{-1}{(a-b)(b-c)} = \frac{-(c-a)}{(a-b)(b-c)(c-a)} \quad -④$$

Let $(c-x)=0 \Rightarrow x=c$

$$1 = C(a-c)(b-c)$$

$$C = \frac{1}{(a-c)(b-c)}$$

$$C = \frac{-(a-b)}{(a-b)(b-c)(c-a)} \quad — (5)$$

Put eqn (3), (4) & (5) in eqn (2)

$$\frac{1}{(a-x)(b-x)(c-x)} = \frac{-(b-c)}{(a-b)(b-c)(c-a)(a-x)} -$$

$$\frac{(c-a)}{(a-b)(b-c)(c-a)(b-x)} - \frac{(a-b)}{(a-b)(b-c)(c-a)(c-x)}$$

$$\frac{1}{(a-x)(b-x)(c-x)} = \frac{1}{(a-b)(b-c)(c-a)}\left[-\frac{(b-c)}{a-x} - \frac{(c-a)}{b-x} - \frac{(a-b)}{c-x}\right]$$

$$\frac{1}{(a-x)(b-x)(c-x)}\left[-\int \frac{b-c}{a-x}\,dx - \int \frac{c-a}{b-x}\,dx - \int \frac{(a-b)}{c-x}\,dx\right.$$

$$= k\int dt$$

$$\frac{(b-c)\int \frac{-1}{a-x}\,dx + (c-a)\int \frac{-1}{b-x}\,dx + (a-b)\int \frac{-1}{c-x}\,dx}{(a-b)(b-c)(c-a)} = k\int dt$$

$$\frac{\left[(b-c)(\ln a-x) + (c-a)(\ln b-x) + (a-b)(\ln c-x)\right]}{(a-b)(b-c)(c-a)} = kt + c$$

# LECTURE 20.

Thurs 9-1-20

# TECHNIQUES OF INTEGARATION:

1) Integaration by partial fraction
2) " by substitution
3) " " parts.

## 1) BY PARTIAL FRACTION METHODE:

TYPES:

i) Linear
ii) Linear repeated
iii) Quadratic
iv) Quadratic repeated.

## i) LINEAR PARTIAL FRACTION METHOD.

e.g; Third order reaction.

$$\int \frac{1}{(a-x)(b-x)(c-x)}\,dx \quad — ①$$

$$\frac{1}{(a-x)(b-x)(c-x)} = \frac{A}{a-x} + \frac{B}{b-x} + \frac{C}{c-x} \quad — ②$$

By multiplying both sides with $(a-x)(b-x)(c-x)$:

$$1 = A(b-x)(c-x) + B(a-x)(c-x) + C(a-x)(b-x) \quad — ③$$

**Let $a-x=0 \Rightarrow a=x$**

Put in eq ③

$$1 = A(b-a)(c-a)$$

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$$A = \frac{1}{(b-a)(c-a)}$$

$$A = -\frac{1}{(a-b)(c-a)} = -\frac{(b-c)}{(a-b)(b-c)(c-a)} \quad \text{---} (4)$$

Let $b - x = 0 \Rightarrow x = b$

$$1 = B(a-b)(c-b)$$

$$B = \frac{1}{(a-b)(c-b)}$$

$$= -\frac{1}{(a-b)(b-c)} = -\frac{(c-a)}{(a-b)(b-c)(c-a)} \quad \text{---} (5)$$

Let $c - x = 0 \Rightarrow x = c$

$$1 = C(a-c)(b-c)$$

$$C = \frac{1}{(a-c)(b-c)}$$

$$= -\frac{1}{(c-a)(b-c)} = -\frac{(a-b)}{(a-b)(c-a)(b-c)} \quad \text{---} (6)$$

Put eq 4, 5, 6 in eq (2).

$$\frac{1}{(a-x)(b-x)(c-x)} = \frac{1}{(a-b)(b-c)(c-a)} \left[ -\frac{(b-c)}{a-x} - \frac{(c-a)}{b-x} - \frac{(a-b)}{c-x} \right]$$

$$= \frac{1}{(a-b)(b-c)(c-a)} \int \left[ \frac{-(b-c)}{(a-x)} - \frac{(c-a)}{(b-x)} - \frac{(a-b)}{(c-x)} \right]$$

$$= \frac{1}{(a-b)(b-c)(c-a)} \left[ \int \frac{-(b-c)}{(a-x)} + \int -\frac{(c-a)}{(b-x)} + \int -\frac{(a-b)}{c-x} \right]$$

$$= \frac{1}{(a-b)(b-c)(c-a)} \left[ b-c \int \frac{-1}{a-x} dx + (c-a) \int \frac{-1}{(b-x)} + (a-b) \int \frac{-1}{c-x} \right]$$

$$= \frac{1}{(a-b)(b-c)(c-a)} \left[ (b-c) \ln(a-x) + (c-a) \ln(b-x) + (a-b) \ln(c-x) \right] + C$$

## ii) LINEAR REPEATED: 2nd case

e.g; third order when two reactants have same initial concentrations.

$$\int \frac{1}{(a-x)^2(c-x)} dx \quad \text{—— ①}$$

$$\frac{1}{(a-x)^2(c-x)} = \frac{A}{(a-x)} + \frac{B}{(a-x)^2} + \frac{C}{(c-x)} \quad \text{—— ②}$$

By multiplying both sides with $(a-x)^2(c-x)$

$$1 = A(a-x)(c-x) + B(c-x) + C(a-x)^2 \quad — ③$$

Let $a-x=0 \Rightarrow x=a$

$$1 = B(c-a)$$

$$B = \frac{1}{c-a} \quad — ④$$

Let, $c-x=0 \Rightarrow x=c$

$$1 = C(a-c)^2$$

$$C = \frac{1}{(a-c)^2} \quad — ④ \; ⑤$$

Equation ③ becomes,

$$1 = A(ac-ax-cx+x^2) + B(c-x) + C(a^2+x^2-2ax)$$

$$1 = Aac - Aax - Acx + Ax^2 + Bc - Bx + Ca^2 + Cx^2 - 2Cax$$

By comparing coefficients:

$x^2$:

$\Rightarrow Dx^2 = Ax^2 + Cx^2$

$0x^2 = x^2(A+C)$

$0 = x^2(A+C)$

$0 = A+C$

$A = -C$

$A = -\dfrac{1}{(a-c)^2}$ — (5)

$x$:

$0 = -Aa - Ac - B - 2Ca$

$B = A(-a-c) - 2Ca$

By putting value of A & C:

$B = -\dfrac{(-a-c)}{(a-c)^2} - \dfrac{2a}{(a-c)^2}$

$= \dfrac{(a+c) - 2a}{(a-c)^2}$

$B = \dfrac{a+c-2a}{(a-c)^2} = \dfrac{c-a}{(a-c)^2}$ — (6)

R.W

$$B = \frac{1}{c-a}$$

$$= -\frac{1}{a-c}$$

$$= -\frac{1}{a-c} \cdot \frac{a-c}{a-c}$$

$$B = \frac{c-a}{(a-c)^2}$$

By putting 4, 5, 6 into eq ②

$$\frac{1}{(a-x)^2(c-x)} = \frac{1}{(a-c)^2}\left[\frac{-1}{a-x} + \frac{(c-a)}{(a-x)^2} + \frac{1}{c-x}\right]$$

$$= \frac{1}{(a-c)^2}\int\left[\frac{-1}{a-x} + \frac{c-a}{(a-x)^2} + \frac{1}{(c-x)}\right]dx$$

$$\frac{1}{(a-c)^2}\left[\int\frac{-1}{a-x}dx - (c-a)\int(a-x)^{-2}(-1)dx - \int\frac{-1}{c-x}dx\right]$$

$$\frac{1}{(a-c)^2}\left[\ln(a-x)-(c-a)\frac{(a-x)^{-2+1}}{-2+1}-\ln(c-x)\right]+c$$

$$\frac{1}{(a-c)^2}\left[\ln\frac{(a-x)}{c-x}-(c-a)\frac{(a-x)^{-1}}{-1}\right]+c$$

$$\frac{1}{(a-c)^2}\left[\ln\frac{a-x}{c-x}+\frac{c-a}{a-x}\right]+c$$

**3rd case:**

$$2A+B\rightarrow \text{Product}$$

| | | | | |
|---|---|---|---|---|
| ini | $a$ | $b$ | zero | 2A |
| | $(a-2x)^2$ | $(b-x)$ | $x$ | |

$$\frac{dx}{dt}=k[A]^2[B]^1$$

$$\frac{dx}{dt}=k(a-2x)^2(b-x)$$

$$\int\frac{dx}{(a-2x)^2(b-x)}=k\int dt \quad \text{— (1)}$$

linear repeated:
2nd case me do reactants ki conc. same ha lekin 3rd case me 2 reactants same ha.

$$\int \frac{1\,dx}{(a-2x)^2(b-x)} \quad \text{— (1)}$$

$$\frac{1}{(a-2x)^2(b-x)} = \frac{A}{(a-2x)} + \frac{B}{(a-2x)^2} + \frac{C}{b-x} \quad \text{— (2)}$$

By multiplying both sides with $(a-2x)^2(b-x)$:

$$1 = A(a-2x)(b-x) + B(b-x) + C(a-2x)^2 \quad \text{— (3)}$$

Let $b-x=0 \Rightarrow x=b$

$$1 = C(a-2b)^2$$

$$C = \frac{1}{(a-2b)^2} \quad \text{— (4)}$$

Let $a-2x=0 \Rightarrow x=a/2$

$$1 = B(b-a/2)$$

$$1 = B\left(\frac{2b-a}{2}\right)$$

$$B = \frac{2}{(2b-a)} = \frac{-2}{(a-2b)}$$

~~$B = -2$~~

$$1 = A(a-2x)(b-x) + B(b-x) + C(a-2x)^2$$

$$1 = A(ab - ax - 2bx + 2x^2) + B(b-x)$$
$$+ C(a^2 + 4x^2 - 4ax)$$

$$1 = Aab - Aax - 2Abx + 2Ax^2 + Bb - Bx$$
$$+ Ca^2 + 4Cx^2 - C4ax$$

By comparing coefficients

$x^2$:

$$0 = 2A + 4C$$
$$2A = -4C$$
$$A = -\frac{4}{2}C$$
$$A = -2C$$

$$0 = 2(A + 2C)$$
$$0 = A + 2C$$
$$A = -2C$$
$$A = -2\left(\frac{1}{(a-2b)^2}\right)$$

So, $A = \dfrac{-2}{(a-2b)^2}$ — 5

$\underline{x}$:

$$0 = -Aa - 2Ab - B - 4Ca$$

$$B = A(-a-2b) - 4Ca$$

By putting value of A & C,

$$B = \frac{-2(-a-2b)}{(a-2b)^2} - \frac{4a}{(a-2b)^2}$$

$$B = \frac{-2(-a-2b) - 4a}{(a-2b)^2}$$

$$B = \frac{2a + 4b - 4a}{(a-2b)^2}$$

$$= \frac{-2a + 4b}{(a-2b)^2}$$

$$B = \frac{2(2b-a)}{(a-2b)^2} \quad \text{--- (6)}$$

R.W

$$B = \frac{-2}{(a-2b)}$$

$$B = \frac{-2}{(a-2b)} \cdot \frac{a-2b}{a-2b}$$

$$= \frac{2(2b-a)}{(a-2b)^2}$$

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By putting value of 4, 5, 6 into eq ② & integrating eq.

$$\frac{1}{(a-2x)^2(b-x)} = \frac{1}{(a-2b)^2}\int\left[\frac{-2}{a-2x} + \frac{2(2b-a)}{(a-2x)^2} + \frac{1}{b-x}\right]dx$$

$$= \frac{1}{(a-2b)^2}\left[\int\frac{-2}{a-2x}\,dx - (2b-c)\int(a-2x)^{-2}(-2)\,dx - \int\frac{-1}{b-x}\,dx\right]$$

$$= \frac{1}{(a-2b)^2}\left[\ln(a-2x) - (2b-a)\frac{(a-2x)^{-2+1}}{-2+1} - \ln(b-x)\right]$$

$$= \frac{1}{(a-2b)^2}\left[\ln(a-2x) + (2b-a)(a-2x)^{-1} - \ln(b-x)\right] + c$$

$$\frac{1}{(a-2b)^2}\left[\ln(a-2x) \mp \frac{(2b-a)}{a-2x} - \ln(b-x)\right] + C$$

$$\frac{1}{(a-2b)^2}\left[\ln\frac{(a-2x)}{b-x} \mp \frac{(2b-a)}{a-2x}\right] + C$$

Thurs 16-Jan

# Integration by parts:

$$\int \underbrace{u}_{I} \cdot \underbrace{v}_{II}\, dx \qquad u = f(x) \quad V = f(x)$$

$$u\int v\,dx - \int\left[\int v\,dx \cdot \frac{d}{dx}u\right]dx$$

1) $\int \underbrace{x}_{I} \cdot \underbrace{\cos x}_{II}\, dx$

$$= x\int \cos x\,dx - \int\left[\int \cos x \cdot \frac{d}{dx}x\right]dx$$

$$\frac{1}{(a-2b)^2}\left[\ln(a-2x) - \frac{2(2b-a)}{a-2x} - \ln(b-x)\right] + C$$

$$\frac{1}{(a-2b)^2}\left[\ln\frac{(a-2x)}{b-x} - \frac{2(2b-a)}{a-2x}\right] + C$$

# Lec #21:

Thurs 16-Jan

General dissussion

# L 22

$= x\sin x - \int\left[\sin x(1)\right]dx$

$= x\sin x - \int \sin x \, dx$

$= x\sin x + \cos x + C$

$\Rightarrow$ trignometric or algebric me say algebric ko first lea jata h!

2) $\int \underbrace{\cos x}_{I} \cdot \underbrace{x}_{II} \, dx$

$= \cos x \int x\,dx - \int\left[\int x\,dx \cdot \frac{d}{dx}\cos x\right]dx$

$= \cos x \cdot \frac{x^2}{2} - \int\left[\frac{x^2}{2} \cdot (-\sin x)\right]dx$

$= \frac{1}{2}x^2\cos x + \frac{1}{2}\int \underbrace{x^2}_{I} \underbrace{\sin x}_{II} \, dx$

3) $\int \underbrace{x^2}_{I} \underbrace{\cos x}_{II} \, dx$

$= x^2\int \cos x\,dx - \int\left[\int \cos x\,dx \cdot \frac{d}{dx}x^2\right]dx$

$= x^2 \cdot \sin x - \int \left[ \sin x . 2x \right] dx$

$= x^2 . \sin x - 2 \int \underbrace{x}_{I} . \underbrace{\sin x}_{II} dx$

yo favourable h
rau k x Ki power km
ho ri. h.

$= x^2 . \sin x - 2 \left\{ x \int \sin x dx - \int \left[ \int \sin x dx . \frac{d}{dx} x \right] dx \right\}$

$= x^2 . \sin x - 2 \left\{ x(-\cos x) - \int \left[ (-\cos x)(1) \right] dx \right\}$

$= x^2 . \sin x - 2 \left\{ -x \cos x + \sin x \right\} + C$

$= x^2 \sin x + 2x \cos x - 2 \sin x + C$

(4) $\int \underbrace{x^3}_{I} . \underbrace{\cos x}_{II} dx$

$= x^3 \int \cos x dx - \int \left[ \int \cos x dx . \frac{d}{dx} x^3 \right] dx$

5) $\int \underbrace{x^2}_{I} \underbrace{e^x}_{II} dx$

$= x^2 \int e^x dx - \int \left[ \int e^x dx \cdot \frac{d}{dx} x^2 \right] dx$

$= x^2 e^x - \int \left[ e^x \cdot 2x \right] dx$

$= x^2 e^x - 2 \int \underbrace{x}_{I} \underbrace{e^x}_{II} dx$

$= x^2 e^x - 2 \left\{ x \int e^x - \int \left[ \int e^x \cdot \frac{d}{dx} x \right] dx \right\}$

$= x^2 e^x - 2 \left\{ x e^x - \int \left[ e^x \cdot (1) \right] dx \right\}$

$= x^2 e^x - 2 \left\{ x \cdot e^x - e^x \right\} + c$

$= x^2 e^x - 2x e^x + 2e^x + c$ Ans.

6) $\int \ln x \, dx$

$\int \underbrace{1}_{II} \cdot \underbrace{\ln x}_{I} dx$

$= \ln x \int 1 dx - \int \left[ \int 1 dx \cdot \frac{d}{dx} \ln x \right] dx$

$= \ln x . x - \int \left[ x \cdot \frac{1}{x} \right] dx$

$= \ln x . x - \int 1 dx$

$= \ln x . x - x \quad + C$

$= x \ln x - x \quad + C$

$= x(\ln x - 1) + C$ — Ans

# INTEGARATION BY SUBSTITUTION:

(i) $\int \frac{a}{2(ax+b)} dx$ — (1)

let $ax+b = t$ — (2)

$\frac{dt}{dx} = a(1) + 0$

$dt = a\,dx$ — (3)

By subtituting (3) & (2) in (1):—

$= \int \frac{1\,dt}{2t}$

$$= \frac{1}{2}\int \frac{1}{t}\,dt$$

$$= \frac{1}{2}\ln|t| + C$$

$$= \frac{1}{2}\ln|ax+b| + C$$

| | → this ~~bond~~ mod always yield a +ve value & gives absolute value.

log −3 = error
log |−3| = Ans: 3

2) $\int \frac{x}{\sqrt{4+x^2}}\,dx$ —— ①

Let $t = 4+x^2$ —— ②

$$\frac{dt}{dx} = 2x$$

$$\frac{1}{2}dt = x\,dx \quad —— ③$$

By substituting ③ & 2 in 1.

$$= \int \frac{1}{2t^{1/2}}\,dt$$

$$= \frac{1}{2} \int \frac{1}{t^{1/2}} dt$$

$$= \frac{1}{2} \frac{t^{-1/2+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{\cancel{1}}{\cancel{2}} \cdot \frac{t^{1/2}}{\cancel{1/2}} + C$$

$$= t^{1/2} + C$$

Back substitution

$$= \sqrt{4+x^2} + C \text{ — Ans.}$$

③ $\int x\sqrt{x-a}\, dx$ — ①

Let

$$t = x - a \quad \text{——— ②}$$

$$\frac{dt}{dx} = 1$$

$$dt = dx \quad \text{——— ③}$$

To find value of $x$

$$t = x - a$$

$$x = t + a \quad \text{——— ④}$$

By substituting 2, 3, & 4 in eqn ①

$$= \int (a+t)\, t^{1/2}\, dt$$

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$$= \int (a t^{1/2} + t^{3/2})\, dt$$

$$= a\int t^{1/2} + \int t^{3/2}\, dt$$

$\frac{3}{2} - \frac{5}{2}$

$\frac{3-5}{2}$

$$= a\frac{t^{1/2+1}}{1/2+1} + \frac{t^{3/2+1}}{3/2+1} + C$$

$$= a\frac{t^{3/2}}{3/2} + \frac{t^{5/2}}{5/2} + C$$

$$= \frac{2at^{3/2}}{3} + \frac{2t^{5/2}}{5} + C$$

$$= 2t^{3/2}\left[\frac{a\cancel{t}}{3} + \frac{t}{5}\right] + C$$

Back substitution

$$= 2(\cancel{a}x-a)^{3/2}\left[\frac{a}{3} + \frac{(x-a)}{5}\right] + C$$

$$= 2(x-a)^{3/2}\left[\frac{5a + 3(x-a)}{15}\right] + C$$

$$= 2(x-a)^{3/2}\left[\frac{5a + 3x - 3a}{15}\right] + C$$

$$= 2(x-a)^{3/2}\left[\frac{2a + 3x}{15}\right] + C$$

4) $\int \frac{\cot\sqrt{x}}{\sqrt{x}}\,dx$ — ①

Let

$z = \sqrt{x}$ — ②

$\Rightarrow \sin^2x + \cos^2x = 1$

$\sin^2x = 1 - \cos^2x$

$\Rightarrow \sin^2x = 1 - \cos 2x$

$\Rightarrow \tan x = \frac{\sin x}{\cos x}$

$\Rightarrow \cot x = \frac{\cos x}{\sin x}$

$\frac{dz}{dx} = \frac{1}{2} x^{-1/2}$

$2dz = x^{-1/2}\,dx$

$2dz = \frac{1}{\sqrt{x}}\,dx$ — ③

Put eqn 2 & 3 in ①

$= \int \cot z \cdot 2dz$

$= 2\int \cot z\,dz$

$= 2\int \cancel{\cot}\ \frac{\cos z}{\sin z}\,dz$

$= 2\ln|\sin z| + c$

∵ function nichay ho or uska derivative upar ho to integration k bad ans. function ata h.

① $\int \frac{1}{x \ln x} dx$ — ①

Let

$t = \ln x$ — ②

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{1}{x} dx$ — ③

By substituting 2, 3 in ①

$= \int \frac{1}{t} dt$

$= \ln|t| + c$

$= \ln|\ln|x|| + c$

② $\int \frac{1}{x(\ln 2x)^3} dx$ — ①

Let $t = \ln 2x$ — ②

$\frac{dt}{dx} = \frac{1}{\cancel{2}x} \cdot \cancel{2}$

$dt = \frac{1}{x} dx$ — ③

By substituting 2, 3 in ②

$$= \int \frac{1}{t^3} dt$$

$$= \int t^{-3} dt$$

$$= \frac{t^{-3+1}}{-3+1} + c$$

$$= \frac{t^{-2}}{-2} + c$$

$$= -\frac{1}{2} t^{-2} + c$$

$$= -\frac{1}{2t^2} + c$$

Back substitution

$$= -\frac{1}{2(\ln 2x)^2} + c \quad \text{— Ans.}$$

③ $\int \frac{-2x}{\sqrt{4-x^2}} dx$ — ①

Let $t = 4 - x^2$ — ②

$$\frac{dt}{dx} = -2x$$

$dt = -2x\,dx$ — ③

By substituting 2 & 3 in (1)

$$= \int \frac{1}{\sqrt{t}} dt$$

$$= \int t^{-1/2} dt$$

$$= \frac{t^{-1/2+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{t^{1/2}}{1/2} + c$$

$$= 2t^{1/2} + c$$

$$= 2\sqrt{4-x^2} + c$$

(4) $$\int \frac{x+b}{(x^2+2bx+c)^{1/2}} dx \quad \text{—(1)}$$

Let $t = x^2+2bx+c$ —(2)

$$\frac{dt}{dx} = 2x+2b$$

$$dt = 2(x+b)\, dx$$

$$\frac{1}{2} dt = (x+b)\, dx \quad \text{—(3)}$$

By substituting 2, 3 in 1

$$= \int \frac{\frac{1}{2}\,dt}{(t)^{1/2}}$$

$$= \frac{1}{2} \int \frac{1\,dt}{t^{1/2}}$$

$$= \frac{1}{2} \frac{t^{-1/2+1}}{-1/2+1} + c$$

$$= \frac{\cancel{1}}{\cancel{2}} \cdot \frac{t^{1/2}}{\cancel{1/2}} + c$$

$$= t^{1/2} + c$$

$$= (x^2 + 2bx + c)^{1/2} + c$$

5 $\int \frac{e^x}{e^x+3}\,dx$ — 1

Let $t = e^x + 3$ . — 2

$$\frac{dt}{dx} = e^x$$

$dt = e^x\,dx$ — 3

By substituting 2 & 3 in 1

$$= \int \frac{1}{t}\,dt$$

$= \ln|t| + C$

$= \ln|e^x + 3| + C$

⑥ $\int \cos^3 x \, (\sin x)^{1/2} dx$ —①

Let

$t = (\sin x)^{1/2}$ —②

$\frac{dt}{dx} = \frac{1}{2} \sin x^{-1/2} \cdot \cos x$

$dt = \frac{1}{2} \sin x^{-1/2} \cdot \cos x \, dx$ —③

$dt = \frac{1}{2 \sin x^{1/2}} \cdot \cos x \, dx$

$\because \sin x^{1/2} = t$

$dt = \frac{1}{2t} \cos x \, dx$

$2t \, dt = \cos x dx$ —③

By substituting 2, 3 in eq ①

$= \int \cos^2 x \, \underbrace{(\sin x)^{1/2}} \, \underbrace{\cos x \, dx}$

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$ — A

since

$t = \sin x^{1/2}$

$t^2 = \sin x$

$\boxed{t^4 = \sin^2 x}$

put value of t in eq (A)

$\cos^2 x = 1 - t^4$ — (4)

By substituting 2, 3, 4 in eq 1

$= \int (1 - t^4) . t . 2t\,dt$

$= \int (t - t^5) . 2t\,dt$

$= 2\int (t^2 - t^6)\,dt$

$= 2\left[\int t^2\,dt - \int t^6\,dt\right]$

$= 2\left[\frac{t^{2+1}}{2+1} - \frac{t^{6+1}}{6+1}\right] + c$

$= 2\left[\frac{t^3}{3} - \frac{t^7}{7}\right] + c$

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$$= 2t^3\left[\frac{1}{3} - \frac{t^4}{7}\right] + c$$

Back substitution

$$= 2(\sin x)^{3/2}\left[\frac{1}{3} - \frac{(\sin x)^2}{7}\right] + c$$

# INTEGRATION BY SUBSTITUTION:

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

1) $\int \sec x \, dx$

$$\int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x} dx \quad —(1)$$

Let

$$\sec x + \tan x = t \quad —(2)$$

$$\frac{dt}{dx} = \sec x \tan x + \sec^2 x$$

$$dt = [\sec x(\tan x + \sec x)] dx \quad —(3)$$

Put eq 2 & 3 in 1)

$$= \int \frac{dt}{t}$$

$= \ln|t| + c$

$= \ln|\sec x + \tan x| + c$

Thus basic integaral of $\int \sec x\,dx$ is $\ln|\sec x + \tan x| + c$

2) $$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx \quad \text{— ①}$$

Here we can not take direct integaral. another substitution is used

$x = a \sin\theta$

$\frac{dx}{d\theta} = a\cos\theta$

$dx = a\cos\theta\,d\theta \quad \text{— ②}$

put value of '$x$' and $dx$ in eq. ①

$$= \int \frac{a\cos\theta\,d\theta}{\sqrt{a^2 - (a\sin\theta)^2}}$$

$$= \int \frac{a\cos\theta\,d\theta}{\sqrt{a^2 - a^2\sin^2\theta}}$$

$\because \cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$$= \int \frac{a\cos\theta\,d\theta}{\sqrt{a^2(1 - \sin^2\theta)}}$$

$$= \int \frac{a\cos\theta\, d\theta}{\sqrt{a^2(1-\sin^2\theta)}}$$

$$= \int \frac{a\cos\theta\, d\theta}{a\sqrt{1-\sin^2\theta}}$$

$$= \int \frac{a\cos\theta\, d\theta}{a\sqrt{\cos^2\theta}}$$

$$= \int \frac{a\cos\theta\, d\theta}{a\cos\theta}$$

$$= \int d\theta \qquad \because x = a\sin\theta$$

$$= \int 1.\, d\theta \qquad \sin\theta = \frac{x}{a}$$

$$\theta = \sin^{-1}\frac{x}{a}$$

$$= \theta + C$$

$$= \sin^{-1}\frac{x}{a} + C$$

3) $$\int \frac{1}{\sqrt{a^2+x^2}}\, dx \quad \text{—— (1)}$$

$$x = a\tan\theta \quad \text{—— (2)}$$

$$\frac{dx}{d\theta} = a\tan\theta$$

$$\frac{dx}{d\theta} = a\sec^2\theta$$

$$dx = a\sec^2\theta\, d\theta \quad \text{—— (3)}$$

put 3, 2 eq. in 1

$$= \int \frac{a \sec^2\theta \, d\theta}{\sqrt{a^2 + a^2 \tan^2\theta}}$$

$$= \int \frac{a \sec^2\theta \, d\theta}{\sqrt{a^2(1+\tan^2\theta)}}$$

$\because 1 + \tan^2\theta = \sec^2\theta$

$$= \int \frac{a \sec^2\theta \, d\theta}{a\sqrt{1+\tan^2\theta}}$$

$$= \int \frac{a \sec^2\theta \, d\theta}{a\sqrt{\sec^2\theta}}$$

$$= \int \frac{\cancel{a} \sec^2\theta \, d\theta}{\cancel{a} \sec\theta}$$

$$= \int \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\sec \tan^{-1}\frac{a}{x} + \cancel{\tan}\frac{a}{x}\right| + C$$

$\because x = a\tan\theta$

$\tan\theta = \frac{a}{x}$

$\theta = \tan^{-1}\frac{a}{x}$

4) $\int_0^a \psi^2 \, dx = 1$

$$\psi = A \sin \frac{n\pi}{a} x$$

$$= \int_0^a \left[A \sin \frac{n\pi}{a} x\right]^2 dx$$

$$* \; A^2 \int_0^a \sin^2 \frac{n\pi x}{a} \, dx = 1$$

$$\because \theta = \frac{n\pi}{a} x$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\sin^2 \frac{n\pi x}{a} = \frac{1}{2}\left(1 - \cos 2\frac{n\pi x}{a}\right)$$

put it in above eq.

$$A^2 \int_0^a \frac{1}{2}\left(1 - \cos \frac{2n\pi x}{a}\right) dx = 1$$

$$\frac{1}{2} A^2 \int_0^a \left(1 - \cos \frac{2n\pi x}{a}\right) dx = 1$$

$$\frac{1}{2} A^2 \left[ \int_0^a 1 \, dx - \int_0^a \cos \frac{2n\pi x}{a} \, dx \right] = 1$$

$$\frac{1}{2} A^2 \left[ |x|_0^a - \left| \frac{\sin \frac{2n\pi x}{a}}{\frac{2n\pi}{a}} \right|_0^a \right] = 1$$

+c nai lagana cor definite integration h. (limits hn)

$$\frac{d}{dx} \sin 3x = \frac{d}{dx} \sin x \cdot \frac{d}{dx} 3x$$

multiply

$$\int \cos 3x \, dx = \frac{\sin 3x}{3} \leftarrow \text{divide}$$

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$$\frac{1}{2}A^2\left[(a-0)-\frac{a}{2n\pi}\left[\sin\frac{2n\pi(a)}{a}-\sin\frac{2n\pi(0)}{a}\right]\right]=1$$

$$\frac{1}{2}A^2\left[a-\frac{a}{2n\pi}\left[\sin 2n\pi-\sin 0\right]\right]=1$$

$\because$ $2n\pi$ is the multiple of sin wave or jo b sin ka multiple hota h wo zero e hota.

$\sin 2n\pi = 0$

$$\frac{1}{2}A^2\left[a-\frac{a}{2n\pi}(0)\right]=1$$

$$\frac{1}{2}A^2 a = 1$$

$$\frac{1}{2}A^2 = \frac{1}{a}$$

$$A^2 = \frac{2}{a}$$

$$A = \sqrt{2/a}$$ — Ans.

# LECTURE 26

31-JAN

1) $\int \operatorname{cosec} x \, dx$

| | | |
|---|---|---|
| sin | cos | |
| tan | cot | aik jesy solve hon gay. |
| sec | cosec | |

$$\int \frac{\operatorname{cosec} x(\operatorname{cosec} x - \cot x)}{(\operatorname{cosec} x - \cot x)} dx \quad -①$$

Let $t = \operatorname{cosec} x - \cot x \quad -②$

$$\frac{dt}{dx} = -\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x$$

$$\frac{dt}{dx} = \operatorname{cosec} x(\operatorname{cosec} x - \cot x)$$

$$dt = \operatorname{cosec} x(\operatorname{cosec} x - \cot x)dx \quad -③$$

put 2 & 3 in 1

$$= \int \frac{dt}{t}$$

$$= \int \frac{1}{t} dt$$

$$= \ln|t| + c$$

$$= \ln|\operatorname{cosec} x - \cot x| + c$$

2) $\int \tan x \, dx$

$$= \int \frac{\sin x}{\cos x} \, dx$$

$$= -\int \frac{-\sin x}{\cos x} \, dx$$

$$= -\ln|\cos x| + C$$

3) $\int \cot x \, dx$

$$= \int \frac{\cos x}{\sin x} \, dx$$

$$= \ln|\sin x| + C$$

$W = P\,dV$

$= P\int_{V_1}^{V_2} dV$

$\because PV = nRT$

$P = \frac{nRT}{V}$

$W = \frac{nRT}{V}\int_{V_1}^{V_2} dV$

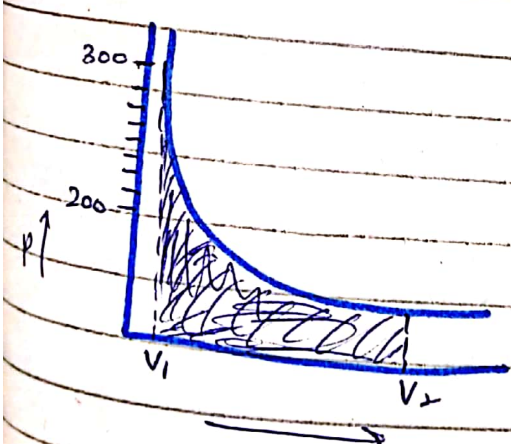
$= \underbrace{nRT}_{\text{Const}} \int_{V_1}^{V_2} \frac{dV}{V}$

$= nRT \ln |V|_{V_1}^{V_2}$

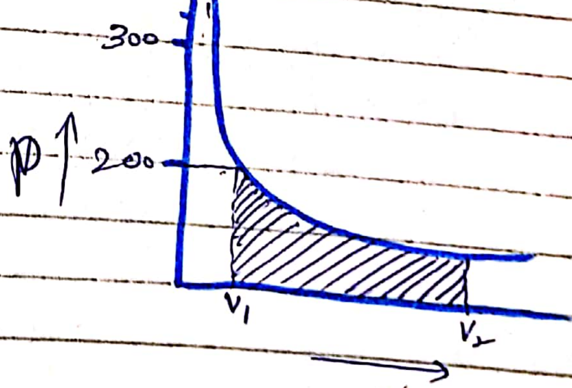
$= nRT \ln (V_2 - V_1)$

$= nRT \ln \frac{V_2}{V_1}$

$\therefore$ In definite integration area under the curve is work done b/w the limits $V_1$ & $V_2$.




Area under the curve is work done. Word done in reversible.




Work done in irreversible