DERIVATIVES: ABINITIO

Abinitio method: means derivative from the first principle

① $y = f(x) = 3x+7$

$f(x) = 3x+7$ ⇒ initial state

$f(x + \delta x) = 3(x+\delta x)+7$ ⇒ Final

x changes to very smal value.

e.g; $x=5$ then 5 to 5.00001

Final state − Initial state

$f(x+\delta x) - f(x) = [3(x+\delta x)+7] - (3x+7)$

$= \cancel{3x} + 3\delta x + \cancel{7} - \cancel{3x} - \cancel{7}$

$f(x+\delta x) - f(x) = 3\delta x$

Interval is δx so by dividing with δx.

$$\frac{f(x+\delta x) - f(x)}{\delta x} = \frac{3\cancel{\delta x}}{\cancel{\delta x}}$$

$$\lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} = 3$$

interval of x tends to 0 means x values is v. small that approach to 0.

$$\frac{dy}{dx} = 3$$

derivative of x & y is 3

② $y = x^2 + 1 = f(x)$ — ①

$f(x) = x^2+1$ initial

$f(x+\delta x) = (x+\delta x)^2 + 1$ final

$$f(x+\delta x) = (x+\delta x)^2 + 1$$

$$= x^2 + (\delta x)^2 + 2x\,\delta x + 1 \quad (2)$$

$$f(x+\delta x) - f(x) = (x^2 + (\delta x)^2 + 2x\,\delta x + 1) - (x^2+1)$$

$$= \cancel{x^2} + (\delta x)^2 + 2x\,\delta x + \cancel{1} - \cancel{x^2} - \cancel{1}$$

$$f(x+\delta x) - f(x) = (\delta x)^2 + 2x\,\delta x$$

By dividing with interval (δx):

$$\frac{f(x+\delta x) - f(x)}{\delta x} = \frac{(\delta x)^{\cancel{2}}}{\cancel{\delta x}} + \frac{2x\,\cancel{\delta x}}{\cancel{\delta x}}$$

$$= \delta x + 2x$$

$$\lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} = \lim_{\delta x \to 0} (\delta x + 2x)$$

$$\frac{dy}{dx} = 0 + 2x$$

$$= 2x$$

Infinitesimal interval

Derivatives tells us

instantaneous rate of change of function (y) with respect to x (interval).

where d means very small interval.

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instantaneous = aik lamhay me kitna change ay ga.

$$\frac{d\psi}{dx} = A \sin \frac{n\pi}{a} x$$

$\frac{d\psi}{dx}$ → instantaneous rate of change in amplitude with respect to very small change.

③ $y = 3x^2 + 7x + 100$

$f(x) y = 3x^2 + 7x + 100$ initial state

$f(x + \delta x) = 3(x + \delta x)^2 + 7(x + \delta x) + 100$

$f(x + \delta x) - f(x) = 3x^2 + 3\delta x^2 \cancel{+ 7x +} + 6x\delta x$
$+ 7x + 7\delta x + 100 - (3x^2 + 7x + 100)$

$= \cancel{3x^2} + 3\delta x^2 + 6x\delta x \cancel{+ 7x} + 7\delta x \cancel{+ 100}$
$\cancel{- 3x^2} \cancel{- 7x} \cancel{- 100}$

$= 3\delta x^2 + 6x\delta x + 7\delta x$

$$\frac{f(x - \delta x) - f(x)}{\delta x} = \frac{\cancel{\delta x}(3\delta x + 6x + 7)}{\cancel{\delta x}}$$

$\lim_{\delta x \to 0}$ $= 3(0) + 6x + 7$

$$\boxed{= 6x + 7}$$

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DERIVATIVES:

These are used to find instantaneous rate of change

$$f(x) = y = \frac{1}{x^2}$$

$$y = x^{-2}$$
$$= -2x^{-2-1}$$
$$= -2x^{-3} = \frac{-2}{x^3}$$

$$f(x+\delta x) = \frac{1}{(x+\delta x)^2}$$

$$f(x+\delta x) - f(x) = \frac{1}{(x+\delta x)^2} - \frac{1}{x^2}$$

$$= \frac{x^2 - (x+\delta x)^2}{x^2(x+\delta x)^2}$$

$$= \frac{x^2 - (x^2 + \delta x^2 + 2x\delta x)}{x^2(x+\delta x)^2}$$

$$= \frac{\cancel{x^2} - \cancel{x^2} - \delta x^2 - 2x\delta x}{x^2(x+\delta x)^2}$$

$$= \frac{-\delta x(\delta x + 2x)}{x^2(x+\delta x)^2}$$

$$\frac{f(x+\delta x) - f(x)}{\delta x} = -\frac{\cancel{\delta x}(\delta x^2 + 2x)}{x^2(x+\delta x)^2 \cdot \cancel{\delta x}}$$

$$= \frac{-(\delta x + 2x)}{x^2(x+\delta x)^2}$$

$$\lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} = \frac{-0 + 2x}{x^2 (x+0)^2}$$

$$= \frac{-2x}{x^4} = \frac{-2}{x^3}.$$

FORMULAS:

① $y = x^n$

$$\frac{dy}{dx} = n x^{n-1}$$

or

$$\frac{d}{dx} x^n = n x^{n-1}$$ (Power rule)

② $y =$ constant

$$\frac{dy}{dx} = 0$$

or

derivation of constant is 0.

$$\frac{d}{dx} c. = 0$$

③ $$\frac{d}{dx} .c.x^n = c \frac{d}{dx} x^n$$

$$= c.n x^{n-1}$$

④ $\frac{d}{dx} \sin x = \cos x$

⑤ $\frac{d}{dx} \cos x = -\sin x$

⑥ $\frac{d}{dx} \ln x = \frac{1}{x}$ or $\frac{d}{dx} \ln x^2+1 = \frac{1}{x^2+1}$

⑦ $\frac{d}{dx} . e^x = e^x$

SIMPLIFICATION FORMULAS:

① Sum-difference rule:

$$\frac{d}{dx}(u \pm v) = \frac{d}{dx}u \pm \frac{d}{dx}v.$$

$u = f(x)$, $v = f(x)$ This is necessary to mention that u & v are function of x otherwise they are considered as constant

② Product rule:

$$\frac{d}{dx}(uv) = v\frac{d}{dx}u + u\frac{dv}{dx}$$

Quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}v}{v^2}$$

e.g; $y = 3x^4 + 7x^3 + 9x^2 + 3x + 100$

$$\frac{dy}{dx} = \frac{d}{dx}(3x^4 + 7x^3 + 9x^2 + 3x + 100)$$

$$= \frac{d}{dx}3x^4 + \frac{d}{dx}7x^3 + \frac{d}{dx}9x^2 + \frac{d}{dx}3x + \frac{d}{dx}100$$

$$= 3\frac{d}{dx}x^4 + 7\frac{d}{dx}x^3 + 9\frac{d}{dx}x^2 + 3\frac{d}{dx}x + 0$$

$$\cancel{= 3x4x^3 + 7x^2 + 9x + 3(1)} \quad \because \frac{d}{dx}x = 1$$

$$= 3 \times 4x^3 + 7 \times 3x^2 + 9 \times 2x + 3 \times 1x^{1-1}$$

$$x^{1-1} = x^0 = 1$$

$$= 12x^3 + 21x^2 + 18x + 3$$

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① $\frac{d}{dx} x = 1$ } used in thermodynamics

$\frac{d}{dT} T = 1$

DIFFERENTIATION:

"Procedure to find derivative is called differentiation."

$y = 3t^2$	$y = 3t^2$
$\frac{dy}{dt} = 6t$	$\frac{dy}{dx} = 0$

DIFFERENTIATION:

① $$y = \frac{2x^3 - 3x^2 + 5}{(x^2+1)}$$

$$\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$$

$$\frac{dy}{dx} = \frac{(x^2+1)\frac{d}{dx}2x^3-3x^2+5-(2x^3-3x^2+5)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(6x^2-6x)-(2x^3-3x^2+5)(2x)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(6x^2-6x)-4x^4+6x^3-10x}{(x^2+1)^2}$$

$$= \frac{6x^4-\cancel{6x^3}+6x^2-6x-4x^4+\cancel{6x^3}-10x}{(x^2+1)^2}$$

$$\Rightarrow \frac{2x^4+6x^2-16x}{(x^2+1)^2}$$

② $y = (2\sqrt{x}+2)(x-\sqrt{x})$

product rule:

$$\because \frac{d}{dx}(uv) = v\frac{d}{dx}(u) + u\frac{d}{dx}(v)$$

$$y = (2(x)^{1/2}+2)(x-x^{1/2})$$

$$\frac{dy}{dx} = (x-x^{1/2})\cdot\frac{d}{dx}(2(x)^{1/2}+2) + 2(x)^{1/2}+2\frac{d}{dx}(x-x^{1/2})$$

$\frac{1}{2}\ \frac{1-2}{2}$ $\qquad\qquad$ $\frac{1}{2}$

$$\frac{dy}{dx} = (x - x^{1/2})\left(2 \times \frac{1}{2} x^{\frac{1}{2}-1}\right) + (2(x)^{1/2}+2)\left(1 - \frac{1}{2}x^{\frac{1}{2}-1}\right)$$

$$= (x - x^{1/2})(x^{-1/2}) + (2(x)^{1/2}+2)\left(1 - \frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= \left(x^1 . x^{-\frac{1}{2}} - x^{\frac{1}{2}} . x^{-1/2}\right) + \left(2(x)^{1/2} - \cancel{2}(x)^{1/2} . \frac{1}{\cancel{2}}(x)^{-1/2} + 2 - 2 \times \frac{1}{2}(x)^{-1/2}\right)$$

$$= (x^{1/2} - 1) + (2(x)^{1/2} - x^0 + 2 - x^{-1/2})$$

$$= (x^{1/2} - 1) + (2(x)^{1/2} - 1 + 2 - x^{-1/2})$$

$$= x^{1/2} - 1 + 2(x)^{1/2} - 1 + 2 - x^{-1/2}$$

$$= 3x^{1/2} - \cancel{2} + \cancel{2} - x^{-1/2}$$

$$= 3x^{1/2} - x^{-1/2} \quad \text{Ans.}$$

$$y = (2\sqrt{x}+2)(x - \sqrt{x})$$

$$= (2(x)^{1/2}+2)(x - (x)^{1/2}) \qquad \text{sum - diff rule.}$$

$$= 2x^{3/2} - \cancel{2x^1} + \cancel{2x} - 2(x)^{1/2}$$

$$= \frac{3}{\cancel{2}} \times \cancel{2} x^{1/2} - \frac{1}{\cancel{2}} \times \cancel{2} x^{-1/2}$$

$$= 3x^{1/2} - x^{-1/2}$$

③ $y = x^3$

$$\frac{dy}{dx} = \frac{d}{dx} x^3 \cdot \frac{d}{dx} x \qquad = \frac{d}{dx} x = 1$$

$$= 3x^2$$

④ $y = (3x^2+100)^5$

$$\frac{dy}{dx} = \frac{d}{dx} (3x^2+100)^5 \cdot \frac{d}{dx} 3x^2+100$$

$$\cancel{\cdot \frac{d}{dx} x^2}$$

$$\cancel{=5(6x}$$

$$= 5(3x^2+100)^4 \cdot 6x \cdot 1$$

$$= 30x(3x^2+100)^4$$

5) $y = \sin x$

$$\frac{dy}{dx} = \frac{d}{dx} \sin x \cdot \frac{d}{dx} x$$

$$= \cos x \cdot 1$$

6) $y = \sin 3x$

$$\frac{dy}{dx} = \frac{d}{dx} \sin 3x \cdot \frac{d}{dx} 3x \cdot \frac{d}{dx} x$$

$= \cos 3x \,.\, 3(1)\,.\,1$

$= 3\cos 3x$

$\because \frac{d}{dx} 3nx$

$3n \frac{d}{dx} x$

$3n(1)$

⑦ $y = \sin 3nx$

$\frac{dy}{dx} = \frac{d}{dx} \sin 3nx \,.\, \frac{d}{dx} 3nx.$

$\frac{d}{dx} x$

$= \cos 3nx \,.\, 3n(1)\,.\,1$

$= 3n \cos 3nx$

⑧ $\psi = A \sin \frac{2n\pi x}{a}$

$\frac{d\psi}{dx} = A \frac{d}{dx} \sin \frac{2n\pi x}{a} \,.\, \frac{d}{dx} \frac{2n\pi x}{a}.$

$\frac{d}{dx} x.$

$= A \left(\cos \frac{2n\pi x}{a}\right) . \frac{2n\pi}{a} (1)\,.(1)$

$\frac{d\psi}{dx} = \frac{2n\pi}{a} A \cos \frac{2n\pi x}{a}$

2nd derivative:

$\frac{d}{dx}\left(\frac{d\psi}{dx}\right) = \frac{d^2\psi}{dx^2}$

$$= \frac{2n\pi A}{a} \frac{d}{dx} \cos\frac{2n\pi x}{a} \cdot \frac{d}{dx} \frac{2n\pi x}{a}$$

$$\frac{d}{dx} x$$

$$= \frac{2n\pi A}{a} \left(-\sin\frac{2n\pi x}{a}\right) \cdot \frac{2n\pi}{a} (1) \cdot (1)$$

$$\frac{d^2\psi}{dx^2} = -\frac{4n^2\pi^2}{a^2} A \left(\sin\frac{2n\pi x}{a}\right)$$

(9) $y = (\ln x + x)^4$

$$\frac{dy}{dx} = \frac{d}{dx}(\ln x + x)^4 \cdot \frac{d}{dx} \ln x + x \cdot \frac{d}{dx} x$$

$$= 4(\ln x + x)^{4-1=3} \cdot \frac{1}{x} + 1 \cdot (1)$$

$$= 4(\ln x + x)^3 \cdot \frac{1}{x} + 1$$

$\because \frac{d}{dx} \ln x = \frac{1}{x}$

$\because \frac{d}{dx} x = 1$

(10) $y = e^{ax}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{ax} \cdot \frac{d}{dx} ax$$

$$= e^{ax} \cdot a$$

$$= a e^{ax}$$ (igon function)

(11) $y = e^{(x^2+1)}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{(x^2+1)} \cdot \frac{d}{dx}(x^2+1)$$

$$= e^{x^2+1} \cdot 2x \cancel{+1}$$

$$= 2x\, e^{(x^2+1)}$$

(12) $y = e^{(x^2+1)^4}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{(x^2+1)^4} \cdot \frac{d}{dx} e^{(x^2+1)} \cdot \frac{d}{dx} x^2+1$$

$$= e^{(x^2+1)^4} \cdot e^{(x^2+1)} \cdot 2x$$

(13) $x^4 + 2x^3 + x^2$

$$\frac{dy}{dx} = \frac{d}{dx} x^4 + \frac{d}{dx} 2x^3 + \frac{d}{dx} x^2$$

$$= \frac{d}{dx} x^4 + 2\frac{d}{dx} x^3 + \frac{d}{dx} x^2$$

$$= 4x^3 + 2 \times 3x^2 + 2x$$

$$= 4x^3 + 6x^2 + 2x.$$

14) $x^{-3} + 2x^{-3/2} + 3$

$$\frac{dy}{dx} = \frac{d}{dx} x^{-3} + \frac{d}{dx} 2x^{-3/2} + \frac{d}{dx} 3$$

$$= \frac{d}{dx} x^{-3} + 2 \frac{d}{dx} x^{-3/2} + 0$$

$$= -3x^{-4} + \cancel{2} \times \frac{-3}{\cancel{2}} x^{-5/2}$$

$$= -3x^{-4} - 3x^{-5/2}$$

$\because \frac{-3}{2} - 1$

$= \frac{-3-2}{2}$

$= -\frac{5}{2}$

15) $\frac{a+x}{a-x}$

$$\frac{dy}{dx} = \frac{(a-x)\frac{d}{dx}(a+x) - (a+x)\frac{d}{dx}(a-x)}{(a-x)^2}$$

$$= \frac{(a-x)\frac{d}{dx}(a+x) - (a+x)\frac{d}{dx}(a-x)}{(a-x)^2}$$

$$= \frac{(a-x).(1) - (a+x)(-1)}{(a-x)^2}$$

$$= \frac{(a-\cancel{x}) + (a+\cancel{x})}{(a-x)^2}$$

$$= \frac{2a}{(a-x)^2}$$

$\frac{d}{dx} 2x$ $1-1$
0

4. $\frac{2x-3}{2x+1}$

$2\frac{d}{dx} x$
$2(1)$

$$\frac{dy}{dx} = \frac{2x+1 \frac{d}{dx}(2x-3) - (2x-3)\frac{d}{dx}(2x+1)}{(2x+1)^2}$$

$$= \frac{(2x+1) \times (2) - (2x-3)(2)}{(2x+1)^2}$$

$$= \frac{4x+2 - (4x-6)}{(2x+1)^2}$$

$$= \frac{\cancel{4x}+2 - \cancel{4x}+6}{(2x+1)^2}$$

$$= \frac{8}{(2x+1)^2}$$

5). $(\overset{u}{x-5})(\overset{v}{3-x})$

$$\frac{dy}{dx} = (3-x)\frac{d}{dx}(x-5) + (x-5)\frac{d}{dx}(3-x)$$

$$= (3-x).(1) + (x-5).(-1)$$

$$= (3-x) - (x-5)$$

$$= 3-x-x+5$$

$$= 8 - 2x + 8 \text{ Ans.}$$

7) $\dfrac{(1+\sqrt{x})(x - x^{3/2})}{\sqrt{x}}$ $\quad \frac{1}{2}$

$= \dfrac{(1 + (x)^{1/2})(x - x^{3/2})}{(x)^{1/2}}$

$= \dfrac{x - x^{3/2} + (x)^{1/2}(x) - (x)^{\frac{1}{2}} \cdot (x)^{\frac{3}{2}}}{x^{1/2}}$

$= x - \cancel{x^{3/2}} + \cancel{x^{3/2}} - (x)^{8/2}$

$= \dfrac{x - (x)^4}{x^{1/2}}$

$\dfrac{dy}{dx} = \dfrac{(x)^{1/2}\frac{d}{dx}(x - x^4) - (x - x^4)\frac{d}{d}x^{1/2}}{(x^{1/2})^2}$

$= \dfrac{x^{1/2} \cdot 1 - 4 - (x - x^4) \cdot \frac{1}{2} \cdot 1}{x}$

$= \dfrac{x^{1/2} \cdot -3 - x + x^4 \cdot \frac{1}{2}}{x}$

$= \dfrac{-3/2 \cdot x^{1/2} - x + x^4}{\cancel{x}}$

$= -3/2 \cdot x^{1/2} - x + x^3 = -\dfrac{3}{2}x^{1/2} - x + x^3$

8) $\dfrac{(x^2+1)^2}{x^2-1}$

$$\frac{dy}{dx} = \frac{(x^2-1)\frac{d}{dx}(x^2+1)^2 - (x^2+1)^2\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{x^2-1 \,.\, 2(x^2+1)^{2-1} - (x^2+1)^2 \,.\, 2x}{(x^2-1)^2}$$

$$= \frac{x^2-1 \,.\, 2x^2+2 - (x^2+1)^2 \,.\, 2x}{(x^2-1)^2}$$

$$= \frac{(x^2-1) \,.\, (2x^2+2) - (x^4+1+2x^2)2x}{(x^2-1)^2}$$

$$= \frac{2x^4 + \cancel{2x^2} - \cancel{2x^2} - 2 - 2x^5 - 2x - 4x^3}{(x^2-1)^2}$$

$$= \frac{-2x^5 + 2x^4 - 4x^3 - 2x - 2}{(x^2-1)^2}$$

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a) $\frac{x^2+1}{x^2-3}$

$$\frac{dy}{dx} = \frac{(x^2-3)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^2-3)}{(x^2-3)^2}$$

$$= \frac{(x^2-3).(2x) - (x^2+1).2x}{(x^2-3)^2}$$

$$= \frac{2x^3 - 6x - (2x^3+2x)}{(x^2-3)^2}$$

$$= \frac{\cancel{2x^3} - 6x - \cancel{2x^3} - 2x}{(x^2-3)^2}$$

$$= \frac{-6x-2x}{(x^2-3)^2} = \frac{-8x}{(x^2-3)^2}$$ Ans

10) $\frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{(1+x)^{1/2}}{(1-x)^{1/2}}$

$\frac{dy}{dx} -$

$$\frac{dy}{dx} = \frac{(1-x)^{1/2}\frac{d}{dx}(1+x)^{1/2} - (1+x)^{1/2}\frac{d}{dx}(1-x)^{1/2}}{(1-x)}$$

10) $\dfrac{\sqrt{1+x}}{\sqrt{1-x}} = \dfrac{(1+x)^{1/2}}{(1-x)^{1/2}}$

$$\frac{dy}{dx} = \frac{(1-x)^{1/2}\frac{d}{dx}(1+x)^{1/2} - (1+x)^{1/2}\frac{d}{dx}(1-x)^{1/2}}{\left[(1-x)^{1/2}\right]^2}$$

$$= \frac{(1-x)^{1/2} \cdot \cancel{d}\,\frac{1}{2}(1+x)^{-1/2}(1) - (1+x)^{1/2} \cdot \frac{1}{2}(1-x)^{-1/2} \cdot (-1)}{(1-x)}$$

$$= \cancel{(1-x)^{1/2} \cdot \frac{1}{2}(1+x) - (1+x)^{1/2}}\cdot$$

$$= \frac{1(1-x)^{1/2}}{2(1+x)^{1/2}} - \frac{(-1)\,1(1+x)^{1/2}}{2(1\cancel{x}-x)^{1/2}}$$

$$= \frac{1}{(1-x)}\cancel{\sqrt{1-x}}\left[\frac{(1-x)^{1/2}}{2(1+x)^{1/2}} + \frac{(1+x)^{1/2}}{2(1-x)^{1/2}}\right]$$

$$= \frac{1}{1-x}\left[\frac{(1-x)^{1/2}(1-x)^{1/2} + (1+x)^{1/2}(1+x)^{1/2}}{2(1+x)^{1/2}(1-x)^{1/2}}\right]$$

$$= \frac{1}{1-x}\left[\frac{\left[(1-x)^{1/\cancel{2}}\right]^{\cancel{2}} + \left[(1+x)^{1/\cancel{2}}\right]^{\cancel{2}}}{2(1+x)^{1/2}(1-x)^{1/2}}\right]$$

$$= \frac{1}{1-x}\left[\frac{1-\cancel{x}+1+\cancel{x}}{2(1+x)^{1/2}(1-x)^{1/2}}\right]$$

$$= \frac{1}{1-x}\left[\frac{\cancel{2}}{\cancel{2}(1+x)^{1/2}(1-x)^{1/2}}\right]$$

$$= \left[\frac{1}{(1+x)^{1/2}(1-x)^{1/2}}\right]\frac{1}{1-x}$$

$$= \frac{1}{(1+x)^{1/2}(1-x^2)^{1/2}}$$

$$= \frac{1}{\sqrt{1+x}\ \sqrt{1-x^2}}$$ Ans.

11) $\dfrac{2x-1}{\sqrt{x^2+1}} = \dfrac{2x-1}{(x^2+1)^{1/2}}$

$$\frac{dy}{dx} = \frac{(x^2+1)^{1/2}\frac{d}{dx}2x-1 - (2x-1)\frac{d}{dx}(x^2+1)^{1/2}}{\left[(x^2+1)^{1/2}\right]^2}$$

$$= \frac{(x^2+1)^{1/2}.(2) - (2x-1)\left(.\frac{1}{2}(x^2+1)^{-1/2}.2x\right)}{(x^2+1)}$$

$$= \frac{(x^2+1)^{1/2}.2 - (2x-1).\frac{1}{2}(x^2+1)^{-1/2}2x}{x^2+1}$$

$$= \frac{1}{x^2+1}\left[2(x^2+1)^{1/2} - \frac{(2x-1)\cancel{2}x}{\cancel{2}(x^2+1)^{1/2}}\right]$$

$$= \frac{1}{x^2-1}\left[2(x^2+1)^{1/2} - \frac{x(2x-1)}{(x^2+1)^{1/2}}\right]$$

$$= \frac{1}{x^2-1}\left[\frac{2\left[(x^2+1)^{1/2}\right]^2 - x(2x-1)}{(x^2+1)^{1/2}}\right]$$

$$= \frac{1}{x^2-1}\left[\frac{2(x^2+1) - (2x^2-x)}{(x^2+1)^{1/2}}\right]$$

$$= \frac{1}{x^2-1}\left[\frac{\cancel{2x^2}+2 - \cancel{2x^2}+x}{(x^2+1)^{1/2}}\right]$$

$$= \frac{1}{x^2-1}\left[\frac{2+x}{(x^2+1)^{1/2}}\right] \text{ Ans.}$$

12) $\sqrt{\frac{a-x}{a+x}} = \left(\frac{a-x}{a+x}\right)^{1/2} = \frac{(a-x)^{1/2}}{(a+x)^{1/2}}$

$$\frac{dy}{dx} = \frac{(a+x)^{1/2}\frac{d}{dx}(a-x)^{1/2} - (a-x)^{1/2}\frac{d}{dx}(a+x)^{1/2}}{\left[(a+x)^{1/2}\right]^2}$$

$$= \frac{(a+x)^{1/2}\cdot\frac{1}{2}(a-x)^{-1/2}(-1) - (a-x)^{1/2}\cdot\frac{1}{2}(a+x)^{-1/2}(1)}{a+x}$$

$$= \frac{1}{a+x}\left[\frac{-(a+x)^{1/2}}{2(a-x)^{1/2}} - \frac{(a-x)^{1/2}}{2(a+x)^{1/2}}\right]$$

$$= \frac{1}{a+x}\left[\frac{-(a+x)^{1/2}.(a+x)^{1/2} - (a-x)^{1/2}.(a-x)^{1/2}}{2(a-x)^{1/2}(a+x)^{1/2}}\right]$$

$$= \frac{1}{a+x}\left[\frac{-\left[(a+x)^{1/2}\right]^2 - \left[(a-x)^{1/2}\right]^2}{2(a-x)^{1/2}.(a+x)^{1/2}}\right]$$

$$= \frac{1}{a+x}\left[\frac{-(a+x) - (a-x)}{2(a-x)^{1/2}.(a+x)^{1/2}}\right]$$

$$= \frac{1}{a+x}\left[\frac{-a-\cancel{x}-a+\cancel{x}}{2(a-x)^{1/2}(a+x)^{1/2}}\right]$$

$$= \frac{1}{a+x}\left[\frac{-\cancel{2}a}{\cancel{2}(a-x)^{1/2}(a+x)^{1/2}}\right]$$

$$= \frac{1}{a+x}\left[\frac{-a}{\sqrt{a-x}\sqrt{a+x}}\right] \quad \text{Ans.}$$

13) $\dfrac{\sqrt{x^2+1}}{\sqrt{x^2-1}} = \dfrac{(x^2+1)^{1/2}}{(x^2-1)^{1/2}}$

$$\frac{dy}{dx} = \frac{(x^2-1)^{1/2}\frac{d}{dx}(x^2+1)^{1/2} - (x^2+1)^{1/2}\frac{d}{dx}(x^2-1)^{1/2}}{\left[(x^2-1)^{1/2}\right]^2}$$

$$= \frac{(x^2-1)^{1/2} \cdot \frac{1}{2}(x^2+1)^{-1/2} 2x - (x^2+1)^{1/2} \cdot \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x}{(x^2-1)}$$

$$= \frac{1}{x^2-1}\left[\frac{(x^2-1)^{1/2}\cancel{2}x}{\cancel{2}(x^2+1)^{1/2}} - \frac{(x^2+1)^{1/2}\cancel{2}x}{\cancel{2}(x^2-1)^{1/2}}\right]$$

$$= \frac{1}{x^2-1}\left[\frac{x\left[(x^2-1)^{1/2}\right]^2 - x\left[(x^2+1)^{1/2}\right]^2}{(x^2+1)^{1/2}(x^2-1)^{1/2}}\right]$$

$$= \frac{1}{x^2-1}\left[\frac{x(x^2-1) - x(x^2+1)}{(x^2+1)^{1/2}(x^2-1)^{1/2}}\right]$$

$$= \frac{1}{x^2-1}\left[\frac{\cancel{x^3} - x - \cancel{x^3} - x}{(x^2+1)^{1/2}(x^2-1)^{1/2}}\right]$$

$$= \frac{1}{x^2-1}\left[\frac{-2x}{\sqrt{x^2+1}\sqrt{x^2-1}}\right] \text{ Ans.}$$

DIFFERENTIATION:

1) $y = \frac{a+x}{a-x}$

Qoutient Rule:

$$\frac{dy}{dx} = \frac{d}{dx} \frac{a+x}{a-x}$$

$$= \frac{(a-x)\frac{d}{dx}(a+x) - (a+x)\frac{d}{dx}(a-x)}{(a-x)^2}$$

$$= \frac{(a-x) \cdot (1) - (a+x)(-1)}{(a-x)^2}$$

$$= \frac{(a-x) - (a+x)(-1)}{(a-x)^2}$$

$$= \frac{(a-x) + (a+x)}{(a-x)^2}$$

$$= \frac{a - \cancel{x} + a + \cancel{x}}{(a-x)^2} = \frac{2a}{(a-x)^2}$$

2) $y = \sqrt{\frac{a+x}{a-x}} = \left(\frac{a+x}{a-x}\right)^{1/2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a+x}{a-x}\right)^{1/2} \cdot \frac{d}{dx}\left(\frac{a+x}{a-x}\right)$$

$$= \frac{1}{2}\left(\frac{a+x}{a-x}\right)^{-1/2} \cdot \frac{(a-x)\frac{d}{dx}(a+x) - a+x\frac{d}{dx}(a-x)}{(a-x)^2}$$

$$= \frac{1}{2}\left(\frac{a+x}{a-x}\right)^{-1/2} \cdot \frac{(a-x)\cdot(1) - (a+x)\cdot(-1)}{(a-x)^2}$$

$$= \frac{1}{2}\left(\frac{a+x}{a-x}\right)^{-1/2} \cdot \frac{(a-\cancel{x}) + a+\cancel{x}}{(a-x)^2}$$

$$= \frac{1}{\cancel{2}}\left(\frac{a+x}{a-x}\right)^{-1/2} \cdot \frac{\cancel{2}a}{(a-x)^2}$$

$$= \frac{(a-x)^{1/2}}{(a+x)^{1/2}} \cdot \frac{a}{(a-x)^2} \qquad \because 2-\frac{1}{2}$$

$$= \frac{a}{(a+x)^{1/2} \cdot (a-x)^{3/2}} \qquad = \frac{4-1}{2} = 3/2$$

③ $y = (a+x)(a-x)^{-1}$

product rule:

$$\frac{dy}{dx} = (a-x)^{-1}\frac{d}{dx}(a+x) + (a+x)\frac{d}{dx}(a-x)^{-1}$$

$$= (a-x)^{-1}(1) + (a+x) \cdot -x^{-2}$$

$$= (a-x)^{-1} + (a+x) \cdot -x^{-2}$$

$= \frac{1-1}{2}$

$= \frac{1-2}{2}$

$$= \frac{1}{2}\left(\frac{a+x}{a-x}\right)^{-1/2} \cdot \frac{(a-x)\frac{d}{dx}(a+x) - a+x\frac{d}{dx}(a-x)}{(a-x)^2}$$

$$= \frac{\frac{1}{2}\left(\frac{a+x}{a-x}\right)^{-1/2} \cdot (a-x)\cdot(1) - (a+x)\cdot(-1)}{(a-x)^2}$$

$$= \frac{1}{2}\left(\frac{a+x}{a-x}\right)^{-1/2} \cdot \frac{(a-\cancel{x}) + a+\cancel{x}}{(a-x)^2}$$

$$= \frac{1}{\cancel{2}}\left(\frac{a+x}{a-x}\right)^{-1/2} \cdot \frac{\cancel{2}a}{(a-x)^2}$$

$$= \frac{(a-x)^{1/2}}{(a+x)^{1/2}} \cdot \frac{a}{(a-x)^2}$$

$\because 2-\frac{1}{2}$

$= \frac{4-1}{2}$

$$= \frac{a}{(a+x)^{1/2} \cdot (a-x)^{3/2}}$$

$= 3/2$

③ $y = (a+x)(a-x)^{-1}$

product rule

$$\frac{dy}{dx} = (a-x)^{-1}\frac{d}{dx}(a+x) + (a+x)\frac{d}{dx}(a-x)^{-1}$$

$$= (a-x)^{-1}(1) + (a+x) \cdot -x^{-2}$$

$$= (a-x)^{-1} + (a+x) \cdot -x^{-2}$$

$$= \frac{1}{a-x} \mp \frac{(a+x)}{x^2}$$

$$= \frac{x^2 - (a+x)(a-x)}{(a-x)\,x^2}$$

$$= \frac{x^2 - (a^2 - x^2)}{(a-x)x^2}$$

$$= \frac{\cancel{x^2} - a^2 + \cancel{x^2}}{(a-x)x^2} = \frac{-a^2}{(a-x)x^2}$$

Chain Rule:

$$y = \sqrt{\frac{a+x}{a-x}}$$

Let $t = \dfrac{a+x}{a-x}$

$\Rightarrow y = t^{1/2}$

$$\frac{dy}{dt} = \frac{d}{dt}\, t^{1/2}$$

$\frac{1}{2} - 1$
$\frac{1-2}{2}$

$$\frac{dy}{dt} = \frac{1}{2} t^{-1/2} \quad -(1)$$

$\Rightarrow t = a+x/a-x$

$$\frac{dt}{dx} = \frac{d}{dx}\left(\frac{a+x}{a-x}\right)$$

$$= \frac{(a-x)\frac{d}{dx}(a+x) - (a+x)\frac{d}{dx}(a-x)}{(a-x)^2}$$

$$= \frac{(a-x)\cdot(1) - (a+x)\cdot(-1)}{(a-x)^2}$$

$$= \frac{(a-x) + (a+x)}{(a-x)^2}$$

$$= \frac{2a}{(a-x)^2}$$

By combining both eq ① & ②.

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \frac{1}{2} t^{-1/2} \cdot \frac{2a}{(a-x)^2}$$

$$= \frac{1}{2} \left(\frac{a+x}{a-x}\right)^{-1/2} \cdot \frac{2a}{(a-x)^2}$$

$$= \frac{1}{\cancel{2}} \frac{(a-x)^{1/2}}{(a+x)^{1/2}} \cdot \frac{\cancel{2}a}{(a-x)^2}$$

$$\frac{1}{2}+2$$

$$= \frac{a}{(a+x)^{1/2} \cdot (a-x)^{3/2}}$$

$$y = \sqrt{\frac{a^2+x^2}{a^2-x^2}}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a^2+x^2}{a^2-x^2}\right)^{1/2} \cdot \frac{d}{dx}\left(\frac{a^2+x^2}{a^2-x^2}\right)$$

$$= \frac{1}{2}\left(\frac{a^2+x^2}{a^2-x^2}\right)^{-1/2} \cdot \frac{(a^2-x^2)\frac{d}{dx}(a^2+x^2) - (a^2+x^2)\frac{d}{dx}(a^2-x^2)}{(a^2-x^2)^2}$$

$$= \frac{1}{2}\left(\frac{a^2+x^2}{a^2-x^2}\right)^{-1/2} \cdot \frac{(a^2-x^2).(2x) - (a^2+x^2)(-2x)}{(a^2-x^2)^2}$$

$$= \frac{1}{2}\frac{(a^2-x^2)^{1/2}}{(a^2+x^2)^{1/2}} \cdot \frac{2a^2x - \cancel{2x^3} + 2a^2x + \cancel{2x^3}}{(a^2-x^2)^2}$$

$$= \frac{1}{\cancel{2}}\frac{(a^2-x^2)^{1/2}}{(a^2+x^2)^{1/2}} \cdot \frac{\overset{2}{\cancel{4}}a^2x}{(a^2-x^2)^2}$$

$$= \frac{2a^2x\,(a^2-x^2)^{1/2}}{(a^2+x^2)^{1/2}\,(a^2-x^2)^2}$$

$$=$$

7 Nov. Lecture 9. Thursday

① $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

$= (\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2(\sqrt{x})\left(\frac{1}{\sqrt{x}}\right)$

$\frac{dy}{dx}$

$= x + \frac{1}{x} - 2$

$\frac{dy}{dx} = \frac{d}{dx} x + \frac{d}{dx} x^{-1} - \frac{d}{dx} 2$

$= \cancel{x}\, 1 - 1x^{-2} - 0$

$= 1 - \frac{1}{x^2}$

By Chain rule:

$y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

let $t = \sqrt{x} - \frac{1}{\sqrt{x}}$

$\Rightarrow y = \cancel{2}t^2$

$\frac{dy}{dt} = 2t$ — ①

$\frac{1}{2} - 1$

$\frac{-2}{2}$

$\frac{-1}{2} - 1$

$\frac{-1-2}{2}$

$\Rightarrow \frac{dt}{dx} = \frac{d}{dx}(x)^{1/2} - \frac{d}{dx} x^{-1/2}$

$= \frac{1}{2} x^{-1/2} + \frac{1}{2} x^{-3/2}$

$$= \frac{1}{2x^{1/2}} + \frac{1}{2x^{3/2}} \quad \text{— } 2$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= 2t \cdot \frac{1}{2x^{1/2}} + \frac{1}{2x^{3/2}}$$

$$= \cancel{2}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) \cdot \frac{1}{\cancel{2}} \left[\frac{1}{x^{1/2}} + \frac{1}{x\sqrt{x}}\right]$$

$$= \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)\left(\frac{1}{\sqrt{x}} + \frac{1}{x\sqrt{x}}\right)$$

$$= \sqrt{x} \cdot \frac{1}{\sqrt{x}} + \frac{1}{x\sqrt{x}} \cdot \sqrt{x} - \frac{1}{\sqrt{x}} \cdot \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x}} \cdot \frac{1}{x\sqrt{x}}$$

$$= \left[1 + \frac{1}{x} - \frac{1}{(\sqrt{x})^2} - \frac{1}{x(\sqrt{x})^2}\right]$$

$$= 1 + \cancel{\frac{1}{x}} - \cancel{\frac{1}{x}} - \frac{1}{x^2}$$

$$= 1 - \frac{1}{x^2} \quad \text{— Ans.}$$

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2) $$y = \underbrace{x}_{u} \underbrace{\sqrt{\frac{a+x}{a-x}}}_{v}$$

Product rule:

$$\frac{dy}{dx} = \left(\frac{a+x}{a-x}\right)^{1/2} \frac{d}{dx} x + x \frac{d}{dx} \left(\frac{a+x}{a-x}\right)^{1/2}$$

DIFFERENTIATION:

Trignometric Functions:

① $\frac{d}{dx}(\sin x) = \cos x$

② $\frac{d}{dx}(\cos x) = -\sin x$

③ $\frac{d}{dx}(\tan x) = \sec^2 x$ ✓.

④ $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ ✓

⑤ $\frac{d}{dx}(\sec x) = \sec x \tan x$ ✓

⑥ $\frac{d}{dx}(\cancel{sec}\operatorname{cosec} x) = \operatorname{cosec} x \cot x$

Q :1: $y = \tan x$

prove that $\frac{d}{dx}\tan x = \sec^2 x$

$y = \frac{\sin x}{\cos x}$ $\qquad \because \tan x = \frac{\sin x}{\cos x}$

By applying quotient rule

$$\frac{dy}{dx} = \frac{\cos x \frac{d}{dx} \sin x - \sin x \frac{d}{dx} \cos x}{(\cos x)^2}$$

$$= \frac{(\cos x)(\cos x) - \sin x \cdot -\sin x}{(\cos x)^2}$$

$$= \frac{(\cos x)^2 + (\sin x)^2}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{(\cos x)^2} \qquad \because \cos^2 x + \sin^2 x = 1$$

$$= \frac{1}{(\cos x)^2} = \frac{1}{\cos x} \cdot \frac{1}{\cos x} \qquad \because \frac{1}{\cos x} = \sec x$$

$$= \sec x \cdot \sec x$$

$$= \sec^2 x$$

② $y = \sin 3x + \cos^2 x$

$$\frac{dy}{dx} = \frac{d}{dx} \sin 3x + \frac{d}{dx} (\cos x)^2$$

$$= \cos 3x \cdot (3) + 2(\cos x)^1 \cdot (-\sin x)$$

$$= 3\cos 3x - 2 \sin x \cos x \quad \text{Ans.}$$

$(\cos(3x))^2$

3) $y = \dfrac{\sin x}{1+\cos x}$

$$\frac{dy}{dx} = \frac{1+\cos x \frac{d}{dx}\sin x - \sin x \frac{d}{dx} 1+\cos x}{(1+\cos x)^2}$$

$$= \frac{(1+\cos x) \cdot (\cos x) - \sin x \cdot (-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$$

$$= \frac{\cos x + 1}{(1+\cos x)^2} \qquad \because \cos^2 x + \sin^2 x = 1$$

$$= \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x} \Rightarrow \text{Ans.}$$

(4) $y = (\tan x)^2 = (\tan x)(\tan x)$

This can be solved by product rule, sum-diff rule, quotient rule, chain rule.

Chain rule:

Let

$u = \tan x$

$y = u^2$

$\dfrac{dy}{du} = 2u$ — i)

$\dfrac{du}{dx} = \dfrac{d \tan x}{dx}$

$= \sec^2 x$ — (ii)

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 2u \cdot \sec^2 x$$

$$= 2\tan x \sec^2 x$$

PRACTICE:

$$y = (a+x)(a-x)^{-1}$$

$$\frac{dy}{dx} = (a+x)\frac{d}{dx}(a-x)^{-1} + (a-x)^{-1}\frac{d}{dx}(a+x)$$

$$= (a+x) \cdot -1(a-x)^{-2}(-1) + (a-x)^{-1}(1)$$

$$= (a+x)(a-x)^{-2} + (a-x)^{-1}$$

$$= \frac{a+x}{(a-x)^2} + \frac{1}{(a-x)^1}$$

$$= \frac{a+\cancel{x} + a-\cancel{x}}{(a-x)^2}$$

$$= \frac{2a}{(a-x)^2} \text{ — Ans.}$$

$$y = \sqrt{\frac{a^2+x^2}{a^2-x^2}} = \left(\frac{a^2+x^2}{a^2-x^2}\right)^{1/2}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a^2+x^2}{a^2-x^2}\right)^{1/2} \cdot \frac{d}{dx}\left(\frac{a^2+x^2}{a^2-x^2}\right)$$

$$= \frac{1}{2}\left(\frac{a^2+x^2}{a^2-x^2}\right)^{-1/2} \cdot \cancel{2x}\frac{d}{dx}\left(\frac{a^2+x^2}{a^2-x^2}\right)$$

$$= \frac{1}{2}\left(\frac{a^2+x^2}{a^2-x^2}\right)^{-1/2} \cdot \frac{(a^2-x^2)\frac{d}{dx}(a^2+x^2) - (a^2+x^2)\frac{d}{dx}(a^2-x^2)}{(a^2-x^2)^2}$$

$$= \frac{1}{2}\left(\frac{a^2+x^2}{a^2-x^2}\right)^{-1/2} \cdot \frac{(a^2-x^2).(2x) - (a^2+x^2).-2x}{(a^2-x^2)^2}$$

$$= \frac{1}{2}\left(\frac{a^2+x^2}{a^2-x^2}\right)^{-1/2} \cdot \frac{(a^2-x^2)(2x) + 2x(a^2+x^2)}{(a^2-x^2)^2}$$

$$= \frac{1}{2}\left(\frac{a^2-x^2}{a^2+x^2}\right)^{1/2} \cdot \frac{2xa^2 - \cancel{2x^3} + 2xa^2 + \cancel{2x^3}}{(a^2-x^2)^2}$$

$$= \frac{1}{\cancel{2}}\left(\frac{a^2-x^2}{a^2+x^2}\right)^{1/2} \cdot \frac{\overset{2}{\cancel{4}}xa^2}{(a^2-x^2)^2}$$

$$= \frac{(a^2-x^2)^{1/2}}{(a^2+x^2)^{1/2}} \cdot \frac{2xa^2}{(a^2-x^2)^2}$$

$$= \frac{(a^2-x^2)^{1/2-2}}{(a^2+x^2)^{1/2}} \cdot 2xa^2$$

$$= \frac{(a^2-x^2)^{-3/2}}{(a^2+x^2)^{1/2}} \cdot 2xa^2$$

$$= \frac{2xa^2}{(a^2+x^2)^{1/2}(a^2-x^2)^{3/2}} \quad \text{Ans.}$$

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14 Nov. **Lecture No. 11** Thursday

(5) $y = \sin^2 x$

$\because \sin^2 x$ means $(\sin x)^2$ & $\sin x^2$ means θ^2 only

$y = (\sin x)^2$

$\frac{dy}{dx} = \frac{d}{dx}(\sin x)^2 \cdot \frac{d}{dx}\sin x$

$= 2(\sin x)^1 \cdot \cos x$ Ans.

(6) $y = \sin x^2$

$\frac{dy}{dx} = \frac{d}{dx}(\sin x^2) \cdot \frac{d}{dx} x^2$

$= \cos x^2 \cdot 2x$

$= 2x \cos x^2$ (Ans)

(7) $y = \sin^2 3x$

$\frac{dy}{dx} = \frac{d}{dx}(\sin 3x)^2 \cdot \frac{d}{dx}(\sin 3x) \cdot \frac{d}{dx} 3x$

$= 2(\cos 3x) \cdot \cos 3x \cdot 3(1)$ Ans.

(8) $k = Ae^{-Ea/RT}$

⇒ Here k & T are variables & remaining $Ae^{-Ea/R}$ is constant.

⇒ This shows rate of change of rate constant with respect to change in Temp.

⇒ with increase in temp the rate of reaction increases.

e.g; $\frac{dk}{dT} = \frac{d(5)}{d(25^\circ)}$

with rise in temp rate of reaction increases v. rapidly

$= \frac{d(10)}{d(50^\circ)}$

$k = Ae^{-Ea/RT}$

$\frac{dk}{dT} = A \frac{d}{dT} e^{-Ea/RT} \cdot \frac{d}{dT} \frac{-Ea}{RT}$

$= A \cdot e^{\frac{-Ea}{RT}} \cdot \frac{-Ea}{R} \frac{d}{dT} \frac{1}{T}$ ∵ $\frac{d}{dT}\left(\frac{1}{T}\right)$

$= Ae^{-Ea/RT} \cdot \frac{-Ea}{R} \cdot \left(-\frac{1}{T^2}\right)$ $= \frac{d}{dT} T^{-1}$

$= -T^{-1-1}$

$= -T^{-2}$

$= -\frac{1}{T^2}$

$$\boxed{\frac{dk}{dT} = Ae^{\frac{-Ea}{RT}} \cdot + \frac{Ea}{RT^2}}$$

(9) $y = e^{\ln(x^2+1)}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{\ln(x^2+1)} \cdot \frac{d}{dx} \ln(x^2+1) \cdot \frac{d}{dx} x^2$$

$$= e^{\ln(x^2+1)} \cdot \frac{1}{x^2+1} \cdot 2x$$

$$= \frac{2x}{x^2+1} \cdot e^{\ln(x^2+1)} \Rightarrow \text{Ans}$$

IMPLICIT DIFFERENTIATION:

The differentiation in which x & y can not be separated.

$$x^2 + \underset{\downarrow}{\overset{\text{product rule}}{xy}} + y^2 = 1$$

$$\frac{d}{dx} x^2 + y \frac{d}{dx} x + x \frac{d}{dy} y \frac{dy}{dx} + \frac{d}{dy} y^2 \frac{dy}{dx} = 0$$

$$2x + y(1) + x(1) \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x+2y) = -2x - y$$

∴ Here chain rule applied

$$= \frac{d}{dy} y \frac{dy}{dx}$$

$$= (1) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

$$= \frac{-(2x+y)}{x+2y} \text{ Ans.}$$

15 Nov — LECTURE No. 12 — Friday

1) $x^3y^5 + 3x = 8y^3 + 1$

$y^5 \frac{d}{dx} x^3 + x^3 \frac{d}{dy} y^5 \frac{dy}{dx} + 3\frac{d}{dx} x = 8\frac{d}{dy} y^3 \frac{dy}{dx} + 0$

$y^5(3x^2) + x^3(5y^4)\frac{dy}{dx} + 3 = 8(3y^2)\frac{dy}{dx}$

$3x^2y^5 + 5x^3y^4\frac{dy}{dx} + 3 = 24y^2\frac{dy}{dx}$

$5x^3y^4\frac{dy}{dx} - 24y^2\frac{dy}{dx} = -3 - 3x^2y^5$

$\frac{dy}{dx}(5x^3y^4 - 24y^2) = -(3 + 3x^2y^5)$

$\frac{dy}{dx} = -\frac{3 + 3x^2y^5}{5x^3y^4 - 24y^2}$ — Ans

2) $x^2 \tan y + y^{10} \sec x = 2x$

$\tan y \frac{d}{dx} x^2 + x^2 \frac{d}{dy} \tan y \frac{dy}{dx} + y^{10} \frac{d}{dx} \sec x + \sec x \frac{d}{dy} y^{10}$

$= 2\frac{d}{dx} x$

15 Nov   LECTURE No. 12   Friday

1) $x^3y^5 + 3x = 8y^3 + 1$

$y^5\frac{d}{dx}x^3 + x^3\frac{d}{dy}y^5\frac{dy}{dx} + 3\frac{d}{dx}x = 8\frac{d}{dy}y^3\frac{dy}{dx} + 0$

$y^5(3x^2) + x^3(5y^4)\frac{dy}{dx} + 3 = 8(3y^2)\frac{dy}{dx}$

$3x^2y^5 + 5x^3y^4\frac{dy}{dx} + 3 = 24y^2\frac{dy}{dx}$

$5x^3y^4\frac{dy}{dx} - 24y^2\frac{dy}{dx} = -3 - 3x^2y^5$

$\frac{dy}{dx}(5x^3y^4 - 24y^2) = -(3 + 3x^2y^5)$

$\frac{dy}{dx} = -\frac{3+3x^2y^5}{5x^3y^4 - 24y^2}$ — Ans

2) $x^2\tan y + y^{10}\sec x = 2x$

$\tan y\frac{d}{dx}x^2 + x^2\frac{d}{dy}\tan y\frac{dy}{dx} + y^{10}\frac{d}{dx}\sec x + \sec x\frac{d}{dy}y^{10}\cdot\frac{dy}{dx}$

$= 2\frac{d}{dx}x$

$\tan y(2x) + x^2(\sec^2 y)\frac{dy}{dx} + y^{10}($

$\sec x(10y^9)\frac{dy}{dx} =$

$2x\tan y + (\sec^2 x^2 y)\frac{dy}{dx} + y^{10}$

$(10y^9\sec x)\frac{dy}{dx} = 2$

$\sec^2 y\, x^2\frac{dy}{dx} + 10y^9\sec x\frac{dy}{dx}$

$\frac{dy}{dx}(\sec^2 y\, x^2 + 10y^9\sec x)$

$\frac{dy}{dx} = \frac{2 - 2x\tan y - y^{10}}{x^2\sec^2 y + 10y^9}$

③ $e^{2x+3y} = x^2 - \ln$

Sol: $\frac{d}{dx}e^{2x+3y}\cdot\frac{d}{dx}(2x+3y$

$$\tan y\,(2x) + x^2(\sec^2 y)\frac{dy}{dx} + y^{10}(\sec x \tan x) + \sec x\,(10y^9)\frac{dy}{dx} = 2$$

$$2x\tan y + (\sec^2 x^2 y)\frac{dy}{dx} + y^{10}(\sec x\tan x) + (10y^9 \sec . x)\frac{dy}{dx} = 2$$

$$\sec^2 y\, x^2)\frac{dy}{dx} + 10y^9 \sec x \frac{dy}{dx} = 2 - 2x\tan y - y^{10}\sec x\tan x$$

$$\frac{dy}{dx}(\sec^2 y\, x^2 + 10y^9\sec x) = 2 - 2x\tan y - y^{10}\sec x\tan x$$

$$\frac{dy}{dx} = \frac{2-2x\tan y - y^{10}\sec x\tan x}{x^2\sec^2 y + 10y^9\sec x}$$ Ans.

③ $e^{2x+3y} = x^2 - \ln(xy^3)$

$\frac{dy}{dx}$ Sol:

$$\frac{d}{dx}e^{2x+3y}\cdot\frac{d}{dx}(2x+3y) = \frac{d}{dx}x^2 - \frac{d}{dx}\ln(xy^3).\ \frac{d}{dx}(xy^3)$$

$$\frac{d}{dx}e^{2x+3y}\cdot 2\frac{d}{dx}x + 3\frac{d}{dy}y\frac{dy}{dx} = \frac{d}{dx}x^2 - \frac{d}{dx}\ln(xy^3). \rightarrow$$

$$y^3 \frac{d}{dx} x + x \frac{d}{dy} y^3 \frac{dy}{dx}$$

$$e^{2x+3y} \cdot \left(2(1) + 3\frac{dy}{dx}\right) = 2x - \frac{1}{xy^3} \cdot y^3(1) + x3y^2\frac{dy}{dx}$$

$$2e^{2x+3y} + 3e^{2x+3y}\frac{dy}{dx} = 2x - \frac{1}{xy^3}\left(y^3 + x(3y^2)\frac{dy}{dx}\right)$$

$$2e^{2x+3y} + 3e^{2x+3y}\frac{dy}{dx} = 2x - \left(\frac{1}{xy^3} \cdot y^3 + \frac{1}{xy^3} \cdot x(3y^2)\frac{dy}{dx}\right)$$

$$= 2x - \left[\frac{1}{x} + \frac{1}{y}(3)\frac{dy}{dx}\right]$$

$$2e^{2x+3y} + 3e^{2x+3y}\frac{dy}{dx} = 2x - \frac{1}{x} - \frac{3}{y}\frac{dy}{dx}$$

$$3e^{2x+3y}\frac{dy}{dx} + \frac{3}{y}\frac{dy}{dx} = 2x - \frac{1}{x} - 2e^{2x+3y}$$

$$\frac{dy}{dx}\left[3e^{2x+3y} + \frac{3}{y}\right] = \frac{2x^2 - 1 - 2xe^{2x+3y}}{x}$$

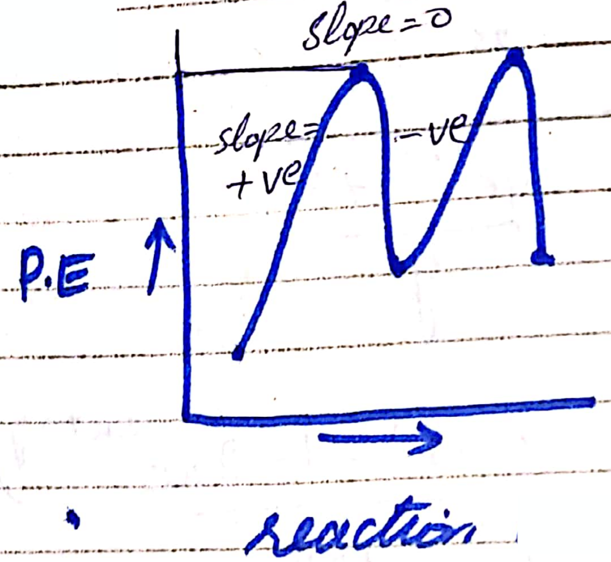
$$\frac{dy}{dx}\left[\frac{3ye^{2x+3y} + 3}{y}\right] = \frac{2x^2 - 1 - 2xe^{2x+3y}}{x}$$

Scanned with CamScanner

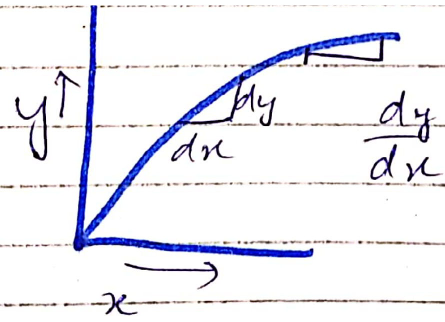
$$\frac{dy}{dx} = \frac{\dfrac{2x^2 - 1 - 2xe^{2x+3y}}{x}}{\dfrac{3ye^{2x+3} + 3}{y}}$$

$$\frac{dy}{dx} = \frac{y\left(2x^2 - 1 - 2xe^{2x+3y}\right)}{x\left(3ye^{2x+3} + 3\right)} \quad \text{Ans.}$$

$$\frac{dy}{dx} = f'(x) = y'$$



$$f(x) = x^3 - 6x^2 + 9x$$



$\frac{dy}{dx}$ = slope

FIRST DERIVATIVE TEST:

$$f(x) = x^3 - 6x^2 + 9x$$

$$= \frac{d}{dx}x^3 - 6\frac{d}{dx}x^2 + 9\frac{d}{dx}x$$

$$f'(x) = 3x^2 - 12x + 9 \quad \text{—(i)}$$

At Extreme points:

$$3x^2 - 12x + 9 = 0$$

$$3(x^2 - 4x + 3) = 0$$

$$x^2 - 4x + 3 = 0$$

Facterization

$$x^2 - 3x - x + 3 = 0$$

$$x(x-3) - 1(x-3) = 0$$

$$(x-3)(x-1) = 0$$

$$x - 3 = 0 \qquad x - 1 = 0$$

$$\boxed{x = +3} \qquad \boxed{x = +1}$$

(3, 2)

in digits ki above or below wali values eq is me lgani h

$\therefore x = 3$

$$f'(2) = 3(2)^2 - 12(2) + 9$$

$$= 12 - 24 + 9$$

$$= -3$$

$$f'(4) = 3(4)^2 - 12(4) + 9$$

$$= 48 - 48 + 9$$

$$= +9$$

Minimum.

$\therefore x = 1$

$f'(0) = 3(0)^2 - 12(0) + 9$

$= +9$ ← Maximum

$f'(2) = -3$

2nd derivative test:

$f'(x) = 3x^2 - 12x + 9$

$f''(x) = \frac{3d}{dx} x^2 - 12 \frac{d}{dx} x + \frac{d}{dx} 9$

$= 6x - 12$

$\therefore x = 1$

$f''(1) = 6(1) - 12$

$= 6 - 12$

$= -6 < 0$

Maximum

$\therefore x = 3$

$f''(3) = 6(3) - 12$

$= 18 - 12$

$= +6 > 0$ Minimum

$f(x) = y$

Q: find y?

$f(x) = x^3 - 6x^2 + 9x$

$\because x = 1$

$y = x^3 - 6x^2 + 9x$

$y = 1 - 6 + 9$

$y = 4$

(x, y)

$(1, 4)$

$\because x = 3$

$y = x^3 - 6x^2 + 9x$

$= (3)^3 - 6(3)^2 + 9(3)$

$= 27 - 54 + 27$

$= 54 - 54$

$y = 0$

(x, y)

$(3, 0)$

3) $E = f(T, V)$

$$dE = \left(\frac{\partial E}{\partial T}\right)_V dT + \left(\frac{\partial E}{\partial V}\right)_T dV$$

④ $H = f(T, P, n_1, n_2 \ldots)$

$$dH = \left(\frac{\partial E}{\partial T}\right)_{(P, n_1, n_2 \cdots)} dT + \left(\frac{\partial E}{\partial P}\right)_{(T, n_1, n_2 \cdots)} dP + \left(\frac{\partial E}{\partial n_1}\right)_{T, P, n_2 \cdots} dn_1$$

EXACT DIFFRENTIAL:

used for / Depends on state function

depends on initial & final state.

e.g; dH. It is represented by 'd'.

INEXACT DIFFRENTIAL:

used for / Depends on path function

depends on path not on initial & final ~~on~~ state

e.g; δw, δq.

It is represented by δ.

① $y = \dfrac{\sqrt[3]{x}}{1+x^2} = \dfrac{(x)^{1/3}}{1+x^2}$

$$= \frac{1+x^2 \frac{d}{dx}(x)^{1/3} - (x)^{1/3}\frac{d}{dx}\,1+x^2}{(1+x^2)^2}$$

$$= \frac{(1+x^2).\left(\frac{1}{3}x^{-2/3}\right) - (x)^{1/3}.\,2x}{(1+x^2)^2}$$

$$= \frac{\frac{1}{3}(1+x^2) - 2x^{4/3}}{(x^{2/3})(1+x^2)^2}$$ —Ans.

2) $y = \sqrt{x^2 + \sqrt{1+4x}}$

Let $u = x^2 + \sqrt{1+4x}$ $\quad = \frac{1}{2} - 1$

then: $\quad \frac{1-2}{2}$

$$y = \sqrt{u} = u^{1/2}$$

$$\frac{dy}{du} = \frac{1}{2}u^{-1/2} \quad -(i)$$

$$\frac{du}{dx} = x^2 + \sqrt{1+4x}$$

$$= x^2 + (1+4x)^{1/2}$$

$$= \frac{d}{dx}x^2 + \frac{d}{dx}(1+4x)^{1/2}.\frac{d}{dx}(1+4x)$$

$$= 2x + \frac{1}{2}(1+4)^{-1/2} \cdot (4)$$

$$= 2x + (1+4)^{-1/2} \cdot 2$$

$$= 2x + 2(1+4)^{-1/2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \left[\frac{1}{2}u^{-1/2}\right] \cdot \left[2x + 2(1+4)^{-1/2}\right]$$

Put value of 'u'

$$= \frac{1}{2}(x^2 + \sqrt{1+4x})^{-1/2} \cdot \left[2x + 2(1+4)^{-1/2}\right]$$

3) $y = \cos(\sin x + x^2)$

$$y = \frac{d}{dx} \cos(\sin x + x^2) \cdot \frac{d}{dx}(\sin x + x^2)$$

$$= -\sin(\sin + x^2) \cdot (\cos x + 2x)$$

$$\frac{dy}{dx} = -\sin(\sin + x^2) \cdot (\cos x + 2x)$$

SOLVE BY abinitio Method:-

① $f(x) = \dfrac{x+1}{x+4}$ —(i)

$\because \delta x = h$ also

$$f(x+\delta x) = \frac{x+\delta x+1}{x+\delta x+4} \quad \text{—(ii)}$$

$$f(x+\delta x) - f(x) = \frac{x+\delta x+1}{x+\delta x+4} - \frac{x+1}{x+4}$$

$$= \frac{(x+\delta x+1)(x+4) - (x+1)(x+\delta x+4)}{(x+\delta x+4)(x+4)}$$

$$= \frac{(x^2+\delta x^2+x+4x+4\delta x+4) - (x^2+\delta x^2+4x+x+\delta x+4)}{(x+\delta x+4)(x+4)}$$

$$= \frac{\cancel{x^2}+\cancel{\delta x^2}+\cancel{x}+\cancel{4x}+4\delta x+\cancel{4}-\cancel{x^2}-\cancel{\delta x^2} - \cancel{4x}-\cancel{x}-\delta x-\cancel{4}}{(x+\delta x+4)(x+4)}$$

$$= \frac{4\delta x - \delta x}{(x+\delta x+4)(x+4)} = \frac{\delta x(4-1)}{(x+\delta x+4)(x+4)}$$

$$= \frac{3\,\delta x}{(x+\delta x+4)(x+4)}$$

$$\frac{f(x+\delta x) - f(x)}{\delta x} = \frac{3\,\cancel{\delta x}}{\cancel{\delta x}\,(x+\delta x+4)(x+4)}$$

$$\lim_{\delta x \to 0} \frac{f(x+\delta x)-f(x)}{\delta x} = \frac{3}{(x+0+4)(x+4)}$$

$$\frac{dy}{dx} = \frac{3}{(x+4)^2} \quad \text{— Ans.}$$

ii) $f(x) = 2 - \sqrt{x}$

$$f(x) = 2 - \sqrt{(x)} \quad \text{—(i)}$$

$$f(x+\delta x) = 2 - \sqrt{x+\delta x} \quad \text{—(ii)}$$

$$f(x+\delta x) - f(x) = 2 - \sqrt{x+\delta x} - (2 - \sqrt{x})$$

$$= \cancel{2} - \sqrt{x+\delta x} - \cancel{2} + \sqrt{x}$$

$$= +\sqrt{x} - \sqrt{x+\delta x}$$

Rationalization:

$$= \sqrt{x} - \sqrt{x+\delta x} \times \frac{\sqrt{x}+\sqrt{x+\delta x}}{\sqrt{x}+\sqrt{x+\delta x}}$$

$$= \frac{\sqrt{x}^2 - \sqrt{(x+\delta x)}^2}{\sqrt{x} + \sqrt{x+\delta x}}$$

$$= \frac{x - (x+\delta x)}{\sqrt{x} + \sqrt{x+\delta x}}$$

$$= \frac{\cancel{x} - \cancel{x} - \delta x}{\sqrt{x} + \sqrt{x+\delta x}}$$

$$= \frac{-\delta x}{\sqrt{x} + \sqrt{x+\delta x}}$$

$$\frac{f(x+\delta x) - f(x)}{\delta x} = \frac{-\cancel{\delta x}}{\cancel{\delta x}\left(\sqrt{x} + \sqrt{x+\delta x}\right)}$$

$$= \frac{-1}{\sqrt{x} + \sqrt{x+\delta x}}$$

$$\lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} = \lim_{\delta x \to 0} \frac{-1}{\sqrt{x} + \sqrt{x+0}}$$

$$\therefore \frac{dy}{dx} = \frac{-1}{2\sqrt{x}} \quad \text{Ans.}$$

Q2: i) $y = \frac{x^2+1}{x^2-3}$

$$\frac{dy}{dx} = \frac{(x^2-3)\frac{d}{dx}x^2+1 - x^2+1\frac{d}{dx}(x^2-3)}{(x^2-3)^2}$$

$$= \frac{(x^2-3).(2x) - (x^2+1).(2x)}{(x^2-3)^2}$$

$$= \frac{(2x^3-6x) - (2x^3+2x)}{(x^2-3)^2}$$

$$= \frac{\cancel{2x^3}-6x-\cancel{2x^3}-2x}{(x^2-3)^2}$$

$$= \frac{-8x}{(x^2-3)^2} \quad \text{Ans.}$$

ii) $y = \sqrt[3]{x^2}\,(2x-x^2)$

iii) $y = \dfrac{4\sqrt{x}}{x^2-2} = \dfrac{4(x)^{1/2}}{x^2-2}$

$$\frac{dy}{dx} = \frac{(x^2-2)\frac{d}{dx}4(x)^{1/2} - 4(x)^{1/2}\frac{d}{dx}(x^2-2)}{(x^2-2)^2}$$

$$= \frac{(x^2-2)\cdot \overset{2}{\cancel{4}}\left(\frac{1}{\cancel{2}}x^{-1/2}\right) - (4x^{1/2})\cdot(2x)}{(x^2-2)^2}$$

$$= \frac{(x^2-2)\cdot 2(x^{-1/2}) - 8x^{3/2}}{(x^2-2)^2}$$

$\because \frac{1}{2}+1$

$\frac{1+2}{2}$

$$= \frac{\frac{2(x^2-2)}{x^{1/2}} - 8x^{3/2}}{(x^2-2)^2}$$

$$= \left[\frac{2(x^2-2) - 8x^{3/2}(x^{1/2})}{x^{1/2}}\right]\frac{1}{(x^2-2)^2}$$

$$= \left[\frac{(2x^2-4) - 8x^{\cancel{4/2}\,2}}{x^{1/2}}\right]\frac{1}{(x^2-2)^2}$$

$\because \frac{3}{2}+\frac{1}{2}$

$\frac{3+1}{2} = \frac{4}{2}$

$$= \left[\frac{2x^2-4-8x^2}{x^{1/2}}\right]\frac{1}{(x^2-2)^2}$$

$$= \left[\frac{-6x^2 - 4}{x^{1/2}} \right] \frac{1}{(x^2-2)^2}$$

$$= \frac{-6x^2 - 4}{(x^{1/2})(x^2-2)^2} \quad \text{— Ans.}$$

iv) $x^2 + y^2 = 4$

Implicit differentiation:

$$\frac{d}{dx} x^2 + \frac{d}{dy} y^2 \frac{dy}{dx} = 0$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{2x}{2y} \qquad \frac{dy}{dx} = y'$$

$$y' = -x/y \quad \text{— Ans}$$

v) $x^2 - xy - x^2 + y = 0$

$$\frac{d}{dx} x^2 - \frac{d}{dx} xy - \frac{d}{d}$$

v) $y^2 - xy - x^2 + 4 = 0$

$$\frac{d}{dy} y^2 \frac{dy}{dx} - \frac{d}{dx} xy - \frac{d}{dx} x^2 + 0 = 0$$

$$\frac{d}{dy} y^2 \frac{dy}{dx} - \left[x \frac{d}{dy} y \frac{dy}{dx} + y \frac{d}{dx} x \right] - \frac{d}{dx} x^2 = 0$$

$$2y \cdot \frac{dy}{dx} - \left[x(1) \frac{dy}{dx} + y(1) \right] - 2x = 0$$

$$2y \cdot \frac{dy}{dx} - x \frac{dy}{dx} - y - 2x = 0$$

$$2y \frac{dy}{dx} - x \frac{dy}{dx} = 2x + y$$

$$\frac{dy}{dx}(\cancel{x}\,2y - x) = 2x + y$$

$$\frac{dy}{dx} = \frac{2x+y}{2y-x} \quad \text{— Ans.}$$