LIMITS:

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 0 \text{ maths error.}$$

(where x goes to 1) at this value we can not find value. it gives maths error. Function is undefined

x	$f(x)$
0.5	1.5
0.7	1.7
0.9	1.9
0.99	1.99
0.999	1.999
1	undefined
1.001	2.001
1.01	2.01
1.1	2.1
1.3	2.3

e.g
$$= \frac{x^2 - 1}{x - 1}$$
$$= \frac{(0.5)^2 - 1}{0.5 - 1}$$
$$= 1.5$$

Tends or

When x approaches 1 then function approaches 2.

$$= \lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x - 1)(x + 1)}{x - 1}$$

$$= \lim_{x \to 1} 1 + 1$$

$$= 1 + 1 = 2$$

Example   Evaluate the following limit.

$$\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - 2x}$$

Solution

First let's notice that if we try to plug in $x = 2$ we get,

$$\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - 2x} = \frac{0}{0}$$

So, we can't just plug in $x = 2$ to evaluate the limit. So, we're going to have to do something else.

The first thing that we should always do when evaluating limits is to simplify the function as much as possible. In this case that means factoring both the numerator and denominator. Doing this gives,

$$\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - 2x} = \lim_{x \to 2} \frac{(x - 2)(x + 6)}{x(x - 2)}$$

$$= \lim_{x \to 2} \frac{x + 6}{x}$$

So, upon factoring we saw that we could cancel an $x - 2$ from both the numerator and the denominator. Upon doing this we now have a new rational expression that we can plug $x = 2$ into because we lost the division by zero problem. Therefore, the limit is,

$$\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - 2x} = \lim_{x \to 2} \frac{x + 6}{x} = \frac{8}{2} = 4$$

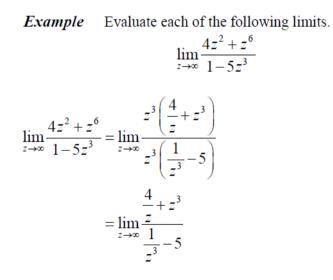*Example*   Evaluate the following limit.

$$\lim_{t \to -\infty} \frac{t^2 - 5t - 9}{2t^4 + 3t^3}$$

Solution

In this case it looks like we will factor a t^4 out of both the numerator and denominator. Doing this gives,

$$\lim_{t \to -\infty} \frac{t^2 - 5t - 9}{2t^4 + 3t^3} = \lim_{t \to -\infty} \frac{t^4 \left(\dfrac{1}{t^2} - \dfrac{5}{t^3} - \dfrac{9}{t^4} \right)}{t^4 \left(2 + \dfrac{3}{t} \right)}$$

$$= \lim_{t \to -\infty} \frac{\dfrac{1}{t^2} - \dfrac{5}{t^3} - \dfrac{9}{t^4}}{2 + \dfrac{3}{t}}$$

$$= \frac{0}{2}$$

$$= 0$$

Example   Evaluate each of the following limits.

$$\lim_{z \to \infty} \frac{4z^2 + z^6}{1 - 5z^3} \qquad\qquad \lim_{z \to -\infty} \frac{4z^2 + z^6}{1 - 5z^3}$$

$$\lim_{z \to \infty} \frac{4z^2 + z^6}{1 - 5z^3} = \lim_{z \to \infty} \frac{z^3 \left(\dfrac{4}{z} + z^3 \right)}{z^3 \left(\dfrac{1}{z^3} - 5 \right)}$$

$$= \lim_{z \to \infty} \frac{\dfrac{4}{z} + z^3}{\dfrac{1}{z^3} - 5}$$

$$\lim_{z \to \infty} \frac{4z^2 + z^6}{1 - 5z^3} = \frac{\infty}{-5} = -\infty$$

Problems

1. $\lim\limits_{x \to 2} \left(8 - 3x + 12x^2\right)$

2. $\lim\limits_{t \to -3} \dfrac{6 + 4t}{t^2 + 1}$

3. $\lim\limits_{x \to -5} \dfrac{x^2 - 25}{x^2 + 2x - 15}$

4. $\lim\limits_{z \to 8} \dfrac{2z^2 - 17z + 8}{8 - z}$

5. $\lim\limits_{y \to 7} \dfrac{y^2 - 4y - 21}{3y^2 - 17y - 28}$

6. $\lim\limits_{h \to 0} \dfrac{(6 + h)^2 - 36}{h}$

7. $\lim\limits_{z \to 4} \dfrac{\sqrt{z} - 2}{z - 4}$

First we will assume that $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ exist and that c is any constant. Then,

1. $\lim\limits_{x \to a} \left[cf(x) \right] = c \lim\limits_{x \to a} f(x)$

   In other words we can "factor" a multiplicative constant out of a limit.

2. $\lim\limits_{x \to a} \left[f(x) \pm g(x) \right] = \lim\limits_{x \to a} f(x) \pm \lim\limits_{x \to a} g(x)$

   So to take the limit of a sum or difference all we need to do is take the limit of the individual parts and then put them back together with the appropriate sign. This is also not limited to two functions. This fact will work no matter how many functions we've got separated by "+" or "-".

3. $\lim\limits_{x \to a} \left[f(x) g(x) \right] = \lim\limits_{x \to a} f(x) \ \lim\limits_{x \to a} g(x)$

   We take the limits of products in the same way that we can take the limit of sums or differences. Just take the limit of the pieces and then put them back together. Also, as with sums or differences, this fact is not limited to just two functions.

4. $\lim\limits_{x \to a} \left[\dfrac{f(x)}{g(x)} \right] = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$,   provided $\lim\limits_{x \to a} g(x) \neq 0$

   As noted in the statement we only need to worry about the limit in the denominator being zero when we do the limit of a quotient. If it were zero we would end up with a division by zero error and we need to avoid that.

5. $\lim\limits_{x \to a} \left[f(x) \right]^n = \left[\lim\limits_{x \to a} f(x) \right]^n$,   where n is any real number

   In this property n can be any real number (positive, negative, integer, fraction, irrational, zero, *etc.*). In the case that n is an integer this rule can be thought of as an extended case of **3**.

   For example consider the case of $n = 2$.

$$\lim_{x \to a}\left[f(x)\right]^2 = \lim_{x \to a}\left[f(x)f(x)\right]$$
$$= \lim_{x \to a}f(x)\lim_{x \to a}f(x) \qquad \text{using property 3}$$
$$= \left[\lim_{x \to a}f(x)\right]^2$$

The same can be done for any integer n.

6. $\lim_{x \to a}\left[\sqrt[n]{f(x)}\right] = \sqrt[n]{\lim_{x \to a}f(x)}$

   This is just a special case of the previous example.

   $$\lim_{x \to a}\left[\sqrt[n]{f(x)}\right] = \lim_{x \to a}\left[f(x)\right]^{\frac{1}{n}}$$
   $$= \left[\lim_{x \to a}f(x)\right]^{\frac{1}{n}}$$
   $$= \sqrt[n]{\lim_{x \to a}f(x)}$$

7. $\lim_{x \to a}c = c, \qquad c$ is any real number

   In other words, the limit of a constant is just the constant. You should be able to convince yourself of this by drawing the graph of $f(x) = c$.

8. $\lim_{x \to a}x = a$

   As with the last one you should be able to convince yourself of this by drawing the graph of $f(x) = x$.

9. $\lim_{x \to a}x^n = a^n$

   This is really just a special case of property **5** using $f(x) = x$.