Review : Common Graphs

The purpose of this section is to make sure that you're familiar with the graphs of many of the basic functions that you're liable to run across in a calculus class.

Example Graph $y = -\frac{2}{5}x + 3$.

Solution

This is a line in the slope intercept form

$$y = mx + b$$

In this case the line has a *y* intercept of (0,*b*) and a slope of *m*. Recall that slope can be thought of as

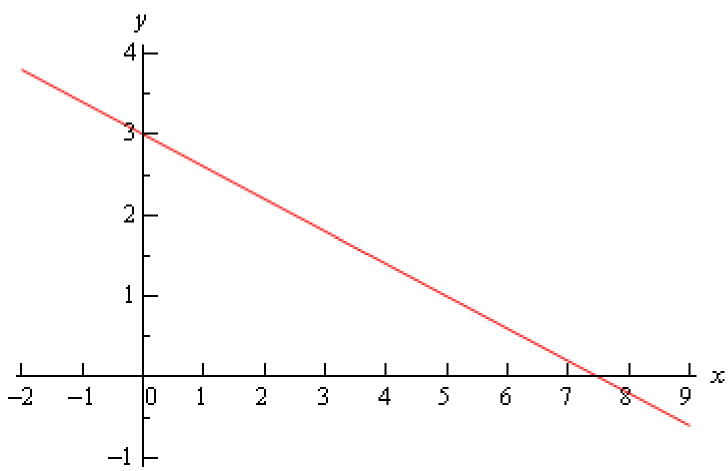
$$m = \frac{\text{rise}}{\text{run}}$$

Note that if the slope is negative we tend to think of the rise as a fall.

The slope allows us to get a second point on the line. Once we have any point on the line and the slope we move right by *run* and up/down by *rise* depending on the sign. This will be a second point on the line.

In this case we know (0,3) is a point on the line and the slope is $-\frac{2}{5}$. So starting at (0,3) we'll move 5 to the right (*i.e.* $0 \to 5$) and down 2 (*i.e.* $3 \to 1$) to get (5,1) as a second point on the line. Once we've got two points on a line all we need to do is plot the two points and connect them with a line.

Here's the sketch for this line.



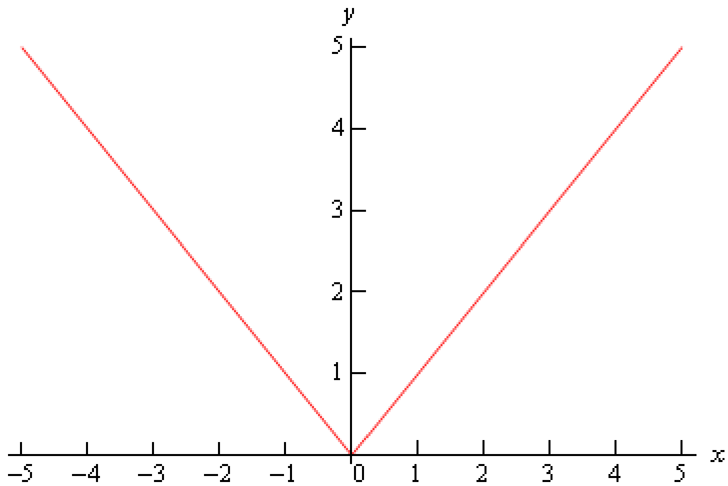
Example Graph $f(x) = |x|$

Solution

There really isn't much to this problem outside of reminding ourselves of what absolute value is. Recall that the absolute value function is defined as,

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

The graph is then,



Example Graph $f(x) = -x^2 + 2x + 3$.

Solution

This is a parabola in the general form.

$$f(x) = ax^2 + bx + c$$

In this form, the *x*-coordinate of the vertex (the highest or lowest point on the parabola) is $x = -\frac{b}{2a}$ and we get the *y*-coordinate is $y = f\left(-\frac{b}{2a}\right)$. So, for our parabola the coordinates of the vertex will be.

$$x = -\frac{2}{2(-1)} = 1$$

$$y = f(1) = -(1)^2 + 2(1) + 3 = 4$$

So, the vertex for this parabola is (1,4).

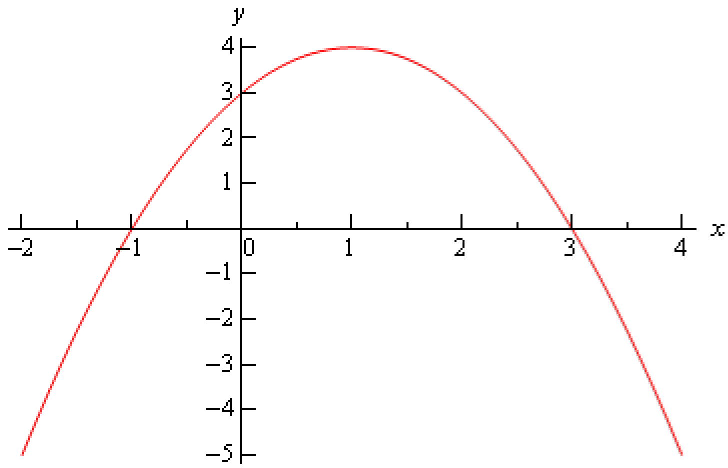
We can also determine which direction the parabola opens from the sign of *a*. If *a* is positive the parabola opens up and if *a* is negative the parabola opens down. In our case the parabola opens down.

Now, because the vertex is above the *x*-axis and the parabola opens down we know that we'll have *x*-intercepts (*i.e.* values of *x* for which we'll have $f(x)=0$) on this graph. So, we'll solve the following.

$$-x^2+2x+3=0$$
$$x^2-2x-3=0$$
$$(x-3)(x+1)=0$$

So, we will have *x*-intercepts at $x=-1$ and $x=3$. Notice that to make our life easier in the solution process we multiplied everything by -1 to get the coefficient of the x^2 positive. This made the factoring easier.

Here's a sketch of this parabola.



Example Graph $f(y)=y^2-6y+5$

Solution
Most people come out of an Algebra class capable of dealing with functions in the form $y=f(x)$. However, many functions that you will have to deal with in a Calculus class are in the form $x=f(y)$ and can only be easily worked with in that form. So, you need to get used to working with functions in this form.

The nice thing about these kinds of function is that if you can deal with functions in the form $y=f(x)$ then you can deal with functions in the form $x=f(y)$ even if you aren't that familiar with them.

Let's first consider the equation.

$$y=x^2-6x+5$$

This is a parabola that opens up and has a vertex of (3,-4), as we know from our work in the previous example.

For our function we have essentially the same equation except the x and y's are switched around. In other words, we have a parabola in the form,

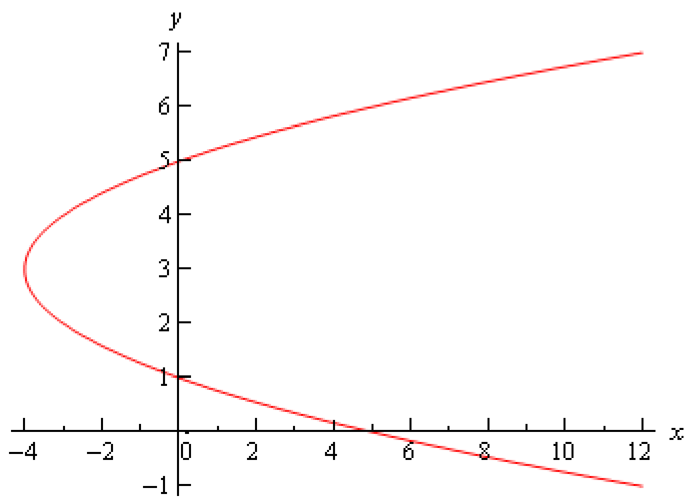
$$x = ay^2 + by + c$$

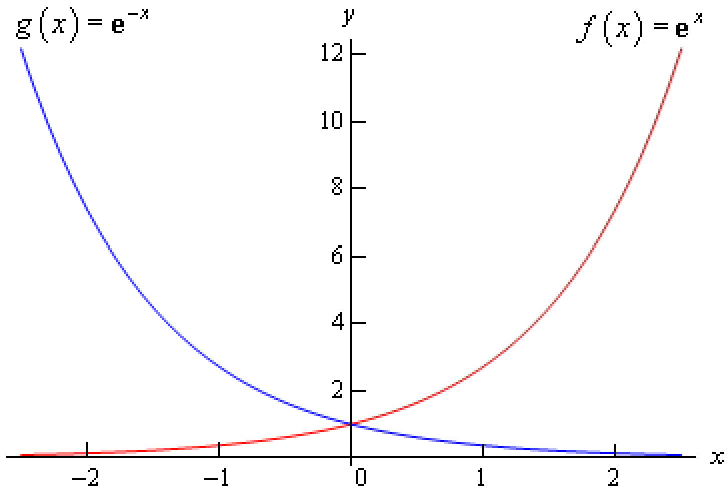
This is the general form of this kind of parabola and this will be a parabola that opens left or right depending on the sign of a. The y-coordinate of the vertex is given by $y = -\frac{b}{2a}$ and we find the x-coordinate by plugging this into the equation. So, you can see that this is very similar to the type of parabola that you're already used to dealing with.

Now, let's get back to the example. Our function is a parabola that opens to the right (a is positive) and has a vertex at (-4,3). The vertex is to the left of the y-axis and opens to the right so we'll need the y-intercepts (*i.e.* values of y for which we'll have $f(y) = 0$)). We find these just like we found x-intercepts in the previous problem.

$$y^2 - 6y + 5 = 0$$
$$(y-5)(y-1) = 0$$

So, our parabola will have y-intercepts at $y = 1$ and $y = 5$. Here's a sketch of the graph.

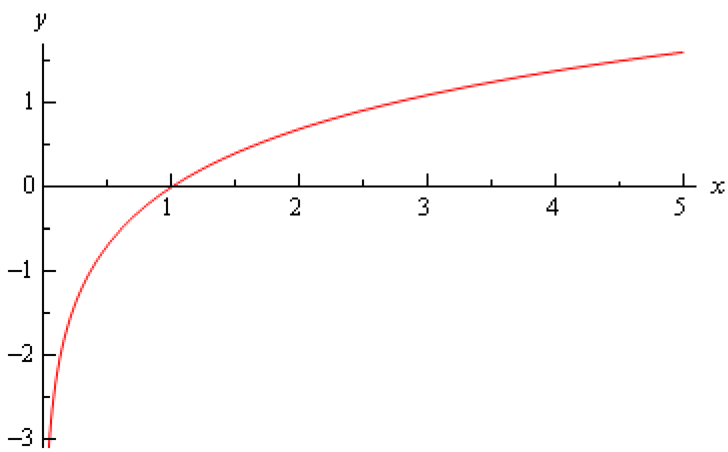




Example Graph $f(x) = \ln(x)$.

Solution

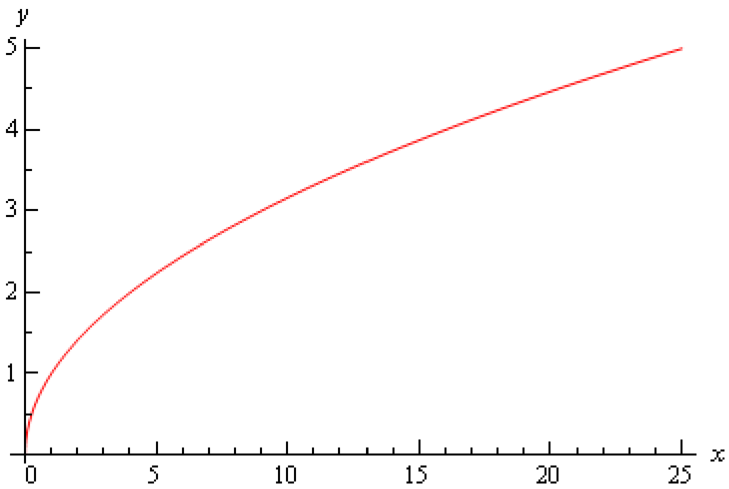
This has already been graphed once in this review, but this puts it here with all the other "important" graphs.



Example Graph $y = \sqrt{x}$.

Solution

This one is fairly simple, we just need to make sure that we can graph it when need be.

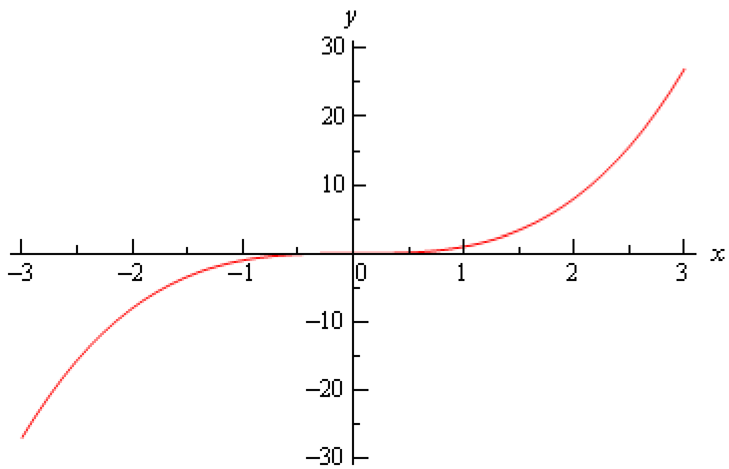


Remember that the domain of the square root function is $x \geq 0$.

Example Graph $y = x^3$

Solution

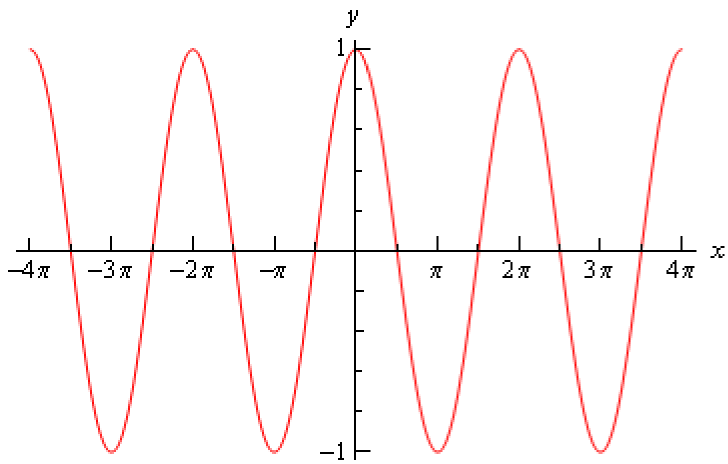
Again, there really isn't much to this other than to make sure it's been graphed somewhere so we can say we've done it.



Example Graph $y = \cos(x)$

Solution

There really isn't a whole lot to this one. Here's the graph for $-4\pi \le x \le 4\pi$.



Let's also note here that we can put all values of x into cosine (which won't be the case for most of the trig functions) and so the domain is all real numbers. Also note that

$$-1 \le \cos(x) \le 1$$

It is important to notice that cosine will never be larger than 1 or smaller than -1. This will be useful on occasion in a calculus class. In general we can say that

$$-R \le R\cos(\omega x) \le R$$

Example 13 Graph $y = \sin(x)$

Solution

As with the first problem in this section there really isn't a lot to do other than graph it. Here is the graph.