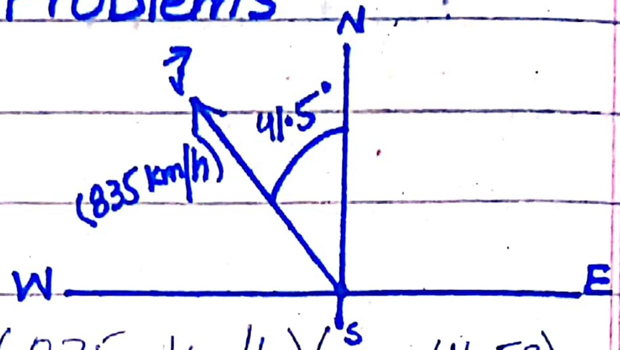
Chapter # 3
Kinematics in Two or Three Dimensions; Vectors

Problems

7. Find.

$V_n, V_w = ?$

$\Delta d = ?$



$V_{north} = (835\ km/h)(\cos 41.5°)$

$\boxed{V_{north} = 625\ km/h}$

$V_{west} = (835\ km/h)(\sin 41.5°)$

$\boxed{V_{west} = 553\ km/h}$

$= \Delta d_{north} =$

$= 625 \times 2.50$

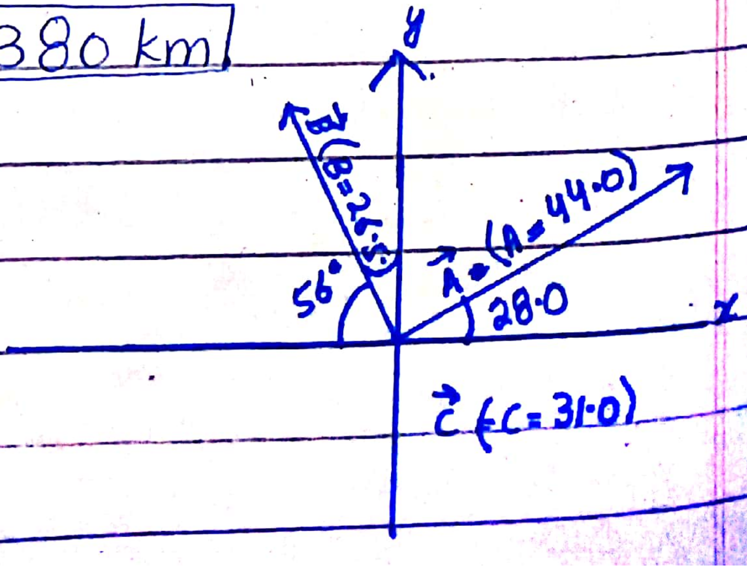
$\boxed{\Delta d = 1560\ km}$

Δd_{west}

$= 553 \times 2.50$

$\boxed{\Delta d = 1380\ km}$

11.



Determine $\vec{B}-\vec{A}$

Determine $\vec{A}-\vec{B}$

Compare both

$\therefore x = \cos$

$\therefore y = \sin$

$A_x = 44.0 \cos 28^\circ = 39.18$

$B_x = -26.5 \cos 56^\circ = -14.82$

$A_y = 44.0 \sin 28 = 18.7$

$B_y = 26.5 \sin 56 = 20.4$

(a)

* $(\vec{B}-\vec{A})_x$

$= (-14.82) - 39.18 = -54$

* $(\vec{B}-\vec{A})_y$

$= 20.4 - 18.7 = 1.7$

Note x component is $-ve$

y component is $+ve$

Then vector is in II Quad.

$\vec{B}-\vec{A} = -54\hat{i} + 1.7\hat{j}$

Magnitude:

$|\vec{B}-\vec{A}| = \sqrt{(-54)^2 + (1.7)^2}$

$|\vec{B}-\vec{A}| = 54$

Direction:

$\theta = \tan^{-1}\left(\frac{1.31}{-54}\right) = 1.54$ above $-ve$ X-axis

(b) $(\vec{A}-\vec{B})_x$

$= (39.18-(-14.82)) = 54$

$(\vec{A}-\vec{B})_y$

$= 18.7-20.4 = -1.7$

Note x component is +ve

y component is -ve

The vector is in IV Quad.

Magnitude: $\vec{A}-\vec{B} = 54\hat{i}-1.7\hat{j}$

$|\vec{A}-\vec{B}| = \sqrt{(54)^2+(1.7)^2}$

$|\vec{A}-\vec{B}| = 54$

Direction:

$\theta = \tan^{-1}\left(\frac{-1.31}{54}\right)$

$= 1.54$ below +ve x-axis

Compare:

Comparing the Results Show

$\vec{B}-\vec{A} = -(\vec{A}-\vec{B})$

12.

Determine $\vec{A}-\vec{C}$

$A_x = 44.0\cos 28 = 39.81$

$A_y = 44.\ \sin 28 = 18.73$

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$C_x = 31.0 \cos 270° = 0.0$

$C_y = 31.0 \sin 270° = -31.0$

$(\vec{A} - \vec{C})_x = 38.85 - 0.0 = 38.85$

$(\vec{A} - \vec{C})_y = 20.66 - (31.0) = 51.66$

$\vec{A} - \vec{C} = 38.8\hat{i} + 51.7\hat{j}$

Magnitude:

$|\vec{A} - \vec{C}| = \sqrt{(38.85)^2 + (51.66)^2}$

$|\vec{A} - \vec{C}| = 64.6$

Direction:

$\theta = \tan^{-1} \frac{51.66}{38.85}$

$\boxed{\theta = 53.1°}$

13. Determine:

(a) $\vec{B} - 2\vec{A}$ (b) $2\vec{A} - 3\vec{B} + 2\vec{C}$

$A_x = 38.85$ $A_y = 20.66$

$B_x = -14.82$ $B_y = \cancel{21}.97$

$C_x = 0.0$ $C_y = -31.0$

(a) $(\vec{B}-2\vec{A})_x$

$= -14.82 - 2(38.85)$

$= -92.52$

$(\vec{B}-2\vec{A})_y$

$= 21.97 - 2(20.66)$

$= -19.35$

Both are negative

lies in III Quad.

$\vec{B}-2\vec{A} = -92.52\hat{i} - 19.4\hat{j}$

Magnitude:

$|\vec{B}-2\vec{A}| = \sqrt{(-92.52)^2 + (-19.35)^2}$

$= \boxed{94.5}$

Direction:

$\theta = \tan^{-1} \frac{-19.35}{-92.52}$

$= 11.8^\circ$ below $-ve$ x-axis.

(b) $(2\vec{A} - 3\vec{B} + 2\vec{C})_x$

$= 2(38.85) - 3(-14.82) + 2(0)$

$= 122.16$

$(2\vec{A} - 3\vec{B} + 2\vec{C})_y$

$= 2(20.66) - 3(21.97) + 2(-31)$
$= -86.59$

x component is +ve
y component is -ve
the vector is in IV Quad.

$2\vec{A} - 3\vec{B} + 2\vec{C} = 122\hat{i} - 86.6\hat{j}$

Magnitude:

$$|2\vec{A} - 3\vec{B} + 2\vec{C}| = \sqrt{(122.16)^2 + (-86.59)^2}$$
$$= 150$$

Direction.

$$\theta = \tan^{-1} \frac{-86.59}{122.16}$$

$= 35.3^\circ$ below +x-axis.

14. Determine
(a) $\vec{A} - \vec{B} + \vec{C}$ (b) $\vec{A} + \vec{B} - \vec{C}$.
(c) $\vec{C} - \vec{A} - \vec{B}$

$A_x = 38.85$ $A_y = 20.66$
$B_x = -14.82$ $B_y = 21.97$
$C_x = 0$ $C_y = -31$

(a) $(\vec{A}-\vec{B}+\vec{C})_x$

$= 38.85-(14.82)+0 = 53.67$

$(\vec{A}-\vec{B}+\vec{C})_y$

$= 20.66-21.97+(-31.0) = -32.31$

x component is positive

y component is Negative

then vector is in 4th Quad.

$\vec{A}-\vec{B}+\vec{C} = 53.7\hat{i} - 32.3\hat{j}$

Magnitude:

$|\vec{A}-\vec{B}+\vec{C}| = \sqrt{(53.7)^2+(-32.31)^2}$

$= 62.6$

Direction:

$\theta = \tan^{-1}\frac{-32.31}{53.67}$

$= 31^\circ$ below $+x$-axis

(b)

$(\vec{A}+\vec{B}-\vec{C})_x$

$= 38.85+(-14.82)-0.0 = 24.03$

$(\vec{A}+\vec{B}-\vec{C})_y$

$= 20.66+21.97-(-31.0) = 73.63$

$\vec{A} + \vec{B} - \vec{C} = 24.0\hat{i} + 73.6\hat{j}$

Magnitude:

$\vec{A} + \vec{B} - \vec{C} = \sqrt{(24)^2 + (73.6)^2}$

$= 77.5$

Direction:

$\theta = \tan^{-1} \frac{73.63}{24.03}$

$\boxed{\theta = 71.9^\circ}$

(c)

$(\vec{C} - \vec{A} - \vec{B})_x$

$= 0.0 - 38.85 - (-14.82)$

$= -24.03$

$(\vec{C} - \vec{A} - \vec{B})_y$

$= -31.0 - 20.66 - 21.97 = -73.63$

Both components are -ve.
vector is in 3rd Quad.

$\vec{C} - \vec{A} - \vec{B} = -24\hat{i} - 73.6\hat{j}$

Magnitude:

$|\vec{C} - \vec{A} - \vec{B}| = \sqrt{(-24.3)^2 + (-73.63)^2}$

$= 77.5^\circ$

Direction:

$$\theta = \tan^{-1} \frac{-73.63}{-24.03}$$

$= 71.9°$ below $-$ve x-axis

∴ Note that answer to (c) is exact opposite of Answer to (b).

17. $\vec{r} = (9.60t\hat{i} + 8.85\hat{j} - 1.00t^2\hat{k})$ m

Determine particles velocity Acceleration.

$$\vec{r} = (9.60t\hat{i} + 8.85\hat{j} - 1.0t^2\hat{k}) \text{ m}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = (9.60\hat{i} - 2.00t\hat{k}) \text{ m/s}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \boxed{-2.00\hat{k} \text{ m/s}^2}$$

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18.

What was the V_{av} of particle in 17 problem b/w $t = 1s$ & $t = 3s$.
Magnitude of V_{ins} at $t = 2s$ = ?

Average velocity from disp. at two times.

$$\vec{V}_{avg} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$$

$$= \frac{[(9.60(3.00)\hat{i} + 8.85\hat{j} - (3.00)^2\hat{k})m] - [(9.60(1.00)\hat{i} + 8.85\hat{j} - (1.00)^2\hat{k})m]}{2.00\ s}$$

$$= (9.60\hat{i} - 4.00\hat{k})\ m/s$$

Magnitude.

$$\vec{v} = \frac{d\vec{r}}{dt} = (9.60\hat{i} - 2.00t\,\hat{k})m/s$$

$$\vec{v}(2.00) = 9.60\hat{i} - (2.00)(2.00)\hat{k})m/s$$

$$= (9.60\hat{i} - 4.00\hat{k})\ m/s$$

$$v = \sqrt{(9.60)^2 + (4.00)^2}\ m/s$$

$$\boxed{v = 10.4\ m/s}$$

* Since the acceleration of this object is constt. the V_{avg} over time interval is equal to V_{ins} at mid point of interval

21.

At $t=0$

particle start from rest at $x=0$, $y=0$ and moves in xy plane with

$\vec{a} = (4.0\hat{i} + 3\hat{j})\,m/s^2$

(a) x and y components of $\vec{V} = ?$

(b) Speed of particle

(c) Position of particle

(d) Evaluate all above at $t = 2s$

(a) Acceleration vector is constant.

Also $\vec{v}_0 = 0$ and $\vec{r}_0 = 0$

(a) $\vec{v} = \vec{v}_0 + \vec{a}t = (4.0t\hat{i} + 3.0t\hat{j})\,m/s$

$$\boxed{V_x = 4.0t\ m/s}$$

$$\boxed{V_y = 3.0t\ m/s}$$

(b) $V = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{(4)^2 + (3)^2}$

$$\boxed{V = 5.0t\ m/s}$$

(c) $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

$$\boxed{\vec{r} = (2.0t^2\hat{i} + 1.5t^2\hat{j})\,m}$$

(d) $V_x(2.0) = 8.0\,m/s$

$V_y(2.0) = 6.0\,m/s$

$V(2.0) = 10.0\,m/s$

$\vec{r}(2.0) = (8.0\hat{j} + 6.0\hat{j})\,m$

24.

particle starts from origin at time t=0 with $v_i = 5\,m/s$ alon x-axis $\vec{a} = (-3.0\hat{i} + 4.5\hat{j})\,m/s^2$.
Determine velocity and position of particle.

acceleration vector is constant. The particle reaches it's maximum x coordinate when the x velocity is 0.

$\vec{v}_0 = 5.0\,m/s\,\hat{i}$ and $\vec{r}_0 = 0$

$\vec{v} = \vec{v}_0 + \vec{a}t = 5.0\hat{i}\,m/s + (-3.0t\hat{i} + 4.5t\hat{j})\,m/s$

$v_x = (5.0 - 3.0t)\,m/s$

$v_x = 0 = (5.0 - 3.0t_{x-max})\,m/s$

$t_{x-max} = \frac{5.0\,m/s}{3.0\,m/s^2} = 1.67\,s$

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$$\vec{v}\,(t_{x\text{-}max}) = 5.0\hat{i}\ m/s\left[-3.0(1.67)\hat{i} + 4.5(1.67)\hat{j}\right]$$
$$= \boxed{7.5\ m/s\ \hat{j}}$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$
$$= 5.0t\,\hat{i}\,m + \frac{1}{2}(-3.0t^2\hat{i} + 4.5t^2\hat{j})m$$

$$\vec{r}\,(t_{x\,max}) = 5.0(1.67)\hat{i}\,m/s + \frac{1}{2}\left[-3(1.67)^2\hat{i} + 4.5(1.67)^2\hat{j}\right]m$$

$$\boxed{\vec{r} = 4.2\hat{i}\,m + 6.3\hat{j}\,m}$$

25. $\vec{r} = (3.0t^2\hat{i} - 6.0t^3\hat{j})\,m$

(a) Determine $\vec{v}$ & $\vec{a}$

(b) Determine $\vec{r}$ and $\vec{v}$ at $t = 2.5s$

Diff. position vector
$$\vec{r} = (3.0t^2\hat{i} - 6.0t^3\hat{j})m.$$
with respect to time.
$$\vec{v} = \frac{d\vec{r}}{dt} = (6.0t\hat{i} - 18.0t^2\hat{j})m/s$$
$$\vec{a} = \frac{d\vec{v}}{dt} = \boxed{(6.0\hat{i} - 36t\hat{j})m/s^2}$$

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(b) $\vec{r}(2.5s)$

$= [3.0(2.5)^2\hat{i} - 6.0(2.5)^2\hat{j}]m$

$= \boxed{(19\hat{i} - 37.4\hat{j})m}$

$\vec{V}(2.5S)$

$= [6.0(2.5)\hat{i} - 18.0(2.5)^2\hat{j}]m/s$

$= \boxed{(15\hat{i} - 112.5\hat{j})m/s}$

26.

p.v from Eq. 3.13(b)

since $\vec{a}$ is constant.

$\vec{V_0} = 0$

both component of velocity $= 0$

$\vec{V} = \vec{V_0} + \vec{a}t = (-14\hat{i} - 7.0\hat{j})m/s + (6.0t\hat{i} + 3.0t\hat{j})m/s.$

$= [(-14 + 6.0t)\hat{i} + (-7 + 3.0t^2)\hat{j}]m/s$

$\vec{V}_{rest} = (0.0\hat{i} + 0.0\hat{j})m/s =$

$= [(-14 + 6.0t)\hat{i} + (-7.0 + 3.0t)\hat{j}]m/s$

$(V_x)_{res} = 0.0 = -14 + 6.0t$

$t = \frac{14}{6.0}s = \frac{7}{3}s$

$(v_y)_{rest} = 0.0 = -7 + 3.0t$

$t = \frac{7}{3} s$

bothe components are 0 at $T = \frac{7}{3}$
object is at rest that time

$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = (0.0\hat{i} + 0.0\hat{j})m +$
$(-14t\hat{i} - 7.0t\hat{j})m +$
$\frac{1}{2}(6.0t^2\hat{i} + 3.0t^2\hat{j})m$

$= \left(-14\left(\frac{7}{3}\right)\hat{i} - 7.0\left(\frac{7}{3}\right)\hat{j}\right)m + \frac{1}{2}\left(6\left(\frac{7}{3}\right)^2\hat{i} + 3\left(\frac{7}{3}\right)^2\hat{j}\right)m$

$= (-16.3\hat{i} - 8.2\hat{j})m$

$\simeq \boxed{(-16.3\hat{i} - 8.2\hat{j})m}$

38.

baseball hitted Speed = 27m/s
angle = 45°

Origion Choosed be that point where the baseball was hit.

Choose upward to be the +ve y direction

Then $y_0 = 1.0\,m$

$y = 13\,m$ at the end of motion.

$V_{y_0} = (27 \sin 45^\circ)\,m/s$

$= 19.09\,m/s$

$a_y = -9.80\,m/s^2$

$y = y_0 + V_{y_0} t + \frac{1}{2} a_y t^2$

$\frac{1}{2} a_y t^2 + V_{y_0} t + (y_0 - y) = 0$

$$t = \frac{-V_{y_0} \pm \sqrt{V_{y_0}^2 - 4\left(\frac{1}{2}a_y\right)(y_0 - y)}}{2 \frac{1}{2} a_y}$$

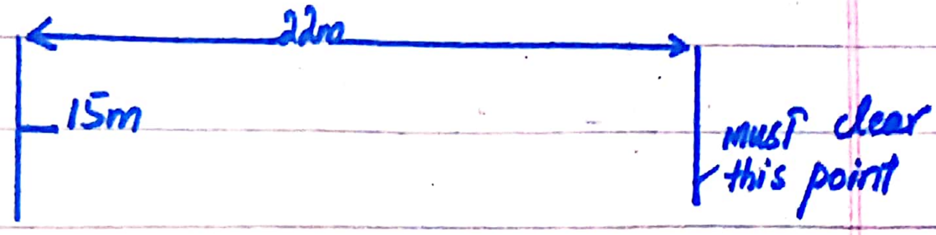
$$= \frac{-19.09 \pm \sqrt{(19.09)^2 - 2(-9.80)(-12)}}{-9.80}$$

$= 0.788\,s,\ 3.108\,s$

$$\Delta x = V_x t \left[(27 \cos 45^{\circ}) m/s\right](3.108 s)$$

$$= \boxed{59.3 \text{ m}}$$

50.



Choose origin to be location where the car leaves the ramp. & choose upward to +ve y. At the end of it's flight over the 8 cars the car must be at

$$y = -1.5 \text{ m}$$

$V_{y_0} = 0$, $a_y = -g$, $V_x = V_0$

$$\Delta x = 22m$$

$$\Delta_x = V_x t \rightarrow t = \Delta x / V_0$$

$$y = y_0 + V_{y_0} t + \frac{1}{2} a_y t^2$$
$$y = 0 + 0 + \frac{1}{2}(-g)(\Delta x / V_0)^2$$

$$V_0 = \sqrt{\frac{-g(\Delta x)^2}{2(y)}}$$

$$= \sqrt{\frac{-(9.80\,m/s^2)(22m)^2}{2(-1.5m)}}$$

$$= 39.76\ m/s$$

$$\approx \boxed{40\ m/s}$$

(b) $y = -1.5m$

$V_{y_0} = V_0 \sin\theta_0$, $a_y = -g$

$V_x = V_0 \cos\theta_0$

$\Delta x = 22m$

The launch angle $\theta_0 = 7.0°$

At constt. Velocity.

$\Delta x = V_x t$

$$t = \frac{\Delta x}{V_0 \cos\theta_0}$$

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The expression for time for vertical motion

$$y = y_0 + V_{y_0} t + \frac{1}{2} a_y t^2$$

$$y = V_0 \sin\theta_0 \frac{\Delta x}{V_0 \cos\theta_0} + \frac{1}{2}(-g)\left(\frac{\Delta x}{V_0 \cos\theta_0}\right)^2$$

$$V_0 = \sqrt{\frac{g(\Delta x)^2}{2(\Delta x \tan\theta_0 - y)(\cos^2\theta_0}}$$

$$= \sqrt{\frac{(9.80 m/s^2)(22 m)^2}{2(22 m)\tan 7.0 + 1.5 m)\cos^2 7.0}}$$

$$= \boxed{24 m/s}$$

60. Two planes approach each other.

Each has Speed = 780 km/h

Spot each other when initially 12 km apart

Time = ?

If each plane has a speed of 780 km/h then their relative speed of approach is 1560 km/hr. If the planes are 12 km apart.
Then time for evasive action is found as follows.

$$\Delta d = vt \rightarrow t = \frac{\Delta d}{v}$$

$$t = \left(\frac{12.0\ km}{1560\ km/hr}\right)\left(\frac{3600\ sec}{1\ hr}\right)$$

$$\boxed{t = 27.7 s}$$