

MAY						
M	T	W	T	F	S	S
31					1	2
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جمادى الاول ١٣٣١

MAY 2010

Context Free Grammars

- Every CFL has CFG
- A Context free grammar is simple recursive method of specifying grammar rules by which strings in a language can be generated.
- To demonstrate that a CFG generates a language we must show two things
 - ① - Every string in the language can be derived from grammar.
 - ② - No other string can be generated from grammar.

Gen. Example of CFG

WEDNESDAY 12
٢٤ جمادى الاول

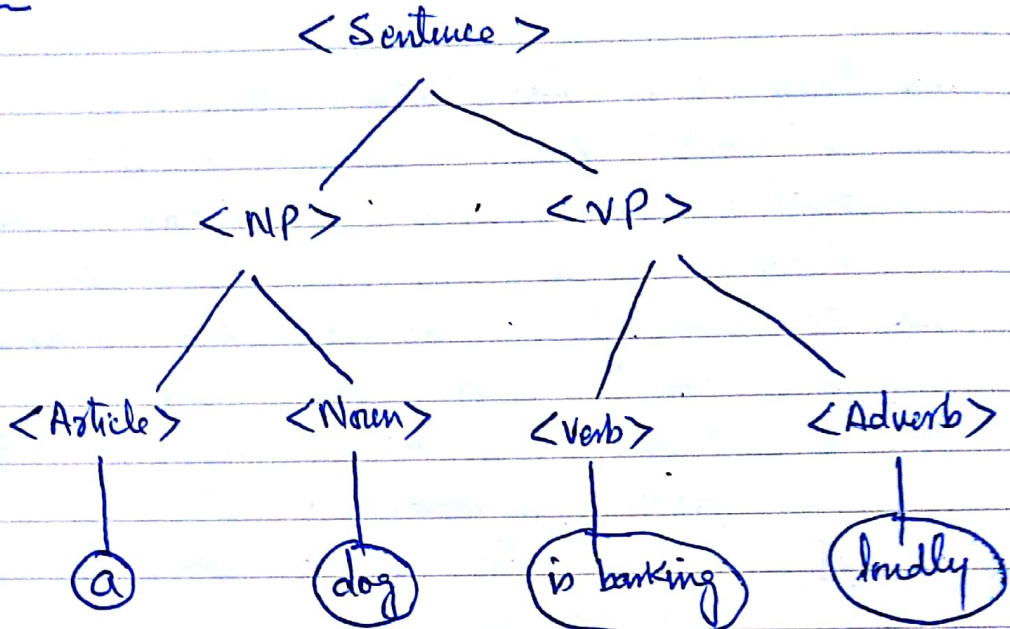
THURSDAY 13
٢٨ جمادى الاول

Set of rules / Productions	{	< Sentence > → < NP > < VP >
		< NP > → < Article > < Noun >
		< VP > → < Verb > < Adverb >
		< Article > → a
		< Article > → Am
		< Noun > → dog
		< Noun > → man
		< Verb > → is talking
		< Verb > → is barking
		< Adverb > → politely
	< Adverb > → loudly	

Now sentence is "A dog is barking loudly"

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Parse Tree



∴ Terminals are encircled

14 FRIDAY
جمادى الاول ٢٩

15 SATURDAY 16 SUNDAY
جمادى الاول ٣٠ جمادى الثاني ١

Defn of CFG

A Context free grammar is a 4-tuple $G = \{V, \Sigma, S, P\}$ where V and Σ are disjoint finite sets. S is an element of V , and P is a finite set rules/productions of the form $A \rightarrow \alpha$, where $A \in V$ and $\alpha \in (V \cup \Sigma)^*$

Defn of CFL

Let $G = \{V, \Sigma, S, P\}$ be a CFG. The language generated by G is $L(G) = \{x \in \Sigma^* \mid S \xrightarrow{*}_G x\}$

A language L is CFL if there is a CFG ' G '

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جمادى الثاني ١٣٣١

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So that $L = L(G)$

Example (The language of Algebraic Expressions)

$$S \rightarrow S+S \mid S-S \mid S*S \mid S/S \mid (S) \mid a$$

the string $a + (a*a) / a - a$ can be obtained from the derivation as

$$\begin{aligned} S &\Rightarrow S-S \Rightarrow S+S-S \Rightarrow a+S-S \Rightarrow a+S/S-S \\ &\Rightarrow a+(S)/S-S \Rightarrow a+(S*S)/S-S \\ &\Rightarrow a+(a*S)/S-S \Rightarrow a+(a*a)/S-S \\ &\Rightarrow a+(a*a)/a-S \Rightarrow a+(a*a)/a-a \end{aligned}$$

MONDAY 17
جمادى الثاني ٢

TUESDAY 18
جمادى الثاني ٣

Example

$$R.E = a^*$$

$$S \rightarrow aS \mid \Lambda$$

Example $R.E = a^* b^*$

$$S \rightarrow AB$$

$$A \rightarrow aA \mid \Lambda$$

$$B \rightarrow bB \mid \Lambda$$

Example

$$(a+b)^*$$

$$S \rightarrow aS \mid bS \mid \Lambda$$

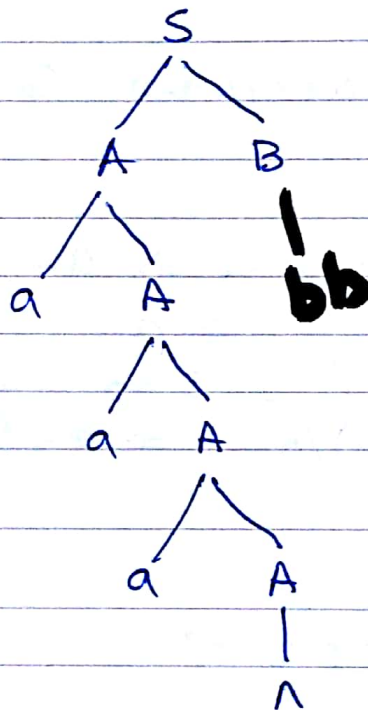
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Example

a^*bb

$$\begin{aligned}
 S &\rightarrow AB \\
 A &\rightarrow aA \mid \Lambda \\
 B &\rightarrow bb \mid \Lambda
 \end{aligned}$$

for $aaabb$



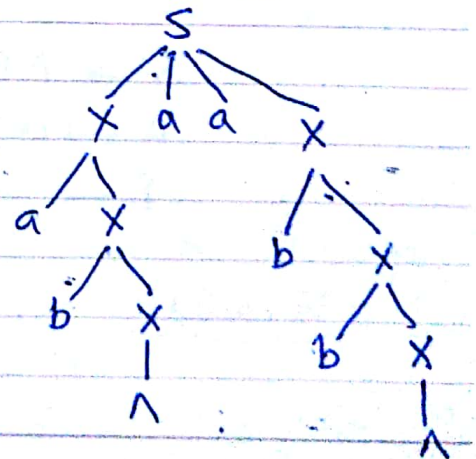
19 WEDNESDAY
٣ جمادى الثاني

20 THURSDAY
٤ جمادى الثاني

Example (string aa must be included)

for abaabb

$$\begin{aligned}
 S &\rightarrow XaaX \\
 X &\rightarrow aX \\
 X &\rightarrow bX \\
 X &\rightarrow \Lambda
 \end{aligned}$$



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جمادى الثاني ١٤٣١

MAY 2010

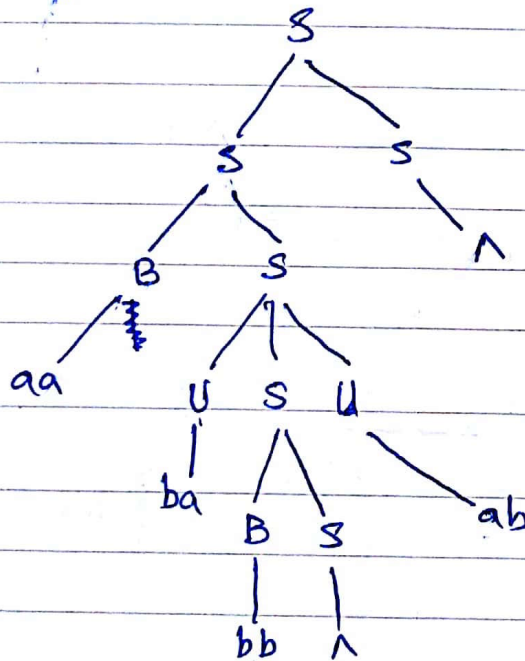
✓ Example (even no. of a's and even no. of b's)

$$S \rightarrow SS \mid BS \mid SB \mid \Lambda \mid USU$$

$$B \rightarrow aa \mid bb$$

$$U \rightarrow ab \mid ba$$

for string $aababbab$



Evening

FRIDAY 21
جمادى الثاني ٦

SUNDAY 23
جمادى الثاني ٨

SATURDAY 22
جمادى الثاني ٤

(H.W) Example

$$S \rightarrow (s) \mid s \supset s \mid \sim s \mid p \mid q$$

Derive 13 letter word $(\sim \sim p \supset (p \supset \sim \sim q))$

Example

$$L = \{x \in \{0,1\}^* \mid n_0(x) = n_1(x)\}$$

$$S \rightarrow SS \mid 0S1 \mid 1S0 \mid \Lambda$$

for string 10001101

Evening

Some steps are skipped here

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08 $S \Rightarrow SS \Rightarrow SSS \Rightarrow 1500S10S1$
 09 $\Rightarrow 1 \wedge 000S110S1$
 10 $\Rightarrow 1000 \wedge 110S1 \Rightarrow 10001101$
 11

12 Theorem

01 if L_1 and L_2 are CFL's, then the
 02 languages $L_1 \cup L_2$, $L_1 L_2$ and L_1^* are also CFLs.
 03

04
 05 Example (A CFG equivalent to R.E)
 Evening

$$(011+1)^*(01)^*$$

24 MONDAY
 9 جمادى الثاني

$$A \rightarrow \underline{011} | 1 \quad \left. \vphantom{A} \right\} \text{ generates } \{011, 1\}$$

25 TUESDAY
 10 جمادى الثاني

$$B \rightarrow AB | \Lambda \quad \left. \vphantom{B} \right\} \text{ generates } (011, 1)^*$$

$$A \rightarrow \underline{011} | 1$$

$$C \rightarrow DC | \Lambda \quad \left. \vphantom{C} \right\} \text{ generates } (01)^*$$

$$D \rightarrow 01$$

12 finally

01 $S \rightarrow BC$

02 $B \rightarrow AB | \Lambda$

03 $A \rightarrow 011 | 1$

04 $C \rightarrow DC | \Lambda$

05 $D \rightarrow 01$
 Evening

Similarly CFG's can be obtained simply

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جمادى الثاني ١٤٣١

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Through some FA by applying simple logic which can be seen in book. So every R.L is CFL.

Derivation & Ambiguity

Derivation

LMD

RMD

{ left most NT expanded first }

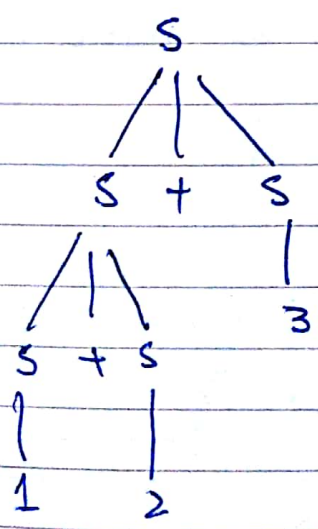
{ Right most NT expanded first }

$S \rightarrow S+S \mid S*S$

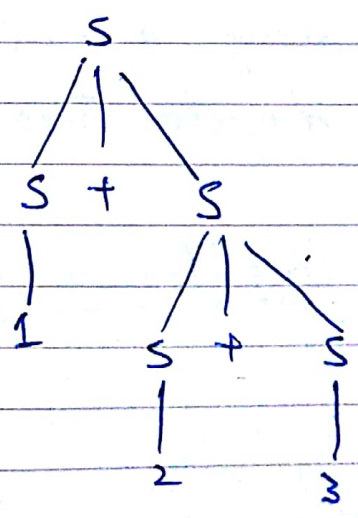
$S \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$

WEDNESDAY 26
جمادى الثاني ١٢

THURSDAY 27
جمادى الثاني ١٣



LMDT



RMDT

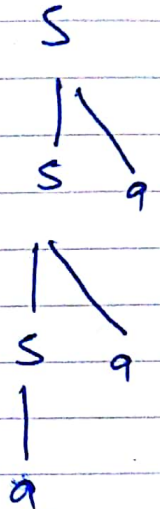
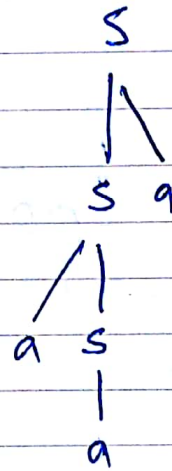
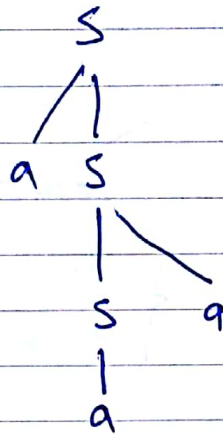
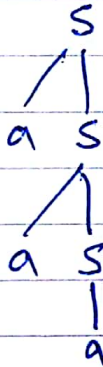
Evening

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Example (Ambiguity)

$$S \rightarrow aS | Sa | a$$

for 'aaa'



28 FRIDAY ١٣ جمادى الثاني

29 SATURDAY ١٤ جمادى الثاني 30 SUNDAY ١٥ جمادى الثاني

"A CFG is ambiguous if there is at least one string in $L(G)$ having two or more distinct derivation trees"

Unambiguous CFG

$$S \rightarrow S+T | T$$

$$T \rightarrow T * F | F$$

$$F \rightarrow (S) | a$$

