			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

Equivalence of states :-

Given $P, Q \in Q$, then
→ if $P = f$ & $q \neq f$ ⇒ P & q are distinguishable and we can not merge them.
→ if (P, q) are non-distinguishable, then they can be merged, means both should be final states or non-final states

Example 1 (DFA Minimization)

$\text{DFA} \longrightarrow \boxed{\text{Algo of ES}} \longrightarrow$ Minimized DFA
or
optimal minimized DFA

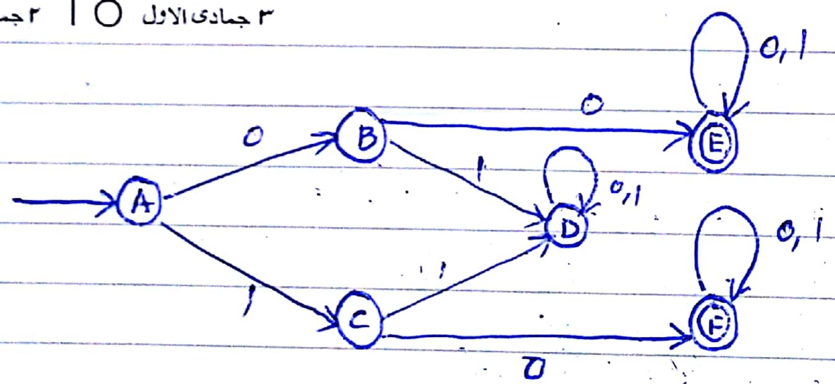
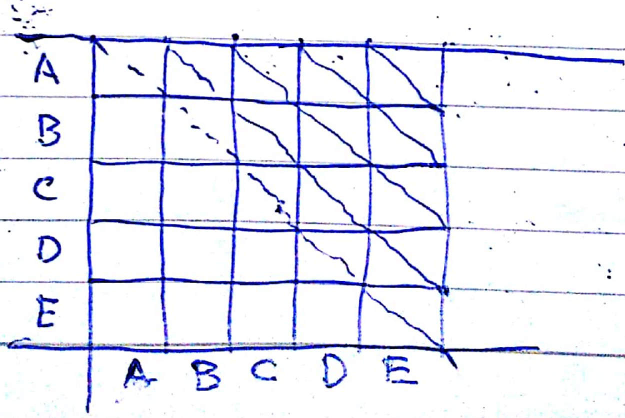


fig A

Then we build a N×N step Table



Since
(B,D) ≅ (D,B)
(A,A) ≅ A
So, we don't need ~~not~~ the crossed portion of the table

Hence the rest of the step table becomes

B				
C				
D				
E				
	A	B	C	D

By applying the Concept, the step table of fig A becomes

B	X^1				
C	X^1				
D	X^2	X^1	X^1		
E	X^0	X^0	X^0	X^0	
F	X^0	X^0	X^0	X^0	
	A	B	C	D	E

$\boxed{\text{Step 0}}$: we marked the cells with X^0 which can not be merged as per rules of equivalence of states. Since the cells are marked in step 0, so we need another iteration to mark cells further.

$\boxed{\text{step 1}}$:

$$\delta[(A,D),0] = (B,D) = ?$$
$$\delta[(A,D),1] = (C,D) = ?$$
$$\delta[(A,C),0] = (B,F) = X_1$$
$$\delta[(A,C),1] = (C,D) = ?$$
$$\delta[(A,B),0] = (B,E) = X^1$$
$$\delta[(A,B),1] = (C,D) = ?$$

$\delta[(B,D),0] = (E,D) = X^1$

$\delta[(B,D),1] = (D,D) = ?$

$\delta[(B,C),0] = (E,F) = ?$

$\delta[(B,C),1] = (D,D) = ?$

$\delta[(C,D),0] = (F,D) = X^1$

$\delta[(C,D),1] = (D,D) = ?$

$\delta[(E,F),0] = (E,F) = ?$

$\delta[(E,F),1] = (E,F) = ?$

Since cells are again marked in step 1, so we need another iteration

$\boxed{\text{Step 2}}$:

$\delta[(A,D),0] = (B,D) = X^2$

$\delta[(A,D),1] = (C,D) = X^2$

$\delta[(B,C),0] = (E,F) = ?$

$\delta[(B,C),1] = (D,D) = ?$

$\delta[(E,F),0] = (E,F) = ?$

$\delta[(E,F),1] = (E,F) = ?$

Since cells are marked again, so we need another iteration

$\boxed{\text{Step 3}}$:

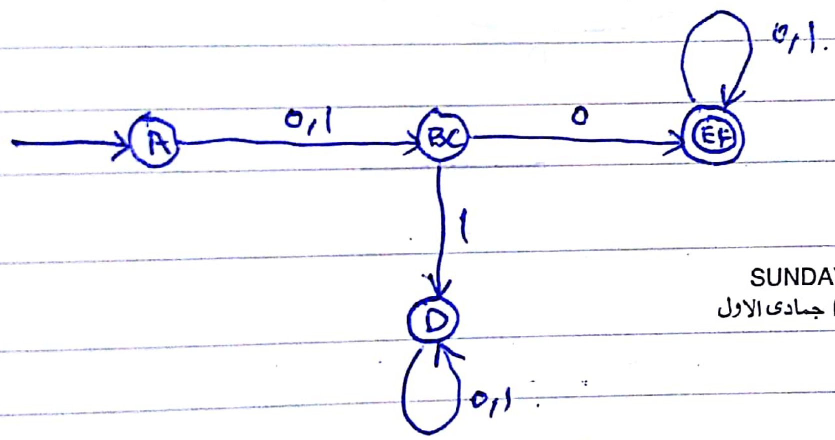
$\delta[(B,C),0] = (E,F) = ?$

$\delta[(B,C),1] = (D,D) = ?$

$\delta[(E,F),0] = (E,F) = ?$

$\delta[(E,F),1] = (E,F) = ?$

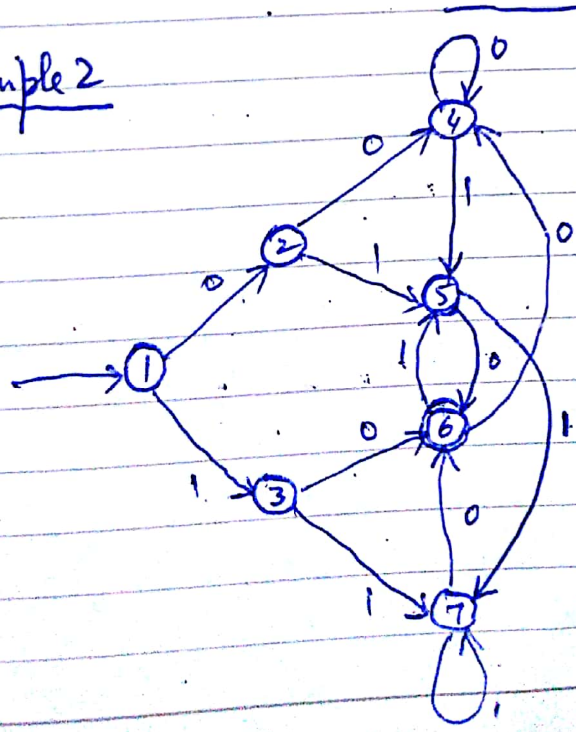
iteration stops here, since no any cell is marked in step 3 and (B,C) and (E,F) are remained as unmarked cells, so we can merge them and final minimized DFA is as below.



Example 2

from Book

Solve it in class



جمادی الاول ١٤٣١

APRIL 2010

APRIL

M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

H.W

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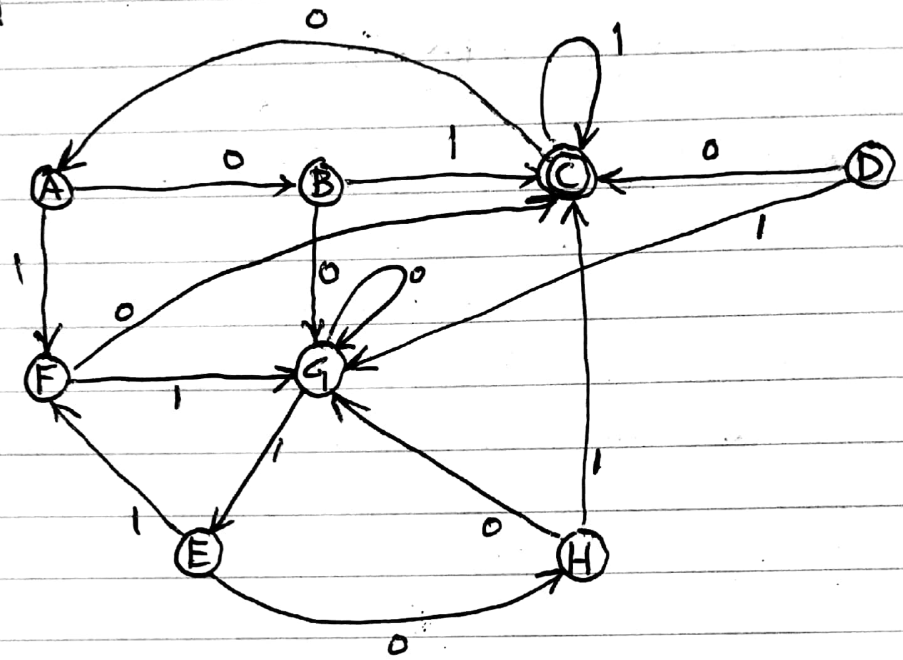

Evening

26 MONDAY
١١ جمادی الاول

27 TUESDAY
١٢ جمادی الاول

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Evening

Statement of Pumping Lemma :-

If language L is regular, then $\forall z \in L$ where $|z| \geq n$ (n is number of states in FSM of L). $\exists$ strings u, v, w such that $z = uvw$ where

$$|uv| \leq n$$
$$|v| > 0$$
$$\text{for any } i \geq 0,\ uv^i w \in L$$

Pumping Lemma extraction :-

Says

→ if an infinite language is regular, it is defined by DFA.

→ The DFA (then must) have some finite number of states say n.

→ Since language is infinite, some strings of language must have length $\geq n$

→ for a string of length $\geq n$ accepted by the DFA, we would have to walk through DFA that must have a cycle.

→ Repeating the cycle an arbitrary number of times must yield another string accepted by the DFA $\Rightarrow$ Pumping property.

Negation of Pumping Property (Contrapositive/Contradiction) :-

$\exists z \in L$; $|z| \geq n$

$\exists$ u, v, w where $z = uvw$; Pumping property
$|uv| \leq n$ and $|v| > 0$
$\exists i \geq 0$; such that $\boxed{uv^2w \notin L}$

then L is not regular

So we Conclude
Regular languages $\longrightarrow$ Pumping property
$\neg$ Pumping property $\Longrightarrow$ $\neg$ Regular languages

General Results :-

1- Pumping lemma used to prove That given infinite language is not regular.

2- Pumping property is for infinite languages

3 - Pumping lemma can not be used to prove that a given languages is regular

Example

P = pallindrome (non regular language)

1. Let P be a regular Language
2. P will have FSM of n state
3. $z = 0^n 1 0^n$ and $z \in P$ & $|z| \geq n$

Then

4. $|z| = |0^n 1 0^n| \geq n$

$n + 1 + n \geq n$

$2n + 1 \geq n$ proved

now

$$z = \underbrace{u}_{0^{n-k}}\ \underbrace{v^2}_{0^{k}}\ \underbrace{w}_{10^{n}} \qquad \therefore k > 0$$

Then

5 – $uv^2w = 0^{n-k}\, 0^{2k}\, 10^n$
$\quad = 0^{n+k(i-1)}\, 10^n$

Then $n + k(i-1) \neq n$ for $i \geq 2$

Hence P is a non regular language

Example

$$L = \{a^n b^n\}$$

1. Let L be a R.L
2. having FSM of m states
3. $z = a^m b^m$

then since $z \in L$, so

4. $|z| \geq m$

$|a^m b^m| \geq m \Rightarrow m + m \geq m \Rightarrow 2m \geq m$

proved

Now for $z = uvw$

$$u = a^{m-k} \quad v = a^{k} \quad w = b^{m}$$

5- $uv^2w \in L$, so

$$a^{m-k}\, a^{ik}\, b^{m} \Rightarrow a^{m+k(i-1)}\, b^{m}$$

which gives for $i \geq 2$, $m + k(i-1) \neq m$

So L is not regular.

Example $L = \{ x\,x \mid x \in \{0,1\}^* \}$

(1)- let L be R.R

(2)- FSM and m states

(3)- $z = (10)^m (10)^m$

(4)- $z \in L$, then $|z| \geq m$

$$|(10)^m (10)^m| \geq m$$

$$2m + 2m \geq m$$

$$4m \geq m$$

(5)- $z = uvw$

$$u = 10^{m-k} \quad v = 10^{k} \quad w = 10^{m}$$

(6)- $uv^2w \in L$, then

$$10^{m-k}\, 10^{2k}\, 10^m \Rightarrow 10^{m+k(2-1)}\, 10^m$$

for $i \geq 2$ $m + k(i-1) \neq m$

Hence L is not regular.

<u>Example</u> $L = 1^{n^2}$ where $n \geq 0$

MAY

M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Example (Repeat)

$$z = 10^m 10^m$$

$$z = uvw$$

$$10^a \quad 0^b \quad 0^{m-a-b} 10^m$$

$uv^2w \in L$

$$10^a 0^{ib} 0^{m-a-b} 10^m$$

$\Rightarrow 10^{a+ib+m-a-b} 10^m \Rightarrow 10^{m+b(i-1)} 10^m$

so for $i \geq 2$, $m + b(i-1) \neq m$

10 MONDAY ٢٥ جمادی الاول

Hence non Regular language.

11 TUESDAY ٢٦ جمادی الاول

08

$\boxed{H.W}$ $L = \{ a^n$ where n is prime no. $\}$

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Evening

Example $L = 1^{n^2}$ where $n \geqslant 0$

1- Let L is R.L

2- FSM $\exists$ N States

3- $z = 1^{N^2} \Rightarrow \{1, 1^4, 1^9, 1^{16}, ---\}$

$1^{N^2} \geqslant$ N proved

5- ~~[illegible]~~

$z_{-1} = 1^{(N-1)^2}$ $\quad z = 1^{N^2}$ $\quad z_{+1} = 1^{(N+1)^2}$

Now $z = |uvw| = n^2 < |uv^2w| = |uvvw|$

$< n^2 + |v| \quad \therefore 0 < |v| \leq N$

$< n^2 + n$

$< n^2 + n + n + 1 \quad \therefore$ square z_{+1}

$= n^2 + 2n + 1 \quad \therefore$ square z_{+1}

$= (n+1)^2$

$= z_{+1}$

~~Hence~~ As uv^2w does not belong to L, So L is not Regular