

Q is the set of states, Σ is the alphabet, q_0 is the initial state and A is the set of accepting states.

Non-Recursive Defn of δ^* for an NFA

If $x = a_1 a_2 \dots a_n$, then saying M can be in state q after processing x means that there are states p_0, p_1, \dots, p_n so that $p_0 = q_0$, $p_n = q$ and M moves as

p_0 (or q_0) to p_1 by processing a_1
 p_1 to p_2 by processing a_2

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p_{n-1} to p_n (or q) by processing a_n

A simple way to say M can get from p_{i-1} to p_i by processing a_i is

$p_i \in \delta(p_{i-1}, a_i)$ for each i with

$1 \leq i \leq n$

we can say that

$p_n \in \delta^*(p_0, a_1 a_2 \dots a_n)$

for $i \geq 1$

$p_i \in \delta^*(p_0, a_1 a_2 \dots a_i)$

Recursive Defn of δ^* for an NFA

~~is~~ let $i = n-1$ and $y = a_1 a_2 \dots a_{n-1}$
Then

$$p_{n-1} \in \delta^*(p, y)$$

$$\Rightarrow p_n \in \delta(p_{n-1}, a_n)$$

In other words if $q \in \delta^*(p, y a_n)$, then
There is a state 'r' ($r = p_{n-1}$) in the set
 $\delta^*(p, y)$, so then $q \in \delta(r, a_n)$

if we consider it reverse order then we can say
if $q \in \delta(r, a_n)$ for some $r \in \delta^*(p, y)$, then
we conclude that $q \in \delta^*(p, y a_n)$

So recursive formula becomes

$$\delta^*(p, y a_n) = \{ q \mid q \in \delta(r, a_n) \text{ for some } r \in \delta^*(p, y) \}$$

or

$$\delta^*(p, y a_n) = \bigcup_{r \in \delta^*(p, y)} \delta(r, a_n)$$

So finally we can say that

1. for any $q \in Q$, $\delta^*(q, \Lambda) = \{ q \}$
2. for any $q \in Q$, $y \in \Sigma^*$, and $a \in \Sigma$

$$\delta^*(q, y a) = \bigcup_{r \in \delta^*(q, y)} \delta(r, a)$$

Definition: Acceptance by NFA

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an NFA, The string $x \in \Sigma^*$ is accepted by M if $\delta^*(q_0, x) \cap A \neq \emptyset$
 the language recognized or accepted by M is the set $L(M)$ of all strings accepted by M
 for any language $L \subseteq \Sigma^*$, L is recognized by M if $L = L(M)$.

Example

Let $M = (Q, \Sigma, q_0, A, \delta)$ where $Q = \{q_0, q_1, q_2, q_3\}$

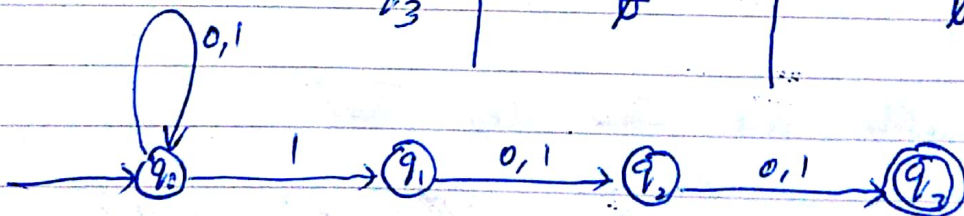
24 WEDNESDAY
 الأربعاء ٩

$\Sigma = \{0, 1\}$; $A = \{q_3\}$ and δ is given

25 THURSDAY
 الخميس ١٠

by the following table

q	$\delta(q, 0)$	$\delta(q, 1)$
q_0	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\{q_2\}$	$\{q_2\}$
q_2	$\{q_3\}$	$\{q_3\}$
q_3	\emptyset	\emptyset



we have to determine the language recognized by the machine, we can conclude from the table that

$$\delta^*(q_0, 0) = \{q_0\} \text{ and } \delta^*(q_0, 1) = \{q_0, q_1\}$$

Now from Recursive transition concept δ^* .

$$\delta^*(q_0, 11) = \bigcup_{x \in \delta^*(q_0, 1)} \delta(x, 1)$$

$$= \bigcup_{x \in \{q_0, q_1\}} \delta(x, 1)$$

$$= \delta(q_0, 1) \cup \delta(q_1, 1)$$

$$= \{q_0, q_1\} \cup \{q_2\}$$

$$= q_0, q_1, q_2$$

Now

$$\delta^*(q_0, 01) = \bigcup_{x \in \delta^*(q_0, 0)} \delta(x, 1)$$

$$= \bigcup_{x \in \{q_0\}} \delta(x, 1)$$

$$= \delta(q_0, 1)$$

$$= \{q_0, q_1\}$$

and

$$\delta^*(q_0, 111) = \bigcup_{x \in \delta^*(q_0, 11)} \delta(x, 1)$$

$$= \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)$$

$$= \{q_0, q_1, q_2, q_3\}$$

FRIDAY 26
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SATURDAY 27
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Evening

Now

$$\delta^*(q_0, 011) = \bigcup_{x \in \delta^*(q_0, 01)} \delta(x, 1)$$

$$= \delta(q_0, 1) \cup \delta(q_1, 1)$$

$$= \{q_0, q_1, q_2\}$$

we have seen that 111 is accepted but 011 is not. So the language of M is

$$\{0, 1\}^* \{1\} \{0, 1\}^2$$

1 MONDAY
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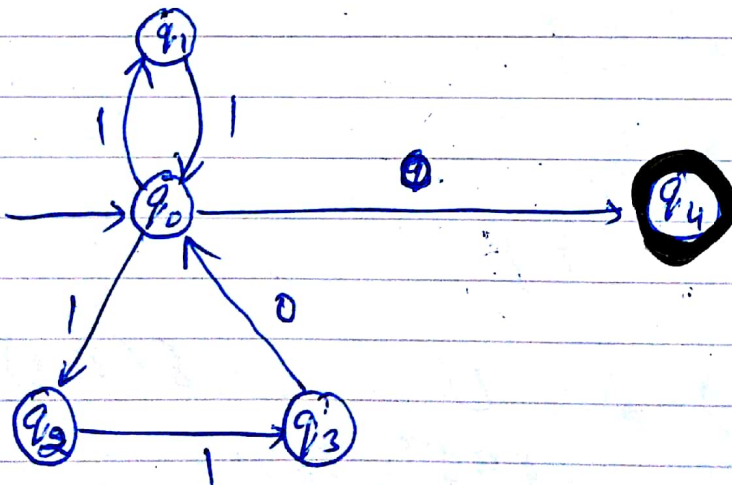
Conversion of a NFA to FA Using Subset

2 TUESDAY
14 ربيع الأول

Construction Method

Example

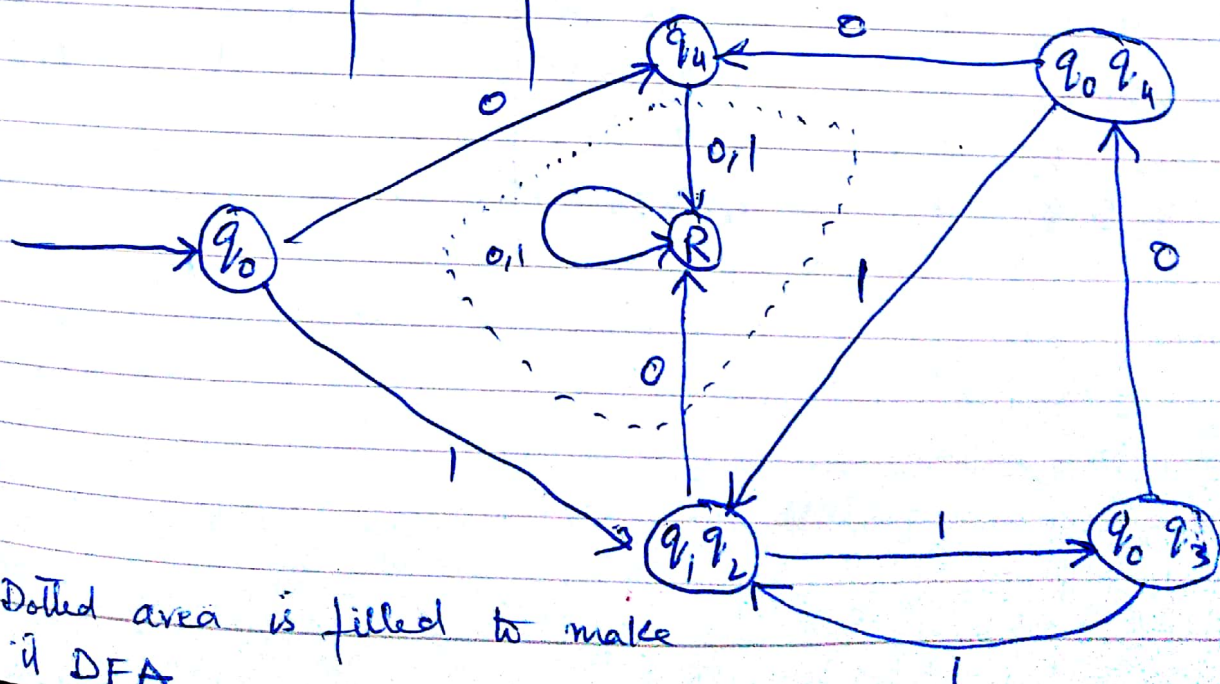
Given a NFA as



Evening

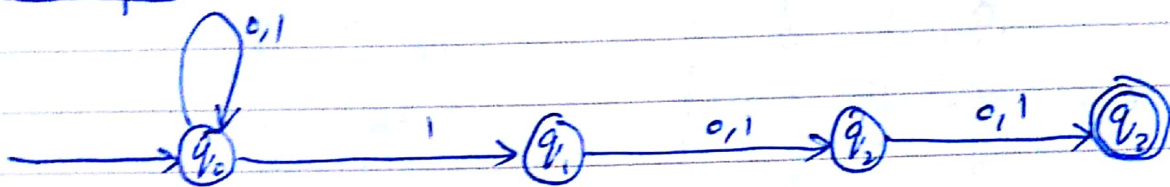
States	input	Resulting states
q_0	0	q_4 ✓
q_0	1	q_1, q_2 ✓
q_4	0	—
q_4	1	—
q_1, q_2	0	—
q_1, q_2	1	q_0, q_3 ✓
q_0, q_3	0	q_0, q_4 ✓
q_0, q_3	1	q_1, q_2 ✓
q_0, q_4	0	q_4 ✓
q_0, q_4	1	q_1, q_2 ✓

WEDNESDAY 3
 ۱۲ ربيع الاول
 THURSDAY 4
 ۱۳ ربيع الاول



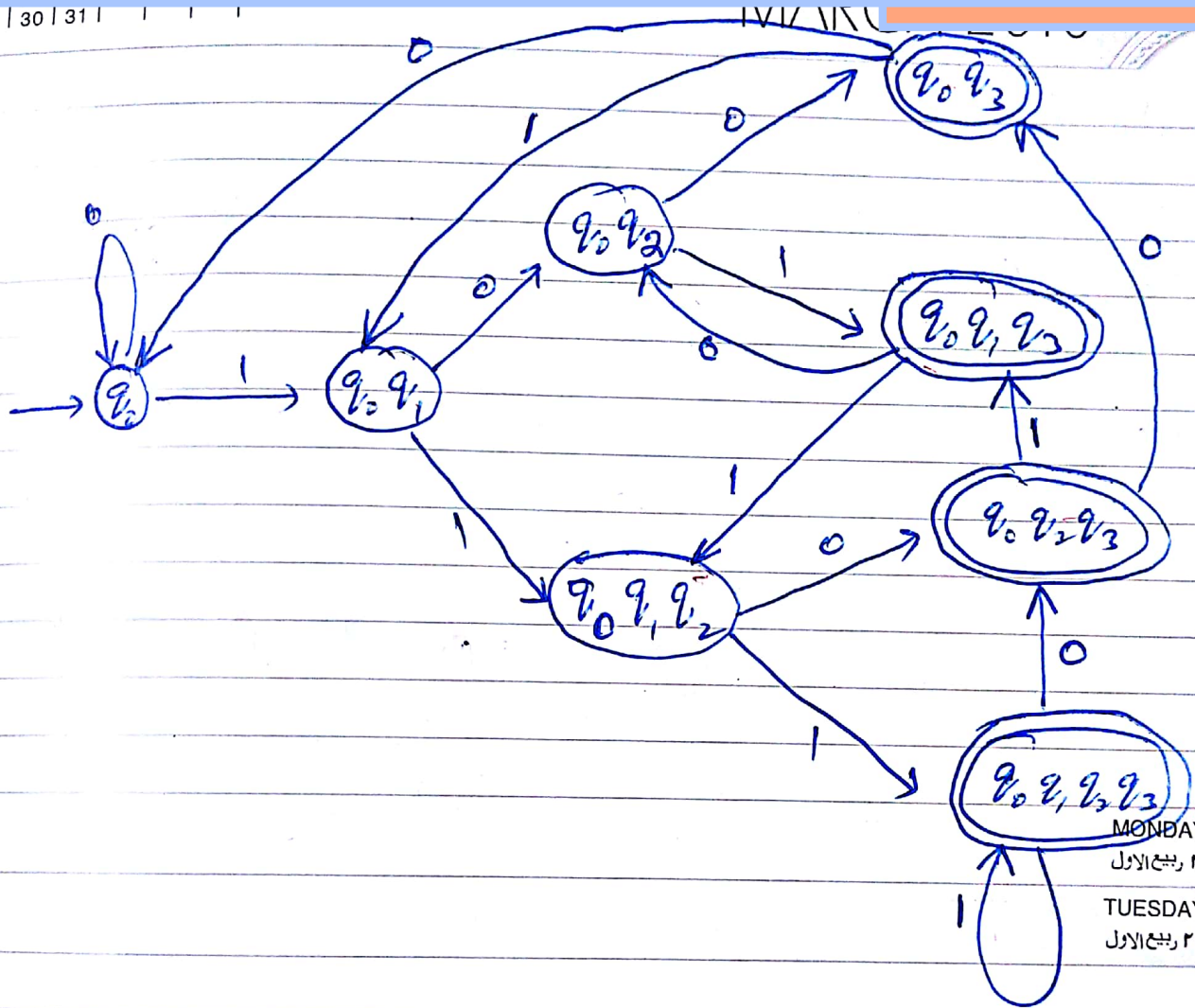
Dotted area is filled to make a DFA.

Example



it should have maximum $2^4 = 16$ states in resultant FA.

Evening	States	input	Resulting States	States	input	Resulting States
	q_0	0	q_0 ✓	$q_0 q_3$	0	q_0 ✓
	q_0	1	$q_0 q_1$ ✓	$q_0 q_3$	1	$q_0 q_1$ ✓
5 FRIDAY الجمعة ١٨				$q_0 q_1 q_3$	0	$q_0 q_2$ ✓
6 SATURDAY السبت ١٩				$q_0 q_1 q_3$	1	$q_0 q_1 q_2$ ✓
7 SUNDAY الاحد ٢٠	$q_0 q_1$	0	$q_0 q_2$ ✓	$q_0 q_1 q_2 q_3$	0	$q_0 q_3$ ✓
	$q_0 q_1$	1	$q_0 q_1 q_2$ ✓	"	1	$q_0 q_1 q_3$ ✓
	$q_0 q_2$	0	$q_0 q_3$ ✓	$q_0 q_1 q_2 q_3$	0	$q_0 q_2 q_3$ ✓
	$q_0 q_2$	1	$q_0 q_1 q_3$ ✓	"	1	$q_0 q_1 q_2 q_3$ ✓
	$q_0 q_1 q_2$	0	$q_0 q_2 q_3$ ✓			
	$q_0 q_1 q_2$	1	$q_0 q_1 q_2 q_3$ ✓			



MONDAY 8
 Tuesday 9
 ربيع الاول ٢١
 ربيع الاول ٢٢

Theorem 4.1

For any NFA $M = \{Q, \Sigma, q_0, A, \delta\}$ accepting a language $L \subseteq \Sigma^*$, there is an FA $M_1 = \{Q_1, \Sigma_1, q_{11}, A_1, \delta_1\}$ that also accepts L .

{ See from book }